3.044 MATERIALS PROCESSING

LECTURE 5

General Heat Conduction Solutions:

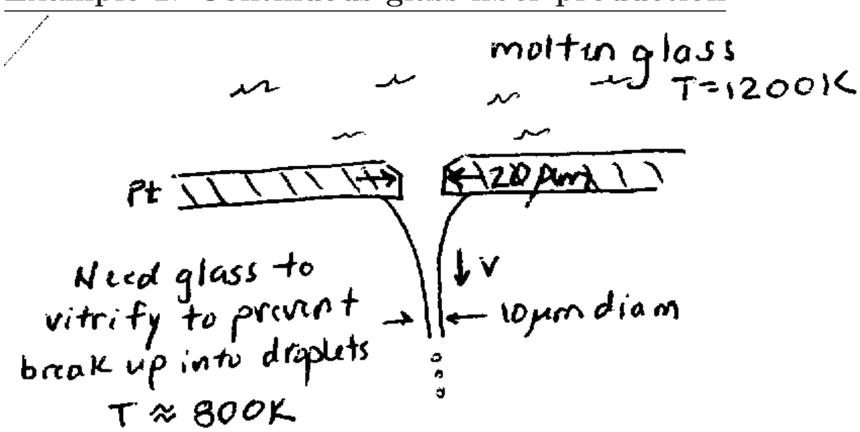
$\frac{\partial T}{\partial t} = \nabla \cdot k \nabla T, \; T(\bar{x}, t)$

Trick one: steady state $\nabla^2 T = 0, \; T(x)$

Trick two: low Biot number $\frac{\partial T}{\partial t} = \alpha \, h(T_s - T_f), \; T(t)$

Transient:
- semi-infinite
- infinite series: book, analytical
- graphical solutions
- computers, numerical, finite elements

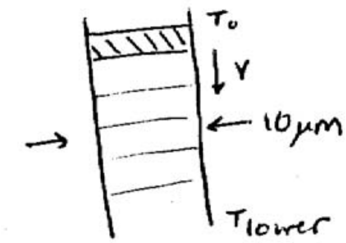
Example 1: Continuous glass fiber production

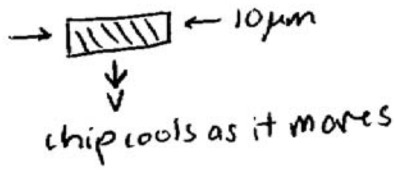


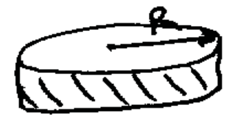
Problem Statement: need to cool from $T = 1200\text{K}$ to $T = 800\text{K}$ in 2ms

Date: February 22nd, 2012.

Geometry/Coordinates: long cylinder of 10μm diameter with a moving reference frame







Boundary Conditions:

1. $T_0 = 1200$K
2. @ $r = R$ convection into ambient air: $q = h(T - T_f)$
3. @ $r = 0$ implicit symmetry condition: $\frac{\partial T}{\partial r} = 0$

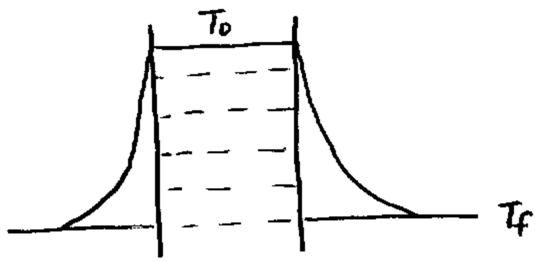
Governing Equation: $Bi = \dfrac{hL}{k}$

$$L = \frac{\text{vol}}{\text{surface area}} = \frac{\pi R^2 l}{2\pi r l} = \frac{R}{2} = 2.5\mu\text{m}$$

