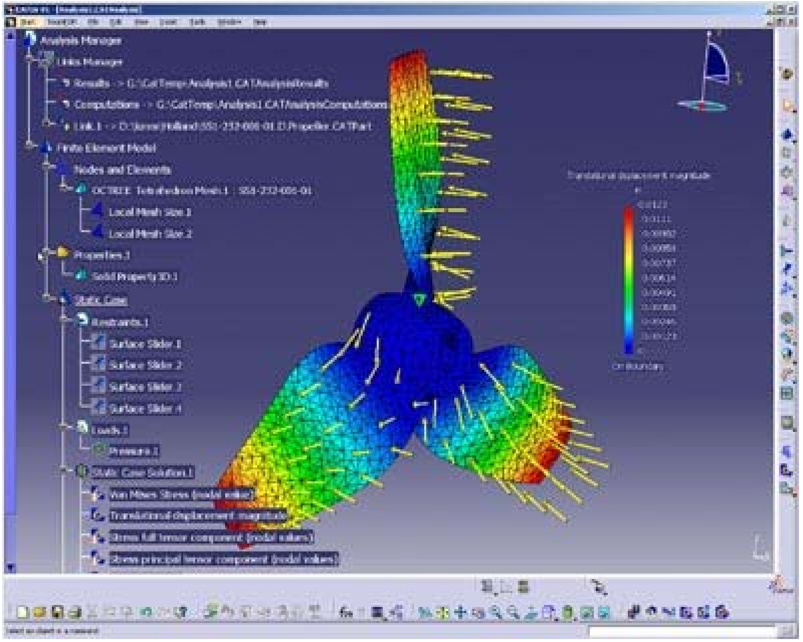
ESD.33 -- Systems Engineering

Session #12
Physics Based Modeling



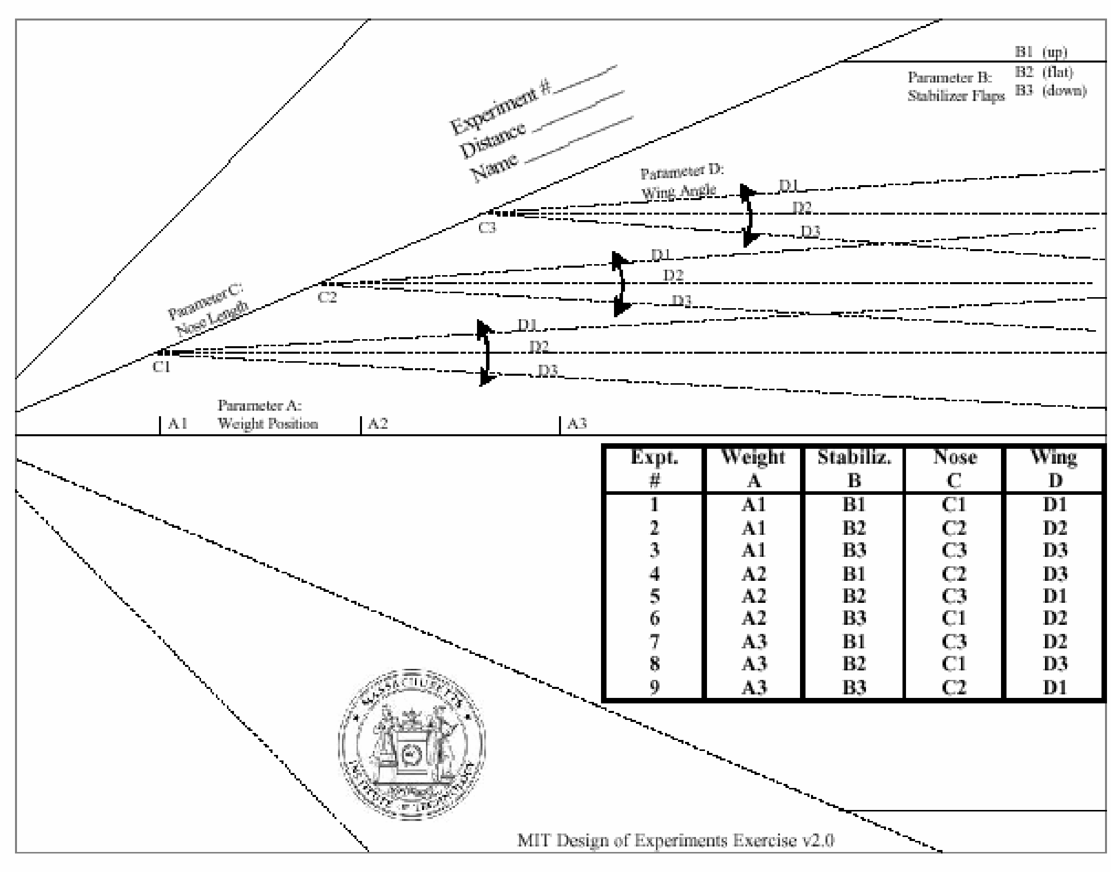
Dan Frey

Plan for the Session

- ➡ Q&A on Assignment #6
- Distributed Object-Based Modeling in Design Simulation Marketplace
- On the Role and Use of Mathematical Models in Engineering Design
- Next steps

Assignment #6

1) Short answers
2) Regression
3) DOE



Expt. #	Weight A	Stabiliz. B	Nose C	Wing D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1

Fractional Factorial Experiments

Three Levels

The design below is also fractional factorial design.

Plackett Burman $(\text{P-B})_{3,9}$ Taguchi $\text{OA}_9(3^4)$

Control Factors			
A	B	C	D
1	1	1	1
1	2	2	2
1	3	3	3
2	1	2	3
2	2	3	1
2	3	1	2
3	1	3	2
3	2	1	3
3	3	2	1

