

# Shading & Material Appearance



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# Lighting and Material Appearance

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- Input for realistic rendering
  - Geometry, Lighting and Materials
- Material appearance
  - Intensity and shape of highlights
  - Glossiness
  - Color
  - Spatial variation, i.e., texture (next Tuesday)



# Unit Issues - Radiometry

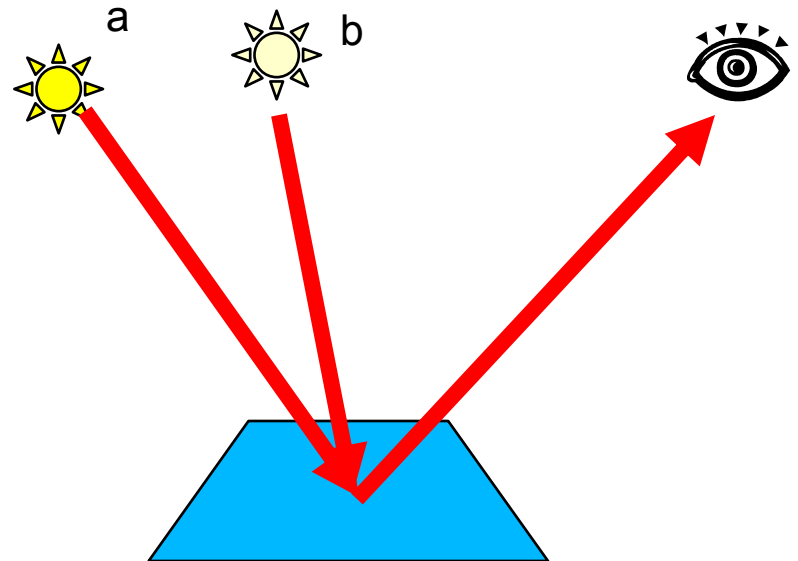
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- We will not be too formal in this class
- Issues we will not really care about
  - Directional quantities vs. integrated over all directions
  - Differential terms: per solid angle, per area
  - Power? Intensity? Flux?
- Color
  - All math here is for a single wavelength only; we will perform computations for R, G, B separately
    - Do not panic, that just means we will perform every operation three times, that is all

# Light Sources

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- Today, we only consider point light sources
  - Thus we do not need to care about solid angles
- For multiple light sources, use linearity
  - We can add the solutions for two light sources
    - $I(a+b) = I(a) + I(b)$
  - We simply multiply the solution when we scale the light intensity
    - $I(s a) = s I(a)$

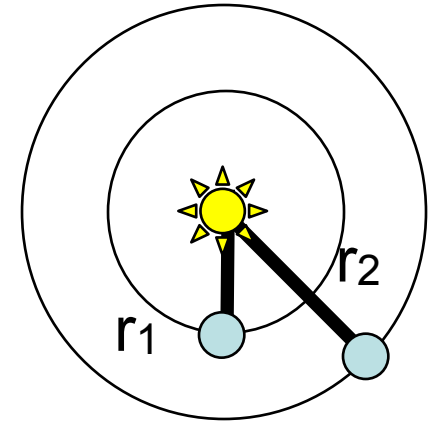


Yet again, linearity  
is our friend!

