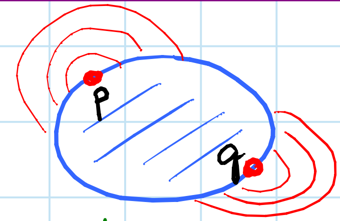
6.849	Class 17	Nov. 8. 2012

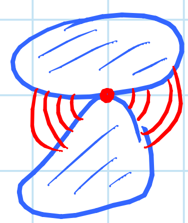
- <u>Pita form</u>: perimeter halving on convex 2D body



— as performed in L17, but can be smooth

<u>Alexandrov–Pogorelov Theorem</u>: [1950/1973]
every convex metric, topologically a sphere, is realized as the surface of a unique convex 3D body ~> BEYOND POLYHEDRA
— proof by limiting argument + Alexandrov

- <u>D-form</u>: glue together 2 convex 2D bodies of equal perimeter



developable, from a dream... [Tony Wills]

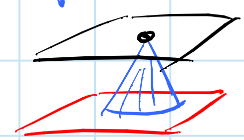
- <u>Seam form</u>: glue together 2D bodies ← flat to satisfy Alexandov-Pogorelov
  — properties: [Demaine & Price – DCG 2009]
  1. = convex hull of seams
  2. creases (other than seams) are line segments with endpoints at (strict) vertices <u>OR</u> tangent to seams
     impossible if 2D bodies convex

  ⇒ D-forms have no seams
  & pita forms have ≤ 1 crease, pq   seam endpoints

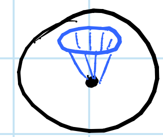
LET'S MAKE SOME D-FORMS!

Proof of ①:

- Minkowski's Theorem: any convex body
  is the convex hull of its extreme points
  a tangent plane hits just the point
- extreme point can't be
  locally flat AND convex:



- curvature = area of tangent
  normals on Gauss sphere

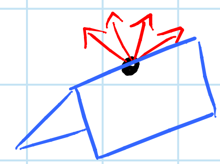


$\Rightarrow$ positive at extreme point of convex body

$\square$

Proof of ②:

- locally flat crease point has
  range of tangent planes
  between two extremes



- point can't be extreme by ①

$\Rightarrow$ all tangent planes hit surface

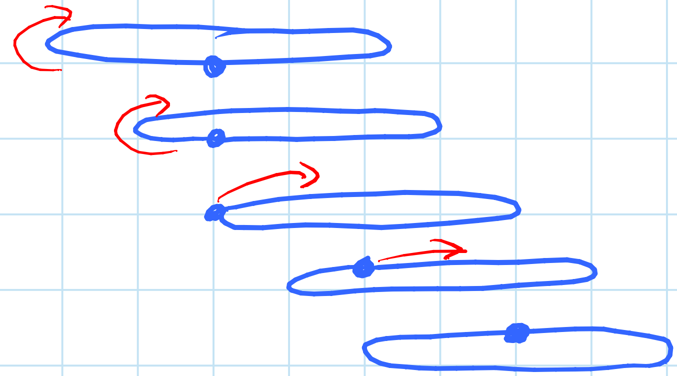
$\Rightarrow$ surface continues along line of intersection,
remaining a crease by tangent planes,
until not locally flat

- if an endpoint is not a vertex,
  still zero curvature

$\Rightarrow$ must be tangent to seam or else
get third normal direction $\Rightarrow$ Gauss area > 0

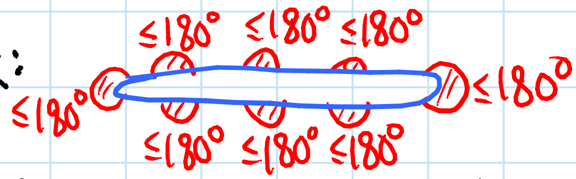
$\square$

- Rolling belt:



half way

  - to work:



(just like a convex body)

- Alexandrov implementation

- Folding nonconvex polyhedra: (see O'Rourke 2010 & Spring 2005)

  Burago–Zalgaller Theorem: [1960; 1996] ↗ genus-Ø case
  every polyhedral metric has an isometric polyhedral realization in 3D.
  noncrossing if metric is orientable or has boundary

  - uses Nash's "spiraling perturbations"
  - is "strongly corrugated"
  - finite # polygons ... but no bound known

$\boxed{\text{OPEN}}$: algorithm to find realization?

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6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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