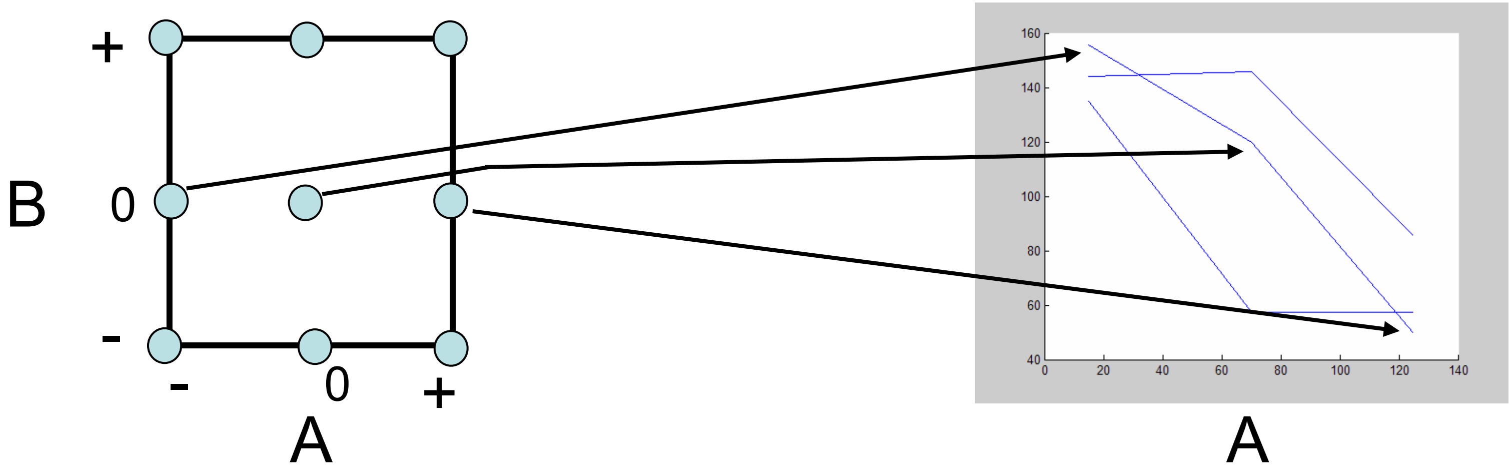
requires only

$$k(p-1)+1=9$$

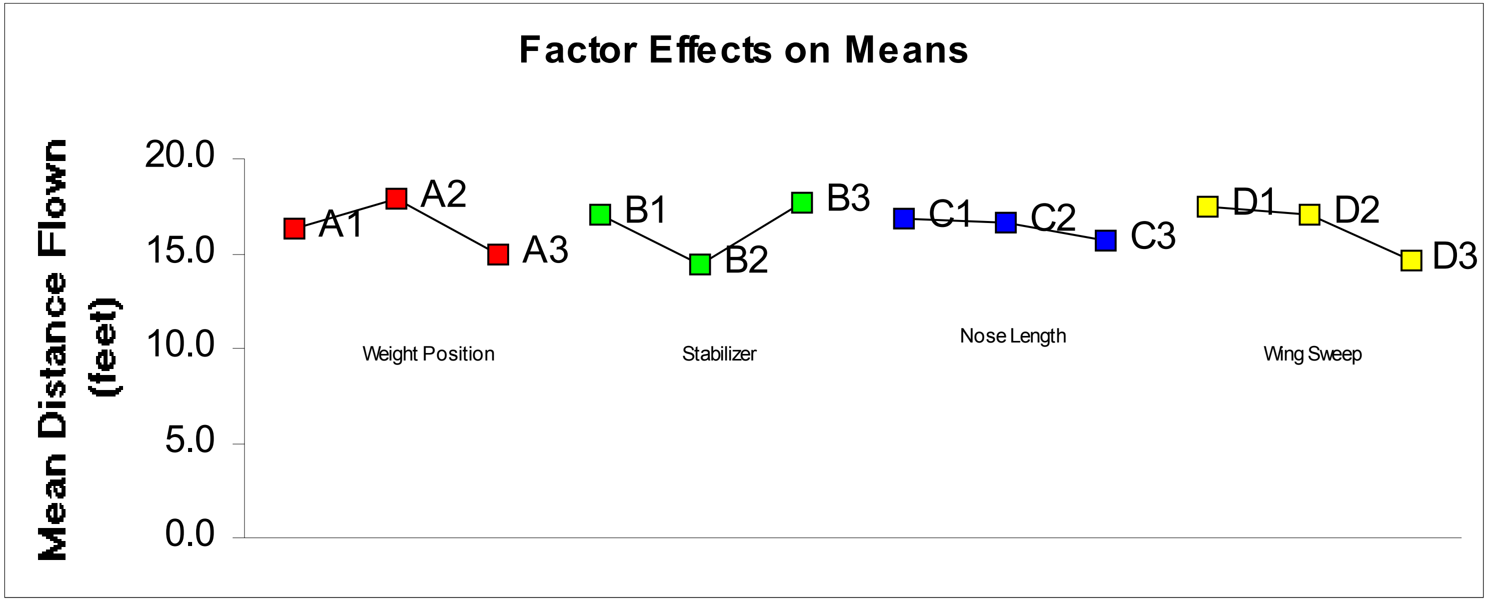
experiments

But it is only Resolution III and also has complex confounding patterns.

Factor Effect Plots



Full Factorial 3^2

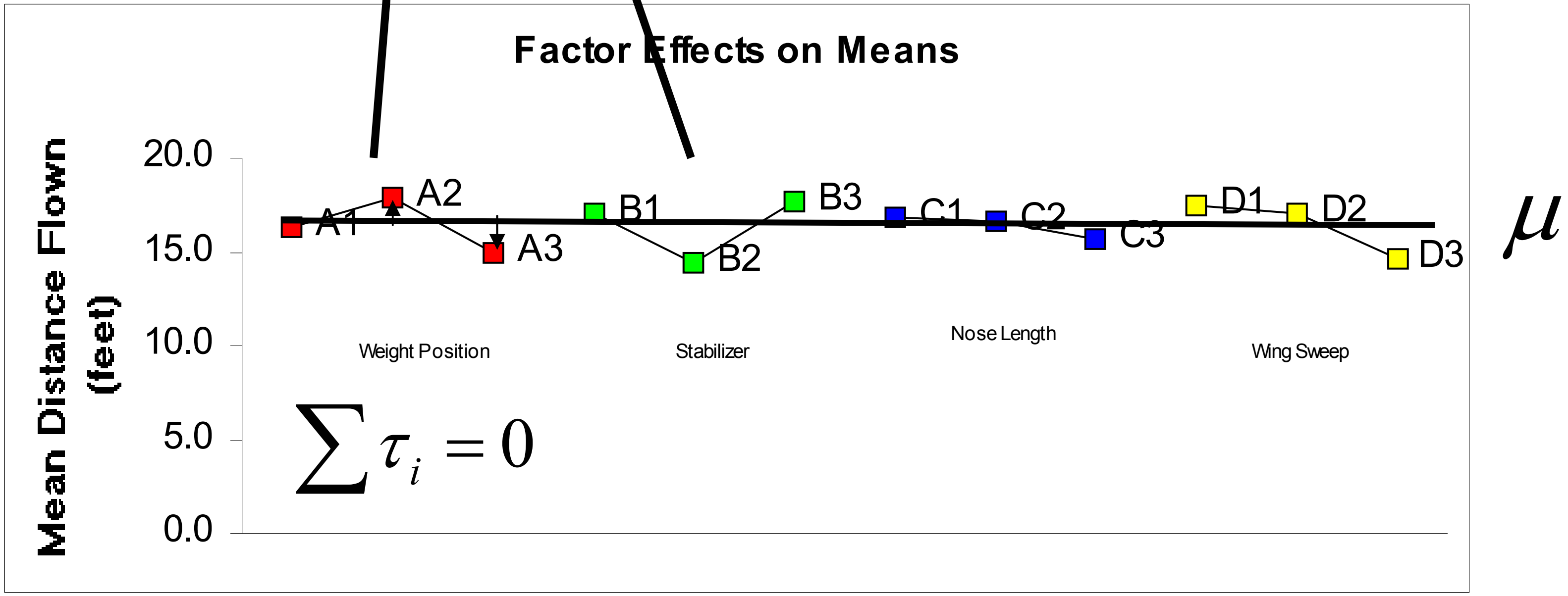


Fractional Factorial L9

Treatment Effects Model (Two Factors)

If it is an “additive model”, we assume there are no interactions

$$y_{ij} = \mu + \tau_i + \beta_j + \cancelto{0}{(\tau\beta)_{ij}} + \varepsilon_{ijk} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \\ k = 1, 2, \ldots, n \end{cases}$$



$$\sum \tau_i = 0$$

What is a Confidence Interval?