# Intensity as Function of Distance

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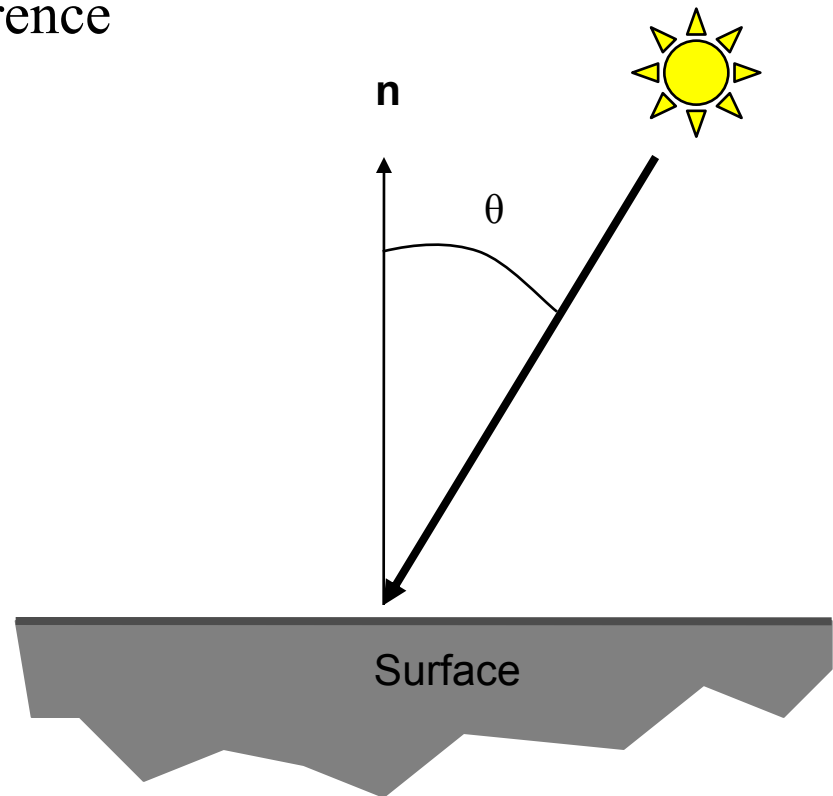
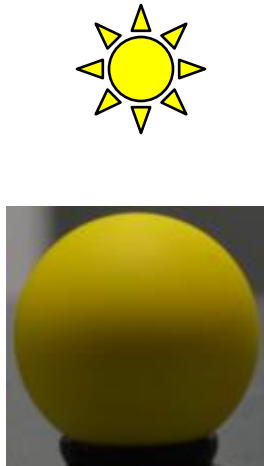
- $1/r^2$  fall-off for isotropic point lights
  - Why? An isotropic point light outputs constant power per solid angle (“into all directions”)
  - Must have same power in all concentric spheres
    - Sphere’s surface area grows with  $r^2 \Rightarrow$  energy obeys  $1/r^2$
- ... but in graphics we often cheat with or ignore this.
  - Why? Ideal point lights are kind of harsh
    - Intensity goes to infinity when you get close – not great!
  - In particular,  $1/(ar^2+br+c)$  is popular



# Incoming Irradiance

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- The amount of light energy received by a surface depends on incoming angle
  - Bigger at normal incidence, even if distance is const.
    - Similar to winter/summer difference
- How exactly?
  - Cos  $\theta$  law
  - Dot product with normal



