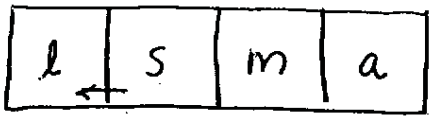
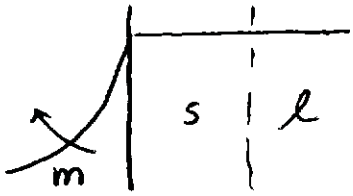
# 3.044 MATERIALS PROCESSING

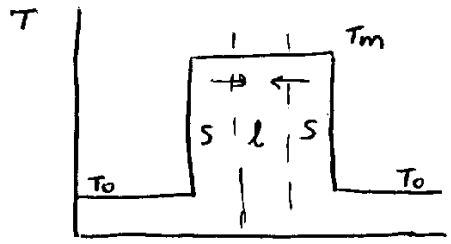
LECTURE 10




1. **Casting:** Thick mold, mold conduction limiting: $s \propto \sqrt{t} \longleftrightarrow t_f \propto \left(\frac{V}{A}\right)^2$

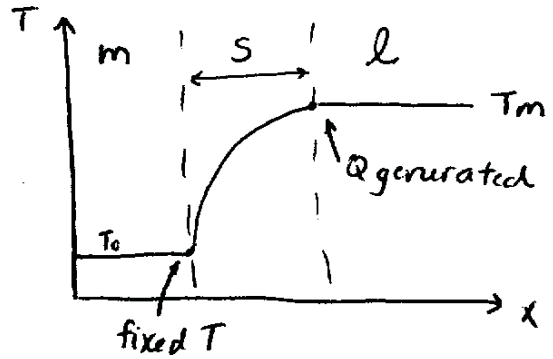



2. **Die Casting:** Thin cooled mold, interface limiting: $s \propto t$




*Date*: March 12th, 2012.

3. **Thick casting in cooled mold**




**s/l interface:** $$\frac{ds}{dt} = \frac{k}{H_f \rho}\left(\left.\frac{\partial T}{\partial x}\right|_x - \left.\frac{\partial T}{\partial x}\right|_l\right), \text{ where } \left.\frac{\partial T}{\partial x}\right|_l \Rightarrow 0$$

$$\Theta_{solid} \cong ERF\frac{x}{2\sqrt{\alpha_s t}}$$

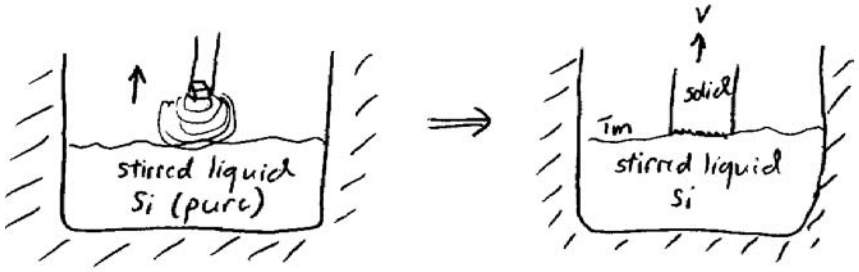
$$\Theta = \frac{T - T_0}{A - T_0}$$

$$\boxed{s(t) = 2\gamma\sqrt{\alpha_s t}}$$

$$\boxed{t_f \propto \left(\frac{V}{A}\right)^2}$$

$$\gamma e^{\gamma^2} ERF\gamma = \frac{(T_m - T_0)c_{p,s}}{H_f\sqrt{\pi}} \Rightarrow \text{Read } \gamma \text{ off chart}$$

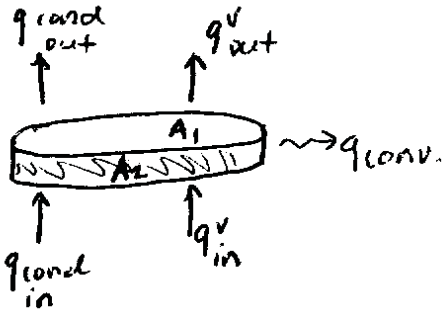
4. **Growing single crystal silicon**




no T gradients in solid or liquid
inert atmosphere: $h \sim 10 - 20\frac{W}{m^2 K}$

$$v = \text{pulling velocity} = \frac{ds}{dt} = \frac{k}{H_f \rho}\left(\left.\frac{\partial T}{\partial z}\right|_s - \left.\frac{\partial T}{\partial z}\right|_l\right), \text{ where } \left.\frac{\partial T}{\partial z}\right|_l \Rightarrow 0$$

**Assume steady state:**




$$\overbrace{\left(-k\left.\frac{\partial T}{\partial z}\right|_{\text{in}} A_1\right)}^{q_{\text{in}}^{\text{cond}}} - \overbrace{\left(-k\left.\frac{\partial T}{\partial z}\right|_{\text{out}} A_1\right)}^{q_{\text{out}}^{\text{cond}}} + q_{\text{in}}^v - q_{\text{out}}^v - \overbrace{(-h(T-T_f)A_2)}^{q_{\text{conv}}} = 0$$

**Solve for $q_{\text{in}}^v$ and $q_{\text{out}}^v$:**

$$\frac{\text{energy}_{\text{in}}}{\text{time}} = H_{\text{in}}\frac{\text{mass}_{\text{in}}}{\text{time}}$$
$$\dot{m}_w = \rho \dot{V}_{ol}$$
$$\dot{V}_{ol} = \rho A_1 v$$
$$\boxed{q_{in}^v = \rho A_1 v H_{in}}$$
$$\boxed{q_{out}^v = \rho A_1 v H_{out}}$$
$$\Delta H = \rho c_p \Delta T$$

**Solve Heat Balance:**

$$\frac{T - T_0}{T_f - T_0} = EXP\left(-\left(\frac{v}{2\alpha} - \sqrt{(v2\alpha)^2 + \frac{2h}{Rk}}\right)z\right)$$

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Spring 2013