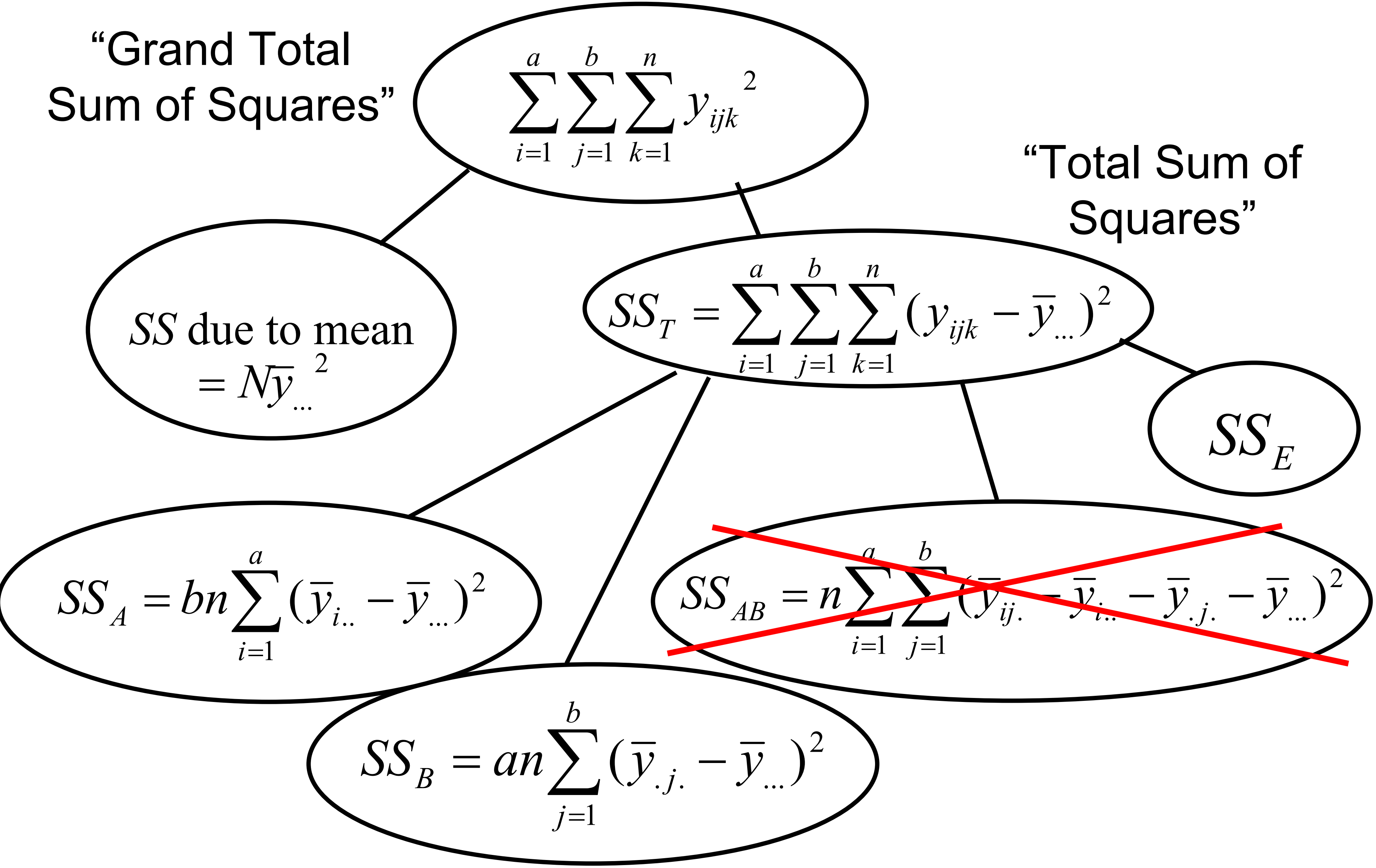
- I throw airplane [A1 B1 C1 D1] 9 times

19 14 16 20 13 23 21 22 19

- My estimate of μ is $\bar{x}$=18.55
- My estimate of σ is s=3.50
- I have 9 samples n=9

$$95\%\ \text{conf. int.} = \bar{x} \pm 1.96 \frac{s}{\sqrt{n}}$$

Breakdown of Sum Squares



"Grand Total Sum of Squares"

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}{}^{2}$$

"Total Sum of Squares"

SS due to mean $= N\bar{y}_{...}{}^{2}$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2$$

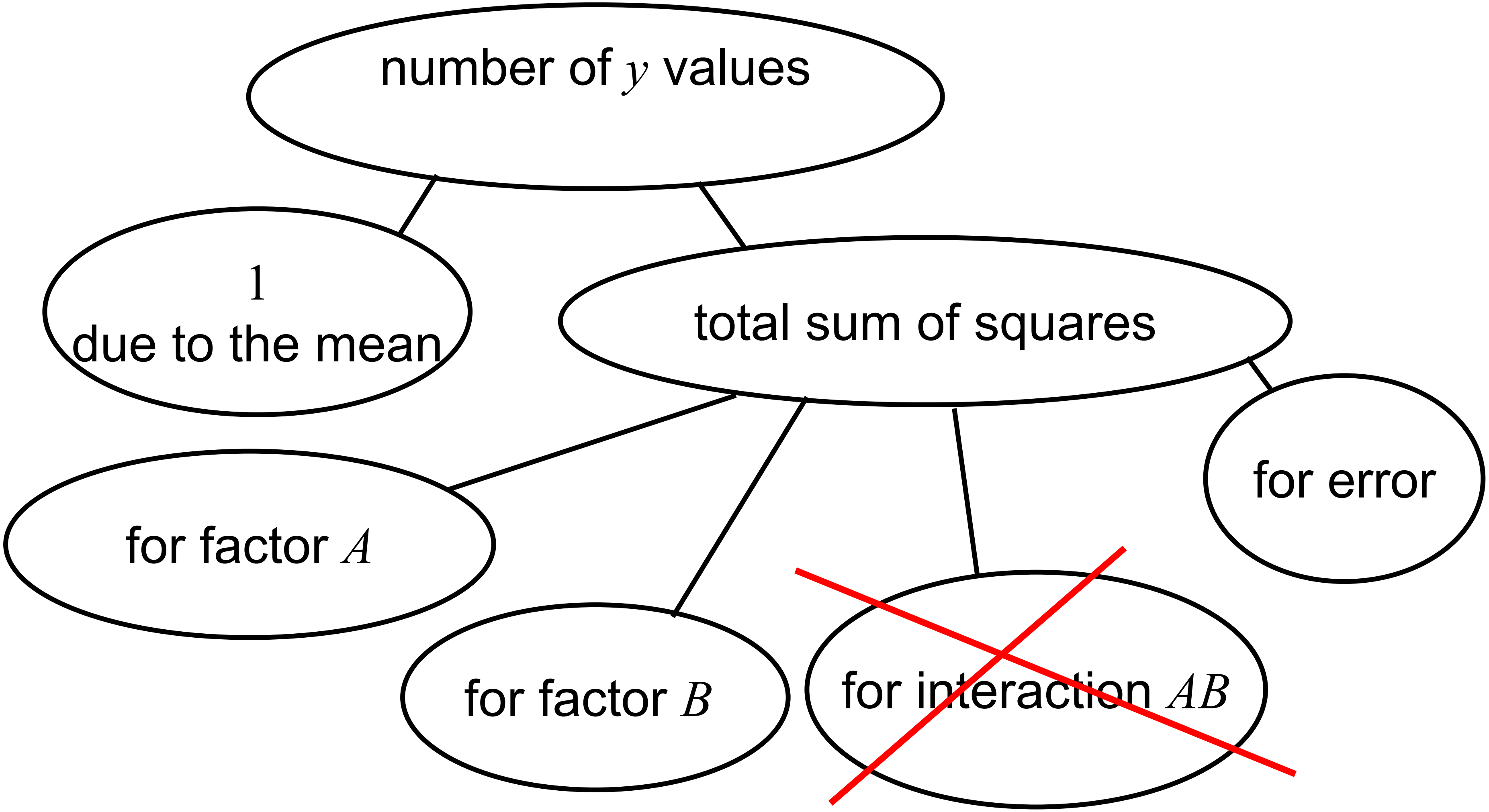
$$SS_E$$

$$SS_A = bn\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$$

$$\xcancel{SS_{AB} = n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{...})^2}$$