$k =$ thermal conductivity of glass $= 1.7\frac{\text{W}}{\text{mK}}$

h is dependent on the fluid into which you draw the fiber into: $h = 10 - 1000$

$$Bi = \frac{(10-1000)(2.5x10^{-}6)}{1.7}\ (10-1000)(10^{-}6) << 0.1 \ \textbf{Newtonian Cooling}$$



$$\frac{T - T_f}{T_0 - T_f} = \exp\left(-\frac{h}{\rho\, c_p\, L} t\right)$$

$$L = \frac{R}{2} = 2.5\mu\text{m}$$

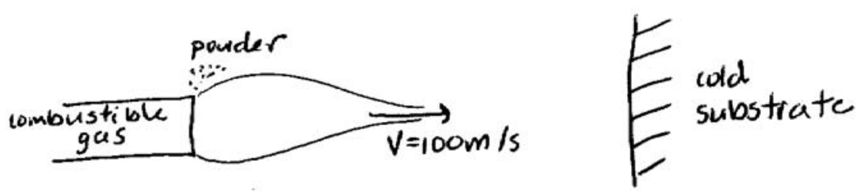
$$\rho = 2500\frac{\text{kg}}{\text{m}^3}$$

$$c_p = 0.12 \frac{\text{kJ}}{\text{kg K}}$$
$$\boxed{h \approx 10 - 1000}$$
$$T_0 = 1200\text{K}$$
$$T_f = 300\text{K}$$
$$T \approx 750\text{K}$$

Solve for h (the only changable variable): $h \approx 260 \, \frac{\text{W}}{\text{m}^2\,\text{K}}$

- oil bath w/ standoff/air gap
- $\boxed{\text{big fans}}$
- other gases

Example 2: Thermal Spray Coatings / Plasma Spray

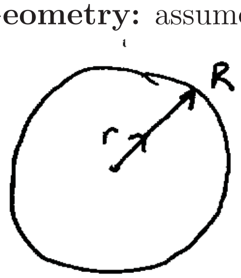


Specific Example:

oxyacetylene torch: $T = 2700°\text{C}$
powder: Ni alloy MAR-M200, $r = 2 - 50\mu\text{m}$

Problem Statement: need a particle to melt in flight ($T = T_m, @\, r = 0$)

Geometry: assume spherical particle



Boundary Conditions:

1. $@\, r = 0$ implicit symmetry condition: $\frac{\partial T}{\partial r} = 0$
2. $@\, r = R$ convection into ambient air: $q = h(T - T_f)$
   $T_f = 2700°\text{C}$, $h \approx 500 \frac{\text{W}}{\text{m}^2\,\text{K}}$

Governing Equation: $Bi = \dfrac{hL}{k} = \dfrac{h\,R}{k\,3}$

$h = 500$
$R =$ use larges particle which takes the longest to melt $= 50\mu$m
$k = 16\frac{\text{W}}{\text{m K}}$

$Bi \approx 10^-4 < 0.1$ **Newtonian Equation**

$$\frac{T - T_f}{T_0 - T_f} = \exp\left(-\frac{h}{\rho\, c_p\, L}t\right)$$
$$T = T_{m,\,\text{Ni}} = 1700\text{K}$$
$$T_f = 3000\text{K}$$
$$T_0 = 300\text{K}$$
$$h = 500$$
$$\rho = 8500\,\frac{\text{kg}}{\text{m}^3}$$
$$c_p = 0.5\,\frac{\text{kJ}}{\text{kg K}}$$
$$\frac{R}{3} = 50\mu\text{m}$$

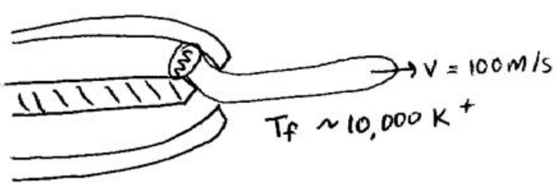
Solve for $\mathbf{t_c}$: $t_c = 0.18s$

recall: v = 100 $\frac{\text{m}}{\text{s}}$ $\Rightarrow$ distance travelled before melting = 18m

How to decrease t and therefore decrease distance travelled?

- preheat the powder $T_0 \uparrow$
- better plasma? $T_f \uparrow$
- smaller R $\rightarrow$ plausible but costs a lot of money
- ~~change material~~
- change h $\rightarrow$ but h is already pretty large

Arc Melter to increase plasma temperature:



3.044 Materials Processing
Spring 2013