

Subject 24-242. Sample problems from the sixth homework, due Thursday, April 29

A set  $A$  of natural numbers is said to be *m-reducible* (for “many-one reducible”) to a set  $B$  just in case there is a total  $\Sigma$  function  $f$  such that, for any  $n$ ,  $n$  is in  $A$  if and only if  $f(n)$  is in  $B$ .

$A$  is *1-reducible* (for “one-one reducible”) to  $B$  just in case there is a one-one total  $\Sigma$  function  $f$  such that, for any  $n$ ,  $n$  is in  $A$  if and only if  $f(n)$  is in  $B$ .

1. Show that the following are equivalent, for any set  $A$ :
  - $A$  is recursively enumerable (that is,  $\Sigma$ )
  - $A$  is 1-reducible to the set of Gödel numbers of valid sentences
  - $A$  is m-reducible to the set of Gödel numbers of valid sentences.
2. Give an example of a  $\Sigma$  partial function that cannot be extended to a  $\Sigma$  total function.