$$SS_B = an\sum_{j=1}^{b}(\bar{y}_{.j.} - \bar{y}_{...})^2$$

Breakdown of DOF



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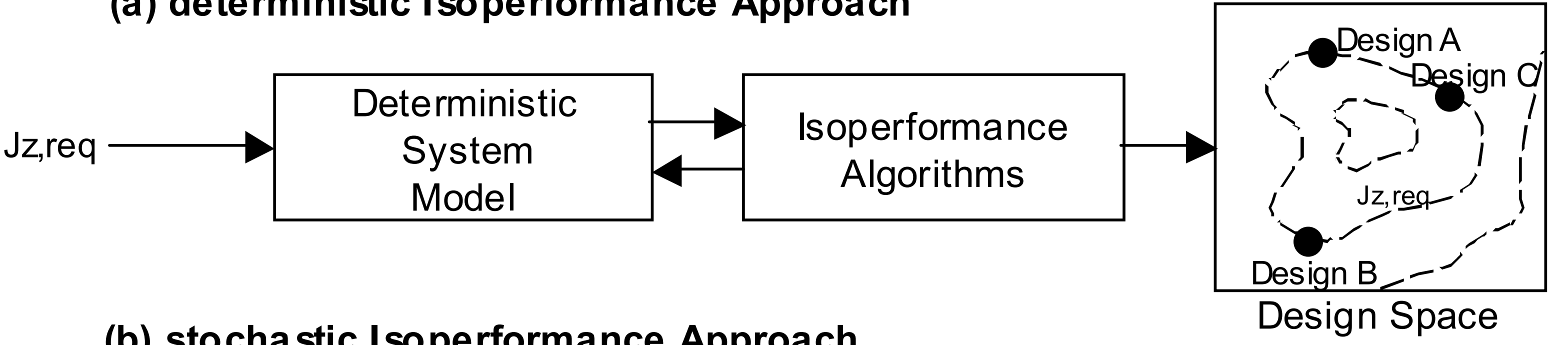
Isoperformance Approaches

(a) deterministic Isoperformance Approach



Jz,req

Deterministic System Model

Isoperformance Algorithms

Design A

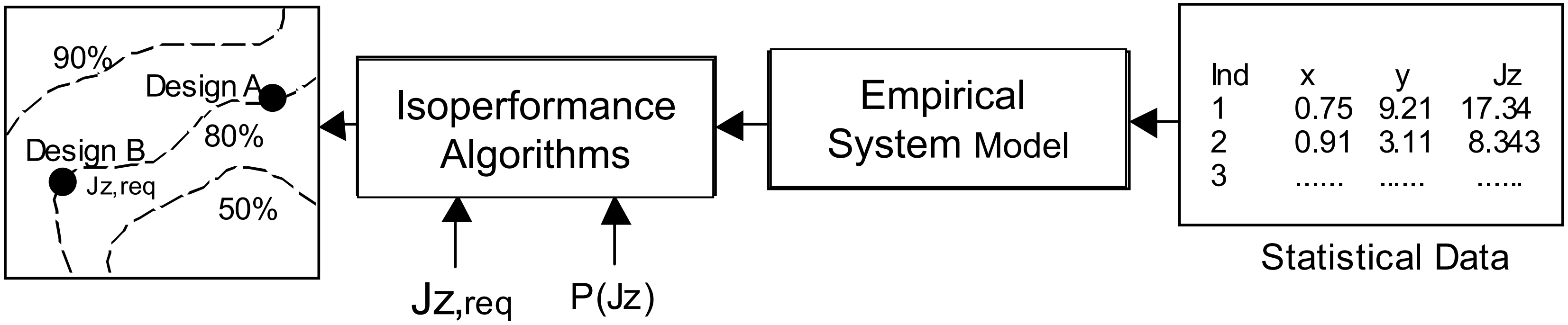
Design C

Jz,req

Design B

Design Space

(b) stochastic Isoperformance Approach



90%

Design A

80%

Design B

Jz,req

50%

Isoperformance Algorithms

$Jz_{,req}$

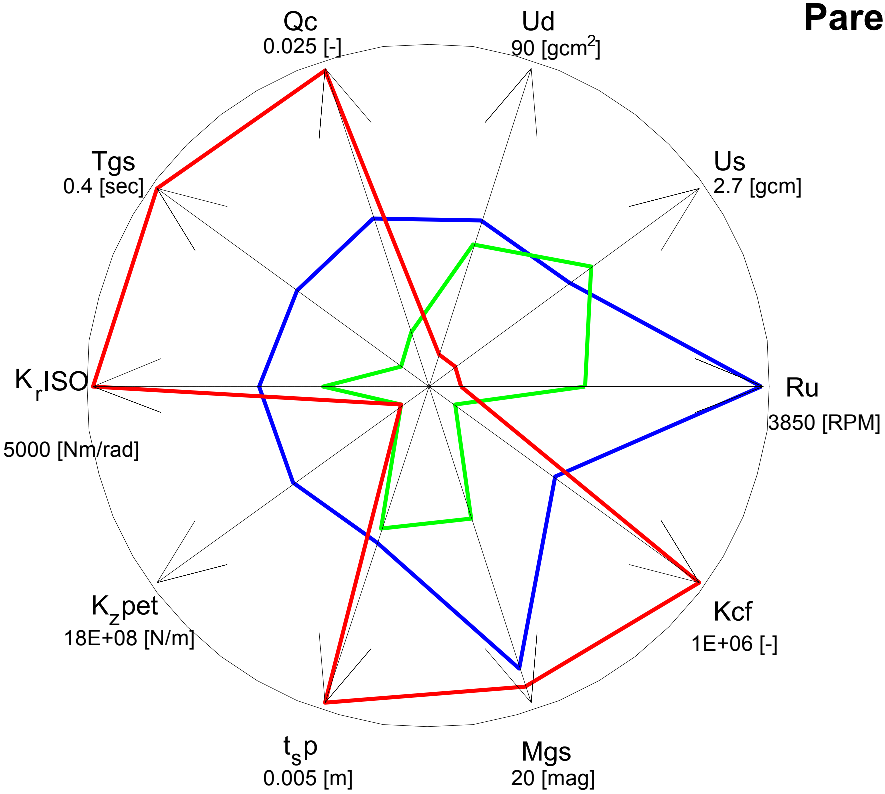
P(Jz)

