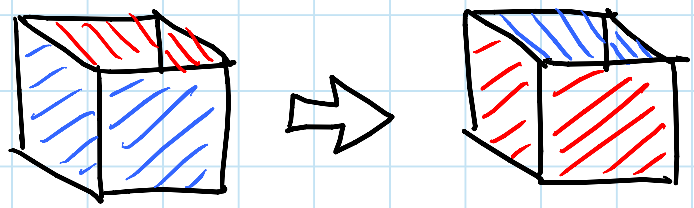
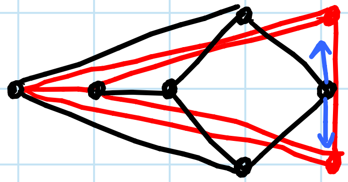
6.849	Class 10	Oct. 11, 2012

- (at end) Finish Hypar Truncated Tetrahedron
- Folded states → folding motions:
  what's wrong with holes?
  - could try to fill hole
  - but folded state doesn't define where hole goes
  - might be impossible without intersection
  - e.g.



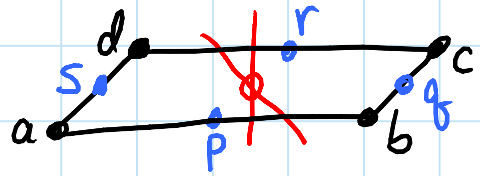
  - possible (even with finite # creases)
  - impossible with "holes" filled...
- Sliding joints in linkages
  - can be simulated by regular linkages via Peaucellier:



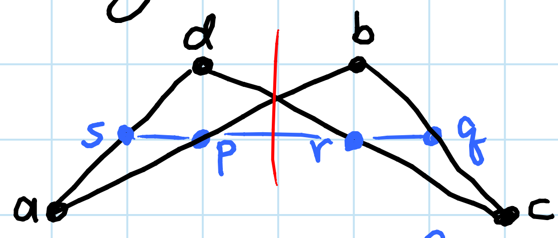
Kempe Universality Theorem:

- Why unbraced contra/parallelogram bad?

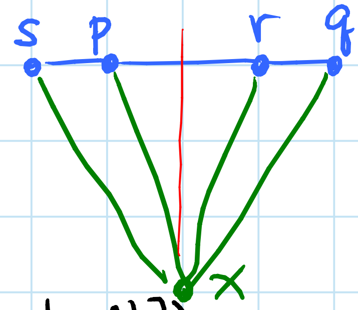
- How contraparallelogram bracing works [Abbott & Barton 2004]
  - $|xp| = |xr|$ & $|xs| = |xq|$
    $\Rightarrow$ x lies on perp. bisectors of pr & sq
    $\Rightarrow$ x would lie interior to parallelogram (actually center)



  - impossible for $|xp| >$ perimeter
  - placing x in contraparallelogram:



$$|sp| \cdot |sr| = \tfrac{1}{4}(|ab|^2 - |ad|^2)$$



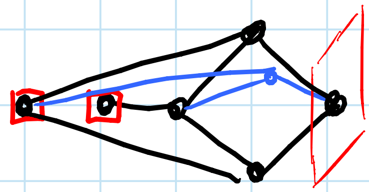
$$|xs|^2 - |xp|^2 = |sp| \cdot |sr|$$

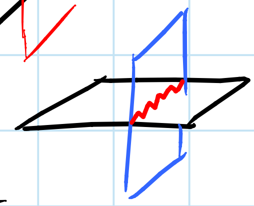
$\Rightarrow$ set $|xs|^2 = |xp|^2 + \tfrac{1}{4}(|ab|^2 - |ad|^2)$

$\boxed{\text{PROJECT}}$: – implement Kempe e.g. for splines
– "Kempe" alphabet
– Kempe sculpture

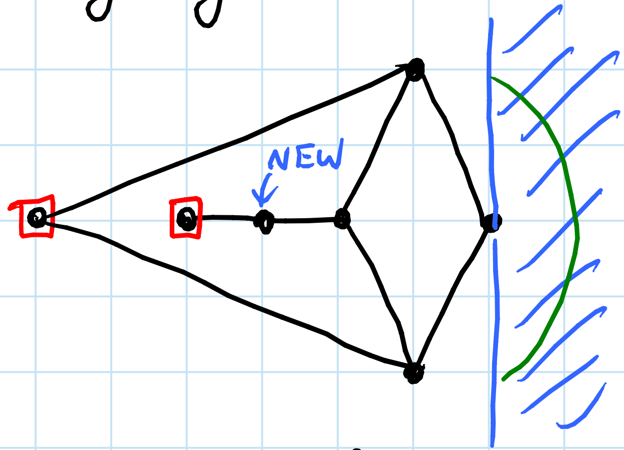
Generalization: [Abbott, Barton, Demaine 2008]
↳ Master's thesis

