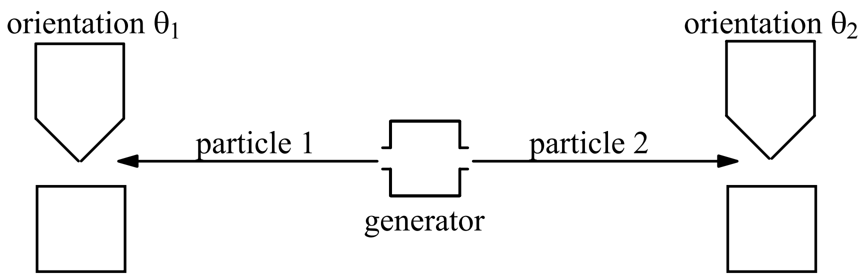
24.111 Philosophy of QM Spring, 2005

Handout #2: Bell's Inequalities



In the experiment depicted above, the generator (which creates pairs of particles, sending one towards each magnet) can be set up in such a way that the two particles exhibit the following behavior: if $\theta_1 = \theta_2$, then either particle 1 goes up and particle 2 down, or particle 1 goes down and particle 2 up. Suppose you try to explain this behavior by means of the following hidden variables hypothesis: First, for each possible magnet orientation θ, each particle either has the property (up, θ)—in which case it will (with certainty) go up through a magnet with this orientation—or it has the property (down, θ)—in which case it will (with certainty) go down through a magnet with this orientation. Second, the particles are generated in such a way that, for any orientation θ, particle 1 has (up, θ) if and only if particle 2 has (down, θ).

Now suppose that we run the experiment many times, letting $\theta_1 = 0°$ or +120°, and letting $\theta_2 = 0°$ or -120°. Then quantum mechanics gives us the following *experimentally confirmed* probabilities:

θ_1	θ_2	Prob(one up, one down)
0°	0°	1
0°	-120°	.25
+120°	0°	.25
+120°	-120°	.25

What predictions does the hidden variables hypothesis make? That depends on the probabilities assigned to the various possible distributions of the relevant properties, which are these:

particle 1	particle 2	probability
(up, 0°),(up, +120°)	(down, 0°),(up, -120°)	p_1
(up, 0°),(up, +120°)	(down, 0°),(down, -120°)	p_2
(up, 0°),(down, +120°)	(down, 0°),(up, -120°)	p_3
(up, 0°),(down, +120°)	(down, 0°),(down, -120°)	p_4
(down, 0°),(up, +120°)	(up, 0°),(up, -120°)	p_5
(down, 0°),(up, +120°)	(up, 0°),(down, -120°)	p_6
(down, 0°),(down, +120°)	(up, 0°),(up, -120°)	p_7
(down, 0°),(down, +120°)	(up, 0°),(down, -120°)	p_8

Since these eight distributions are the only possible distributions, we must have $p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8 = 1$. Further, brief inspection shows us that the h.v. hypothesis yields the following table of probabilities:

θ_1	θ_2	Prob(one up, one down)
0°	0°	1
0°	-120°	$p_2+p_4+p_5+p_7$
+120°	0°	$p_1+p_2+p_7+p_8$
+120°	-120°	$p_2+p_3+p_6+p_7$

If the h.v. hypothesis is to yield the same predictions as quantum mechanics, then the two tables must be identical—in which case we must have

$$p_2+p_4+p_5+p_7 = .25$$
$$p_1+p_2+p_7+p_8 = .25$$
$$p_2+p_3+p_6+p_7 = .25$$

Adding these equations together, it follows that $p_1+3p_2+p_3+p_4+p_5+p_6+3p_7+p_8 = .75$. But this cannot be, since the left-hand side equals $1+2p_2+2p_7 \geq 1$. So the h.v. hypothesis we constructed to explain the observed perfect correlations (i.e., the fact that when $\theta_1 = \theta_2$, the two outcomes are always different) must be false.

A thorny question remains: How *else* can we explain these perfect correlations? Answer that, and you'll be famous.