Empirical System Model

Ind	x	y	Jz
1	0.75	9.21	17.34
2	0.91	3.11	8.343
3	……	……	……

Statistical Data

Pareto-Optimal Designs p^*_{iso}



Design A

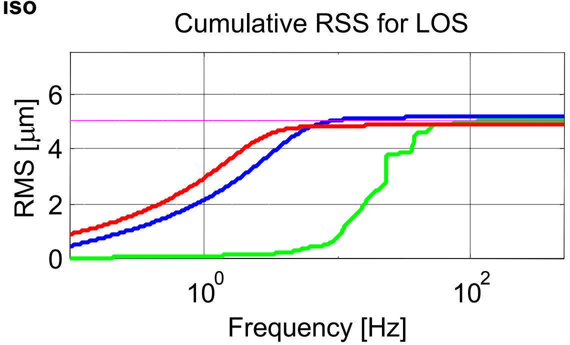
Best “mid-range” compromise

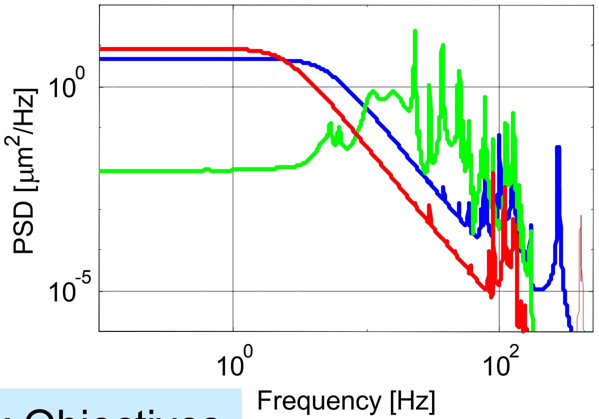
Design B

Smallest FSM control gain

Design C

Smallest performance uncertainty





A: min(J_{c1})
B: min(J_{c2})
C: min(J_{r1})

	Performance		Cost and Risk Objectives		
	Jz,1	Jz,2	Jc,1	Jc,2	Jr,1
Design A	20.0000	5.2013	0.6324	0.4668	+/- 14.3218 %
Design B	20.0012	5.0253	0.8960	0.0017	+/- 8.7883 %
Design C	20.0001	4.8559	1.5627	1.0000	+/- 5.3067 %

DOME as an SE Case Study

- What were the key implementation challenges?
- How were they resolved?
- What role did standards play?
- What do you think of the engineering solution?
- How will it be rolled out?

DOME as an SE Tool

- DOME can support many SE tools
  - Design Structure Matrix
  - Utility theory
  - Acceptability
  - Analytical Hierarchy Process
- And DOME may also support a new social dynamic of model building

Distributed Simulation Service Marketplace

“it is generally understood that in practice individual participants perform their work in an informal marketplace, bartering service exchange relationships to get what they need to resolve their part of the problem”

Plan for the Session

- Q&A on Assignment #6
- Distributed Object-Based Modeling in Design Simulation Marketplace
- ➡ On the Role and Use of Mathematical Models in Engineering Design
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Definitions

- **Mathematical (or Symbolic) Model** – A construct that comprises an abstract representation of a real system. Models are constructed by people, often for the purpose of system design.
- **Computational Simulation** – A mathematical model implemented in a digital computer
- **Law of Nature** – A fundamental understanding of causality in a physical system often expressed in mathematical form or as an algorithm executable by a computer

Classification of Models

advantages / disadvantages?

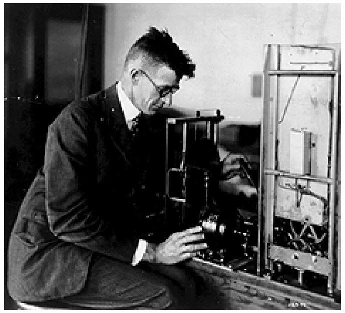
advantages / disadvantages?

Real World

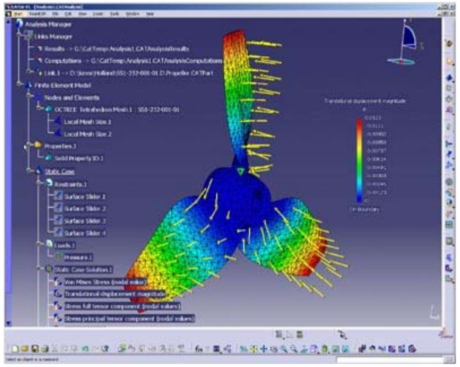
Physical or Iconic

Analog

Mathematical or Symbolic

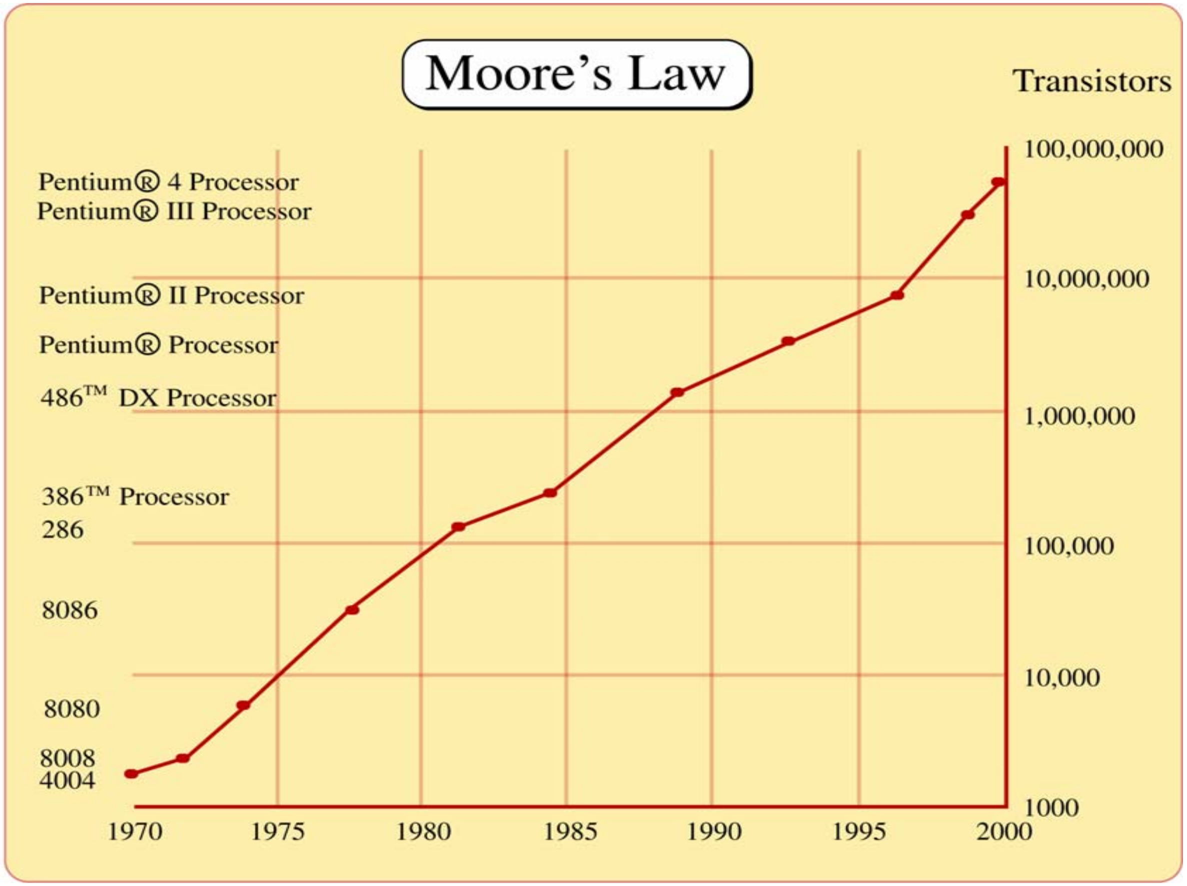


$$-x \cdot \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-h^{3}}{12\mu} \cdot \frac{\mathrm{d}p}{\mathrm{d}x}$$