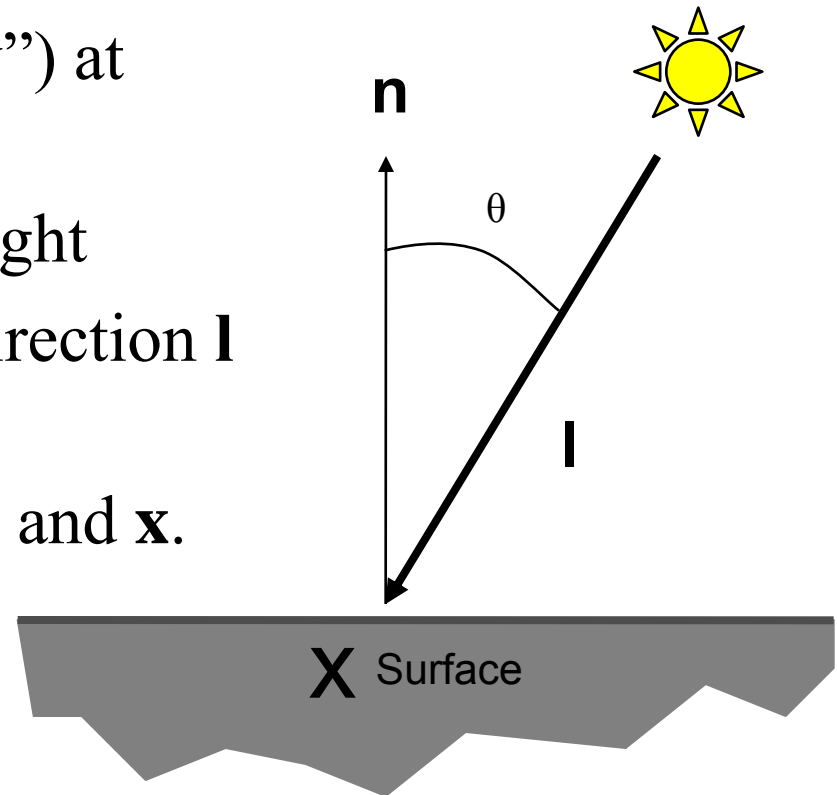


# Incoming Irradiance for Pointlights

- Let's combine this with the  $1/r^2$  fall-off:

$$I_{in} = I_{light} \cos \theta / r^2$$

- $I_{in}$  is the irradiance (“intensity”) at surface point  $\mathbf{x}$
- $I_{light}$  is the “intensity” of the light
- $\theta$  is the angle between light direction  $\mathbf{l}$  and surface normal  $\mathbf{n}$
- $r$  is the distance between light and  $\mathbf{x}$ .



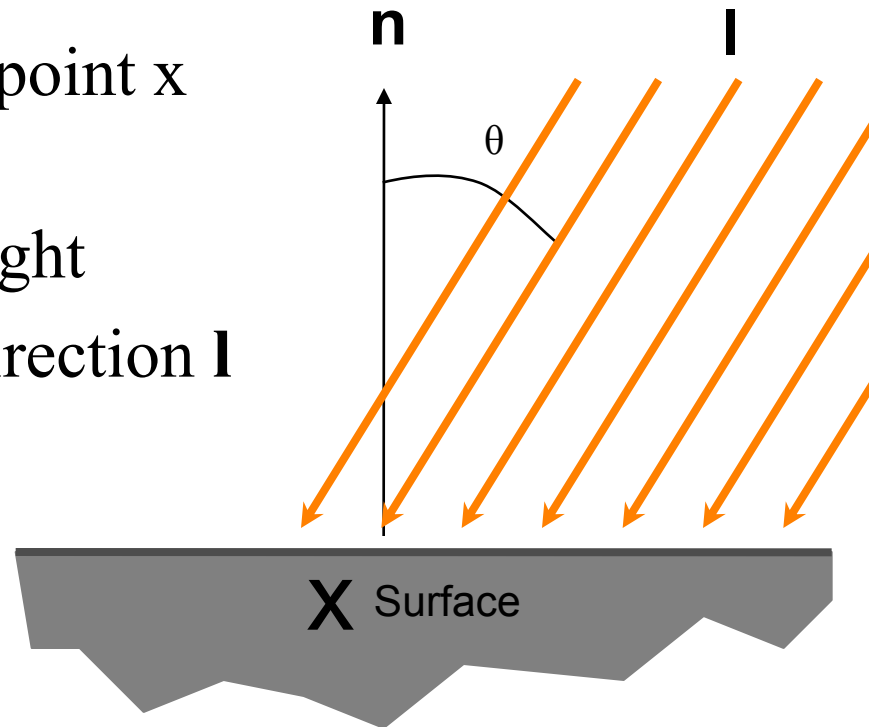
# Directional Lights

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- “Pointlights that are infinitely far”
  - No falloff, just one direction and one intensity

$$I_{in} = I_{light} \cos \theta$$

- $I_{in}$  is the irradiance at surface point  $\mathbf{x}$  from the directional light
- $I_{light}$  is the “intensity” of the light
- $\theta$  is the angle between light direction  $\mathbf{l}$  and surface normal  $\mathbf{n}$ 
  - Only depends on  $\mathbf{n}$ , not  $\mathbf{x}$ !