- Higher dimensions:
  - 3D Peaucellier restricts to plane
  - two restrict to line
  - Kempe construction in xy plane
  - copy angles/lengths into/out of xy

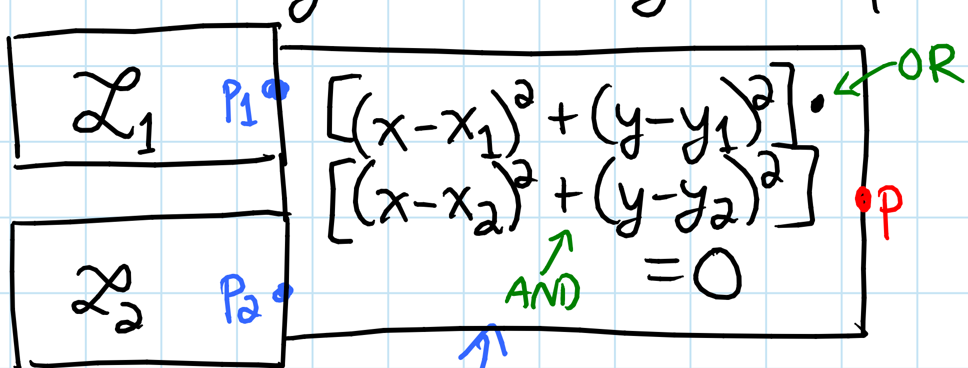




- Semi-algebraic set = finite union/intersection of polynomial inequalities $p(x,y) \geq 0$
  - includes splines (piecewise polynomial)
  - inequality by modified Peaucellier:

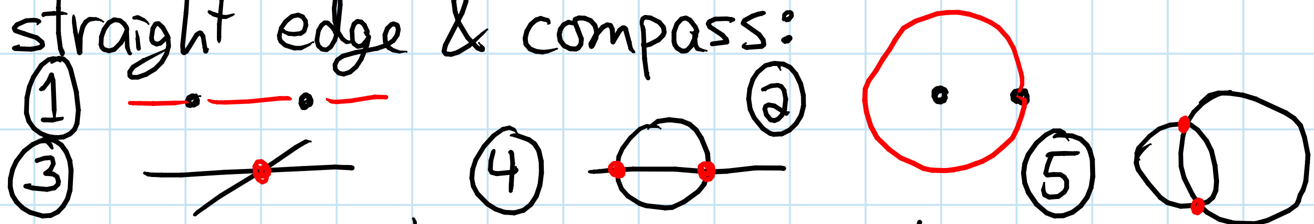


  - intersection: overlay two linkages at p
  - union:



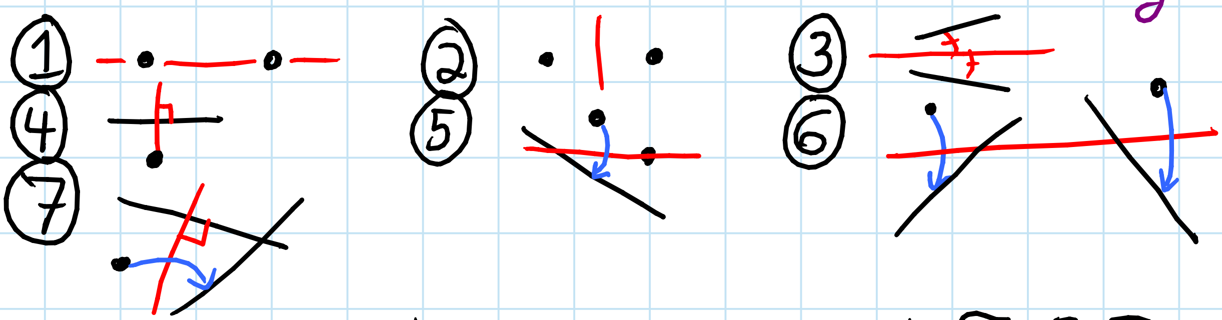
Kempe on 3 points

- Axioms:
  - straight edge & compass:



    - can compute $\emptyset$, 1, +, −, *, /, $\sqrt{\ }$ ($\Rightarrow$ solve quadratics) & that's all
    - can't trisect 60° or compute $\sqrt[3]{2}$ [Wanzel 1837]

  - single-fold origami: [Huzita 1989; Hatori 2002; Justin 1989; Lang 2010]



    - can compute $\emptyset$, 1, +, −, *, /, $\sqrt{\ }$, $\sqrt[3]{\ }$ ($\Rightarrow$ solve cubics & quartics) & that's all [Huzita & Scimemi 1989; Emert, Meeks, Nelson 1994]
    - can trisect angles but can't quintisect [Abe]
    - can compute $\sqrt[3]{2}$ [Messer 1985]
    - Reference Finder software [Lang]

  - two-fold origami [Alperin & Lang – OSME 2006]
    - can quintisect angles

  - n-fold origami [Alperin & Lang; Demaine & Demaine 2000]
    - can solve any polynomial

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012