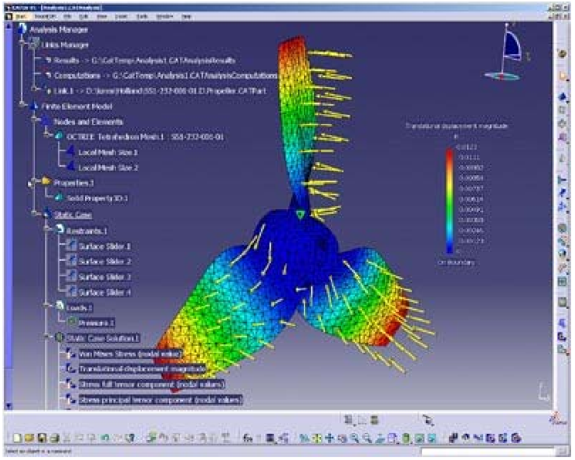
Mathematical Models Are Rapidly Growing in Power

- Moore’s Law – density ↑ 2X / 18 months
- Better algorithms being developed



Mathematical Models are Becoming Easier to Use

- A wide range of models are available
  - Finite Element Analysis
  - Computational fluid dynamics
  - Electronics simulations
  - Kinematics
  - Discrete event simulation


Analysis Manager
Links Manager
Results -> G:\CatTemp\Analysis1.CATAnalysisResults
Computations -> G:\CatTemp\Analysis1.CATAnalysisComputations
Finite Element Model
Nodes and Elements
Local Mesh Size.1
Local Mesh Size.2
Properties.1
Solid Property 3D.1
Static Case
Restraints.1
Surface Slider.1
Surface Slider.2
Surface Slider.3
Surface Slider.4
Loads.1
Pressure.1
Static Case Solution.1
Von Mises Stress (nodal values)
Translational displacement magnitude
Stress full tensor component (nodal values)
Stress principal tensor component (nodal values)

- Sophisticated visualization & animation make results easier to communicate
- Many tedious tasks are becoming automate (e.g., mesh generation and refinement)

Mathematical Models are Becoming Better Interconnected

- Product Data Management Solutions
- Distributed Object-based Modeling Environment
  - Enables a decentralized design simulation marketplace
  - Intends to do for models what HTML does for text

Why Models Can Go Wrong

- Right model → Inaccurate answer
  - Rounding error
  - Truncation error
  - Ill conditioning
- Right model → Misleading answer
  - Numerical stability, chaotic systems
- Right model → No answer whatsoever
  - Failure to converge
  - Algorithmic complexity
- Not-so right model → Inaccurate answer
  - Unmodeled effects
  - Bugs in coding the model

Computational Complexity and Big-O Notation

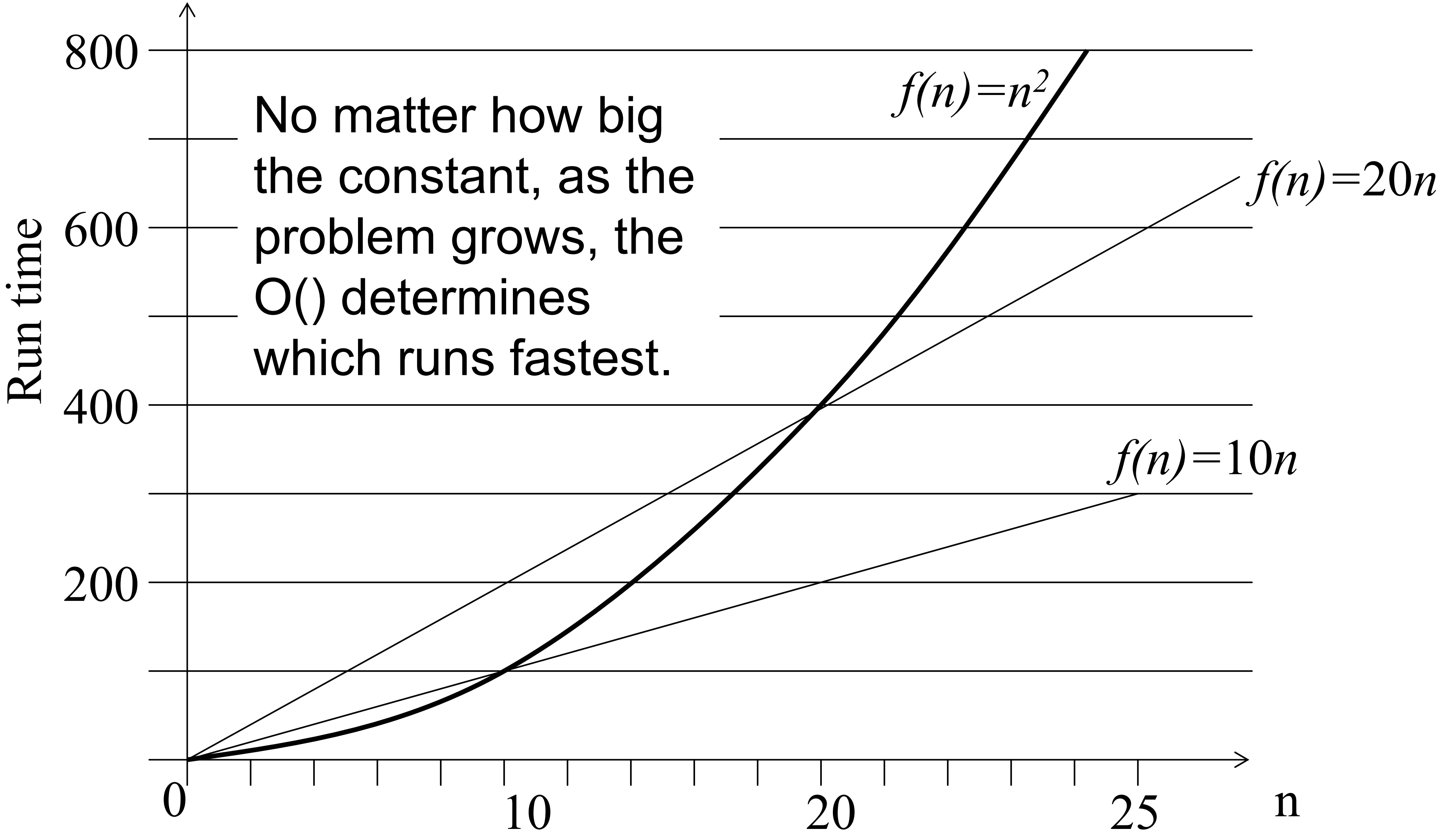
- n is the number of variables in the problem
- $O(f(n))$ means that as $n \rightarrow \infty$, the execution time t is at most $cf(n)$ for some constant c.
- Since only the growth rate matters, we can ignore the multiplicative constants and the lower order terms, e.g.,

  $3n\log(n)+6n+2.7$ is $O(n\log(n))$

  $n^3 + 10000000000n$ is $O(n^3)$

  3^n is $O(3^n)$

Effect of Multiplicative Constant



No matter how big the constant, as the problem grows, the O() determines which runs fastest.