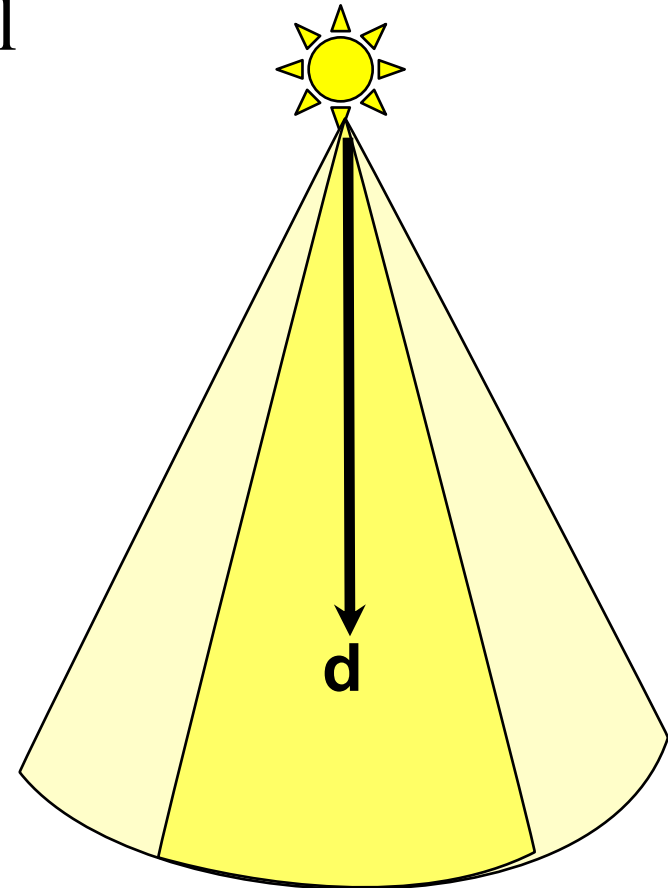




# Spotlights

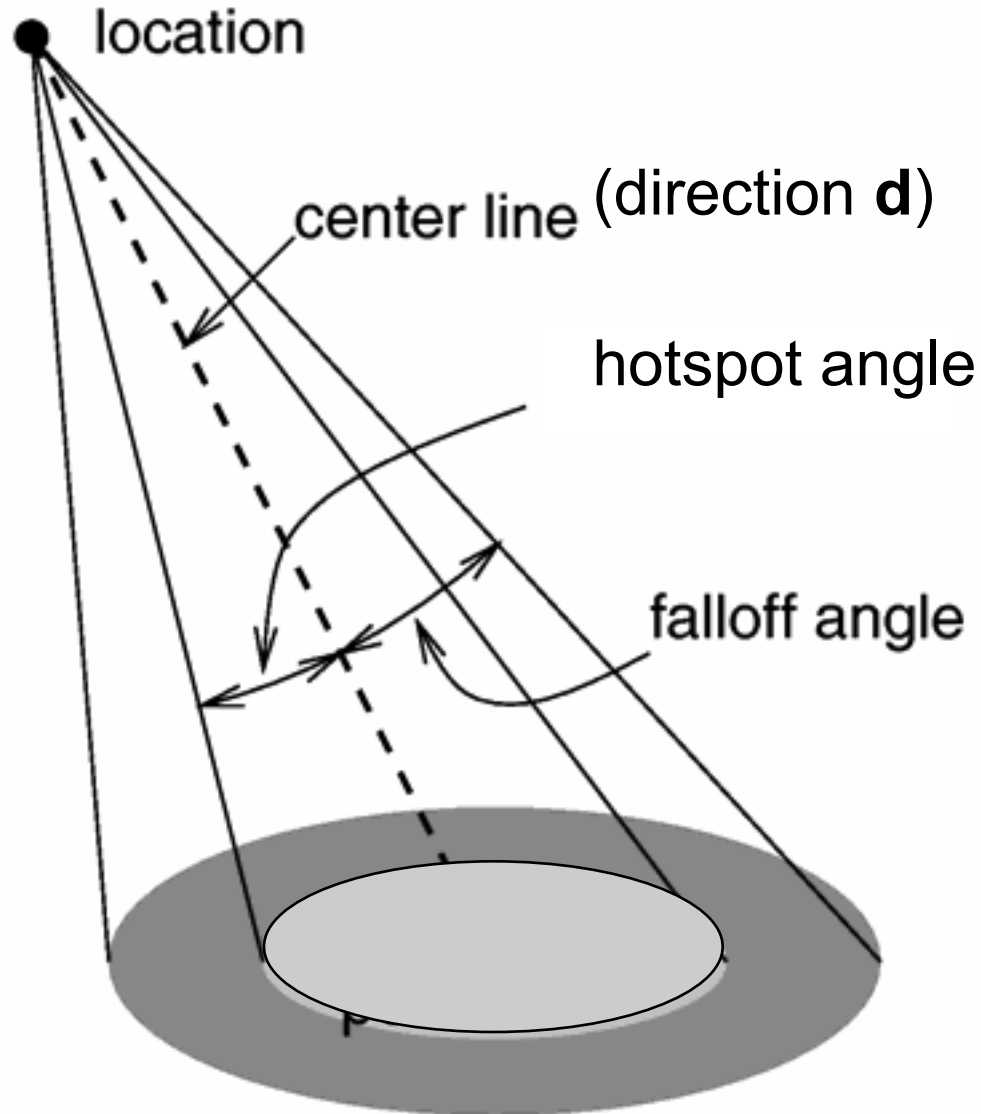
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- Pointlights with non-uniform directional emission
- Usually symmetric about a central direction  $\mathbf{d}$ , with angular falloff
  - Often two angles
    - “Hotspot” angle:  
No attenuation within the central cone
