(1) **The S-Exhaustivity Generalization (predicted by Sauerland's Theory)**: utterance of a sentence, S, as a default, licenses the inference that (the speaker believes that) every sentence is false if it is Sauerland-Excludable given S and Alt(S).

p is *Sauerland-Excludable* given S and C if $p \in C$, p is stronger than S and $\neg\exists q \in C$ [(q is stronger than S) and ($S \wedge \neg p$ entails q)].

Homework:

Prove that the Sauerland-Exhaustivity Generalization is indeed predicted by Sauerland's theory.

Solution to question 1:

Let p be Sauerland-Excludable given S and Alt(S). We need to prove that

$B_s(S) \wedge \bigcap PI \wedge B_s(\neg p)$ is not contradictory

Assume otherwise: (and try to derive a contradiction)

(a) $B_s(S) \wedge \bigcap PI \wedge B_s(\neg p)$ is contradictory.

We conclude:

(b) $B_s(S) \wedge B_s(\neg p)$ entails $\neg\bigcap PI$

(c) $B_s(S) \wedge B_s(\neg p)$ entails $\bigcup\neg PI$ (De Morgan)

(d) $B_s(S) \wedge B_s(\neg p)$ entails $\bigcup\{B_s(q): q \in Alt(S)$ and q stronger than s$\}$

Let w^0 be a world in which s believes nothing but S and $\neg p$ (and their logical consequences).

(e) w^0 satisfies $\bigcup\{B_s(q): q \in Alt(S)$ and q stronger than s$\}$. (given the entailment in (d))

For a world to satisfy a disjunction, it must satisfy one of the disjuncts.

So

(f) there must be a $q_i \in Alt(S)$, stronger than s such that q' is a logical consequence of S and $\neg p$.

Hence,

(g) p is not Sauerland-Excludable. □

Note: it is easier to prove the other direction, i.e. $\forall p \in ALT(S)(B_s(\neg p)$ is a Secondary Implicature of S by Sauerland's algorithm $\rightarrow$ p is Sauerland Excludable given S and ALT(S)).