Growth Rate of some Functions

$O(\log n)$, $O(\log^2 n)$, $O(n^{0.5})$, $O(n)$,
$O(n \log n)$, $O(n^{1.5})$, $O(n \log^2 n)$, $O(n^2)$,
$O(n^3)$, $O(n^4)$

Polynomial Functions

$O(n^{\log n})$, $O(2^n)$, $O(3^n)$, $O(4^n)$, $O(n^n)$,
$O(n!)$

Exponential Functions

NP-Complete

- The class <u>NP-Complete</u> is a set of problems that, we believe, has no polynomial time algorithms.
- Therefore, they are hard problems.
- If a problem is NP-complete, there is no hope to solve the problem efficiently when *n* becomes large.

Computational Complexity and Moore’s Law

- Consider a problem that requires 3^n flops
- World’s fastest computer ~ 36 Teraflops/sec
- In a week, you can solve a problem where

  n=log(60*60*24*7*36*10^{12})/log(3)=40
- If Moore’s Law continues for 10 more years

  n=log($2^{10/1.5}$*60*60*24*7*36*10^{12})/log(3)=44
- We will probably not reach n=60 in my lifetime

Why Models Can Go Wrong

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These may be the biggest problems of all

Errors in Scientific Software

- Experiment T1
  - Statically measured errors in code
  - Cases drawn from many industries
  - ~10 serious faults per 1000 lines of commercially available code
- Experiment T2
  - Several independent implementations of the same code on the same input data
  - One application studied in depth (seismic data processing)
  - Agreement of 1 or 2 significant figures on average

Hatton, Les, 1997, "The T Experiments: Errors in Scientific Software", *IEEE Computational Science and Engineering*.

Models in Engineering vs Science

There is a fundamental difference between models used in science and engineering… Science uses models to gain an understanding of the way things are… Engineering uses models … in an attempt to achieve what ought to be… The validated models of science are used in engineering … to establish bounds and to improve products…

Blanchard and Fabrycky, section 7.2

Uses of Models in Engineering

- Engineers seek to answer these questions:
  - – Will the system, as designed, work?
  - – Which of two system designs is the better?
  - – Do I adequately understand the system?

Hazelrigg, 1999, "On the Role and Use of Models in Engineering Design", *ASME J. of Mechanical Design.*

Definitions

- **Accuracy** – The ability of a model to faithfully represent the real world
- **Resolution** – The ability of a model to distinguish properly between alternative cases
- **Validation** – The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model. (AIAA, 1998)
- **Verification** – The process of determining that a model implementation accurately represents the developer’s conceptual description of the model and the solution to the model. (AIAA, 1998)

Model Validation in Engineering

- A model of an engineering system can be validated using data to some degree within some degree of confidence
- Physical data on that specific system cannot be gathered until the system is designed and built
- Models used for design are ***never fully validated*** at the time design decisions must be made

Definitions

- **Decision** – (1) A choice made from among a set of alternatives, (2) An irrevocable allocation of resources
- **Outcome** – The actual result of a decision
- **Information** – The basis upon which decisions are made
- **Uncertainty** – A lack of precise knowledge regarding the inputs to a model or process, or the model or process itself, or about future events
- **Risk** – The result of uncertainty on the outcome of a decision

Quantifying Uncertainty due to Model Data

- Express uncertainty mathematically
  - Probability density functions
  - Moments
  - Ranges
- Propagate through the system
- Evaluate the effects
  - On performance
  - On decisions to be made

Monte Carlo Simulations
What are They Good at?

$$\text{Accuracy} \propto {}^{1}\!/_{\sqrt{\text{N}}}$$

$$\boxed{\text{N} \equiv \#\,\text{Trials}}$$

- Above formulae apply regardless of scale
- So, Monte Carlo is good for:
  - Rough approximations <u>or</u>
  - Simulations that run quickly
  - <u>Even if</u> the system has many random variables

Continuous-Stirred Tank Reactor

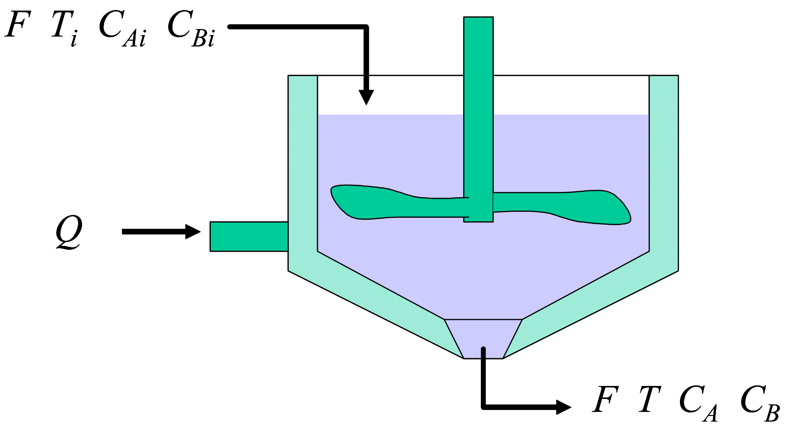
- Objective is to generate chemical species B at a rate of 60 mol/min

$$Q = F\rho C_p (T - T_i) + V(r_A H_{RA} + r_B H_{RB})$$

$$C_A = \frac{C_{Ai}}{1 + k_A^0 e^{-E_A/RT}\tau} \quad C_B = \frac{C_{Bi} + k_A^0 e^{-E_A/RT}\tau C_A}{1 + k_B^0 e^{-EB/RT}\tau}$$

$$-r_A = k_A^0 e^{-E_A/RT} C_A$$

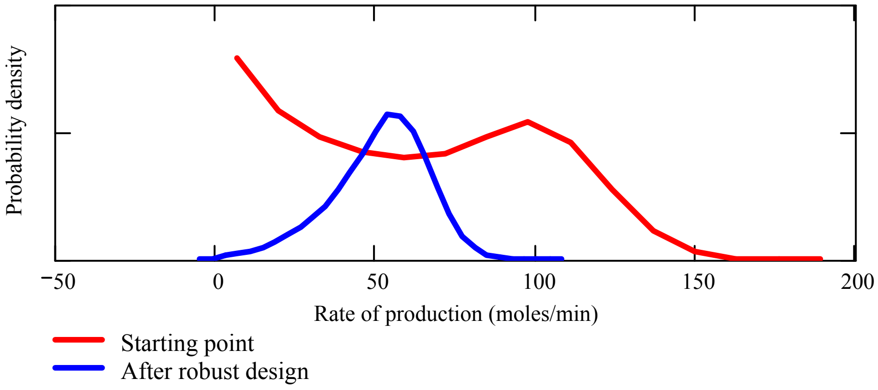
$$-r_B = k_B^0 e^{-E_B/RT} C_B - k_A^0 e^{-E_A/RT} C_A$$



Adapted from Kalagnanam and Diwekar, 1997, “An Efficient Sampling Technique for Off-Line Quality Control”, *Technometrics* (39 (3) 308-319.

Continuous-Stirred Tank Reactor Probabilistic Analysis

- 10% uncertainty in 6 input variables (10%=2σ)
- 66% uncertainty in the outputs (starting point)
- 30% uncertainty in the outputs (robust design)



Models for Decision Support

- What are the critical elements to making informed engineering decisions?