    - “Falloff” angle: Light attenuates from full intensity to zero intensity between the hotspot and falloff angles
- Plus your favorite distance falloff curve



# Spotlight Geometry

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Adapted from  
POVRAY documentation

# Questions?

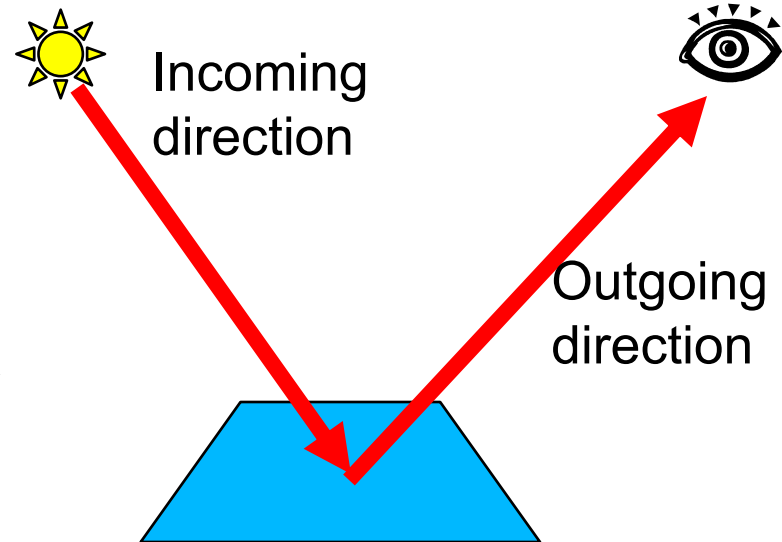
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# Quantifying Reflection – BRDF