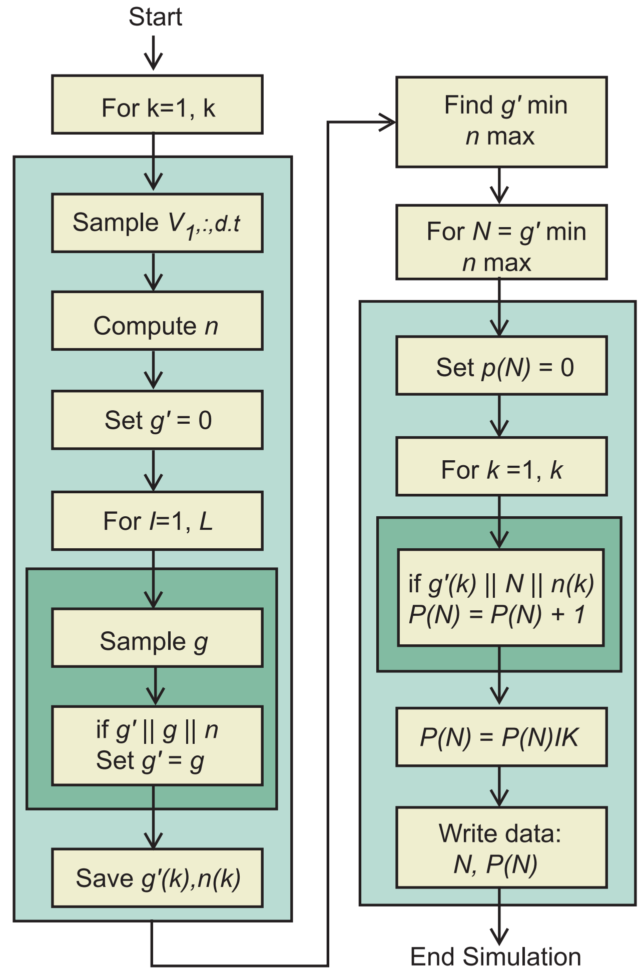
- Are there engineering decisions that can be made with confidence despite large uncertainties?

Hazelrigg’s M&M Game

- “Guess” the number of M&Ms in a jar
- Competitive game
- Person who guesses closest WITHOUT GOING OVER wins

Simulations of the Game

Logic for the Simulation of the Candy Jar Game



Start

For k=1, k

Sample $V_{1,:,d.t}$

Compute *n*

Set $g' = 0$

For *l*=1, *L*

Sample *g*

if $g' \| g \| n$
Set $g' = g$

Save $g'(k), n(k)$

Find g' min
n max

For $N = g'$ min
n max

Set $p(N) = 0$

For *k* =1, *k*

if $g'(k) \| N \| n(k)$
$P(N) = P(N) + 1$

$P(N) = P(N)IK$

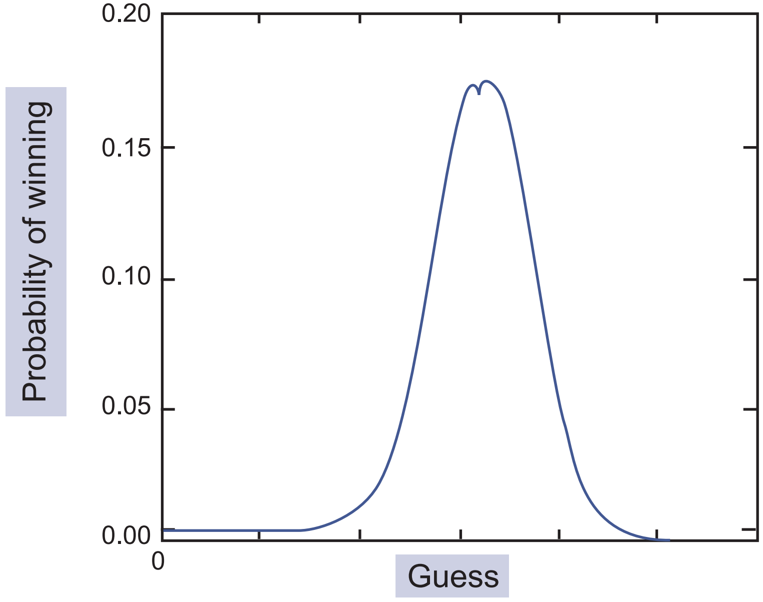
Write data:
N, *P(N)*

End Simulation

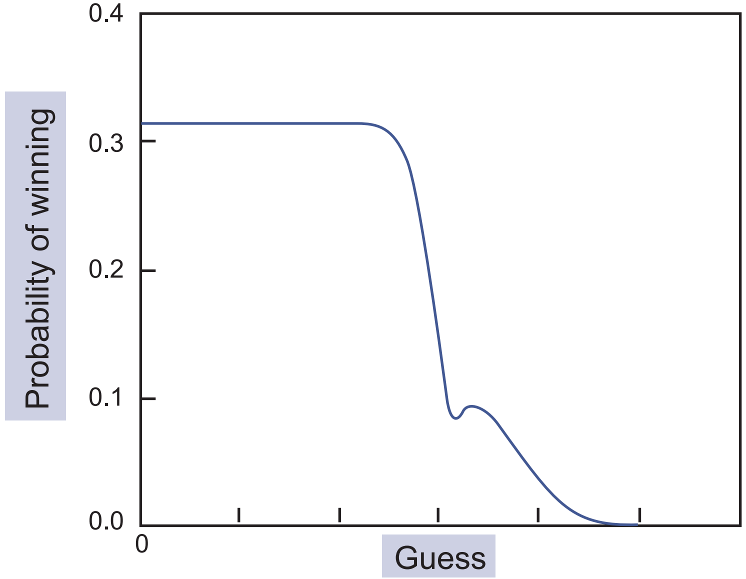
L = number of other players in the game

K = number of simulations

Simulation Results for Case 1



Simulation Results for Case 2



Modeling and Simulation Summary

- Mathematical modeling is an essential part of modern systems engineering
- Models have many pitfalls
- Most of the pitfalls get worse as systems grow in complexity and performance
- Strategies
  - Know the limits of your models
  - Make estimates of your uncertainty
  - Know the effect uncertainty on your decisions
  - In critical areas, develop validated models
  - Combine models with experimentation

Plan for the Session

- Q&A on Assignment #6
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Proposed Adjustment

- Exam #2 moved to “out of class”
- Exam #2 afforded three hours
- Aircraft engines moved earlier
- Tim Davis given more time

T 20 JUL		Robust Design	Taguchi_Clausing_Robust Quality.pdf Ulrich_Eppinger_Product Design and Dev ch13.pdf		#6
R 22 JUL		Extreme Programming	Beck_Extreme Programming.pdf Williams_Pair Programming.pdf		
T 27 JUL		**Examination #2**			
R 29 JUL	**Case Studies**	Aircraft Engines (GE) Automobiles		#7	
T 3 AUG		Work on Aircraft Engines Assignment Project Oxygen	Steele_TBD.pdf		
R 5 AUG		NORAD Command and Control (Mitre)	Folk_TBD.pdf	#8	#7
T 10 AUG		Tactical Tomahawk	Cummings_TBD.pdf		
R 12 AUG		Course Summary / Feedback			#8

Next Steps

- Complete HW #6 on DOE
  - Due 8:30AM Tues 20 July
- See you at Tuesday's session
  - On the topic "Robust Design"
  - 8:30AM Tuesday, 20 July
- Reading assignment for Thursday
  - Taguchi_Clausing_Robust Quality.pdf
  - Ulrich_Eppinger_Product Design and Dev ch13.pdf