- Bidirectional Reflectance Distribution Function
- Ratio of light coming from one direction that gets reflected in another direction
  - Pure reflection, assumes no light scatters into the material
- Focuses on angular aspects, not spatial variation of the material
- **How many dimensions?**



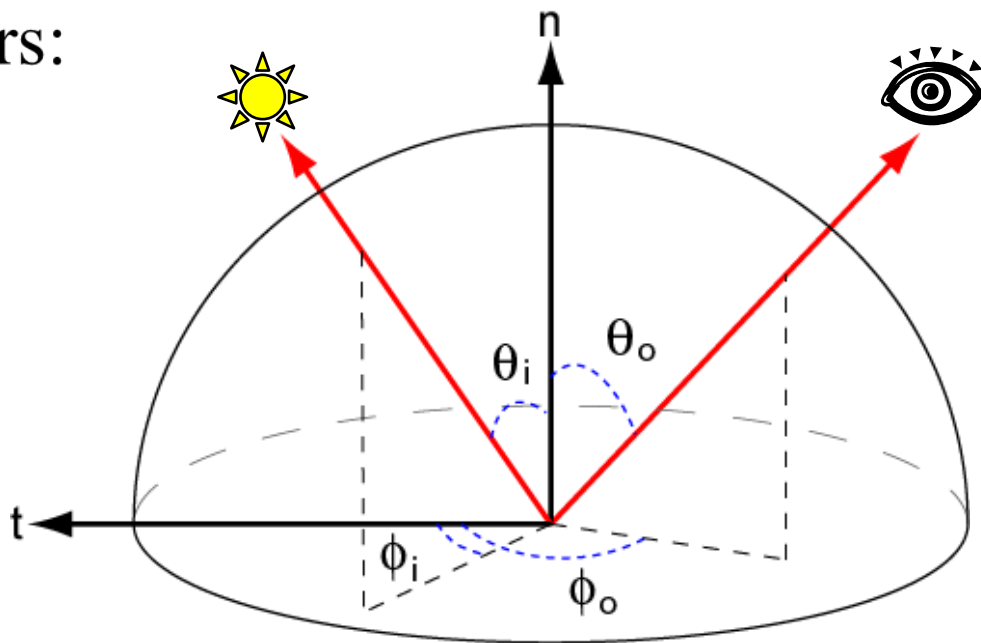
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# BRDF $f_r$

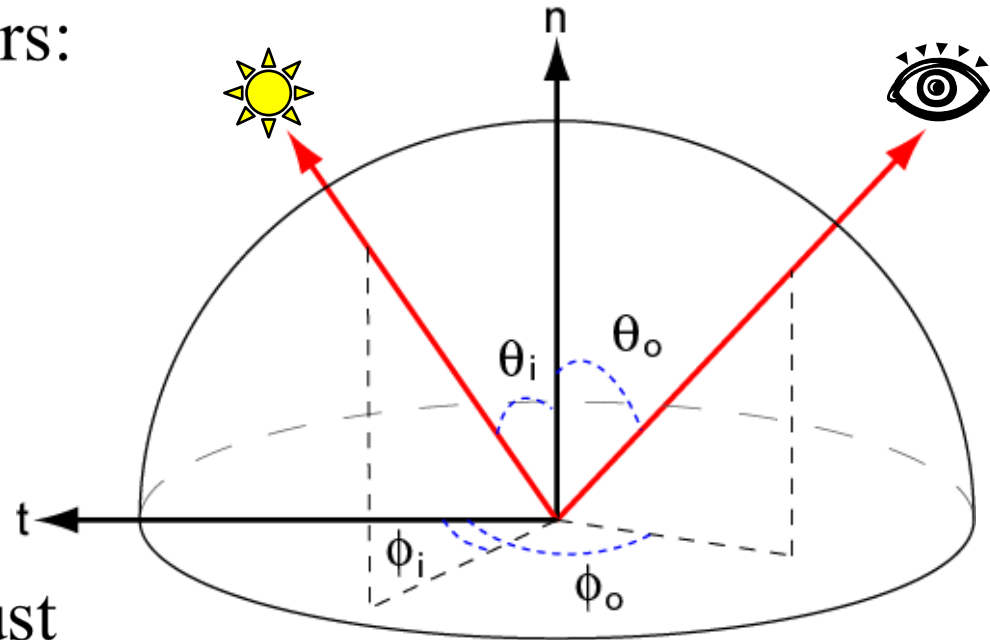
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- Bidirectional Reflectance Distribution Function
  - 4D: 2 angles for each direction
  - $\text{BRDF} = f_r(\theta_i, \phi_i; \theta_o, \phi_o)$
  - Or just two unit vectors:  
 $\text{BRDF} = f_r(\mathbf{l}, \mathbf{v})$ 
    - $\mathbf{l}$  = light direction
    - $\mathbf{v}$  = view direction



# BRDF $f_r$

- Bidirectional Reflectance Distribution Function
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  - Or just two unit vectors:  
 $\text{BRDF} = f_r(\mathbf{l}, \mathbf{v})$ 
    - $\mathbf{l}$  = light direction
    - $\mathbf{v}$  = view direction
  - The BRDF is aligned with the surface; the vectors  $\mathbf{l}$  and  $\mathbf{v}$  must be in a local coordinate system

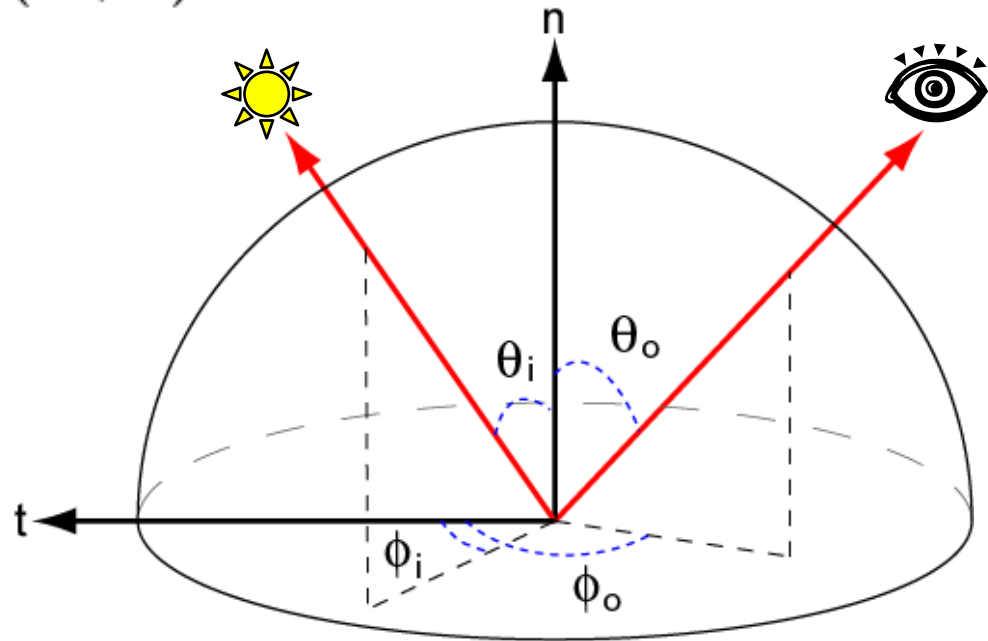


# BRDF $f_r$

- Relates incident irradiance from every direction to outgoing light.  
How?

$$I_{\text{out}}(\mathbf{v}) = I_{\text{in}}(\mathbf{l}) f_r(\mathbf{v}, \mathbf{l})$$

**$\mathbf{l}$  = light direction  
(incoming)**  
 **$\mathbf{v}$  = view direction  
(outgoing)**





# BRDF $f_r$

- Relates incident irradiance from every direction to outgoing light.  
How?

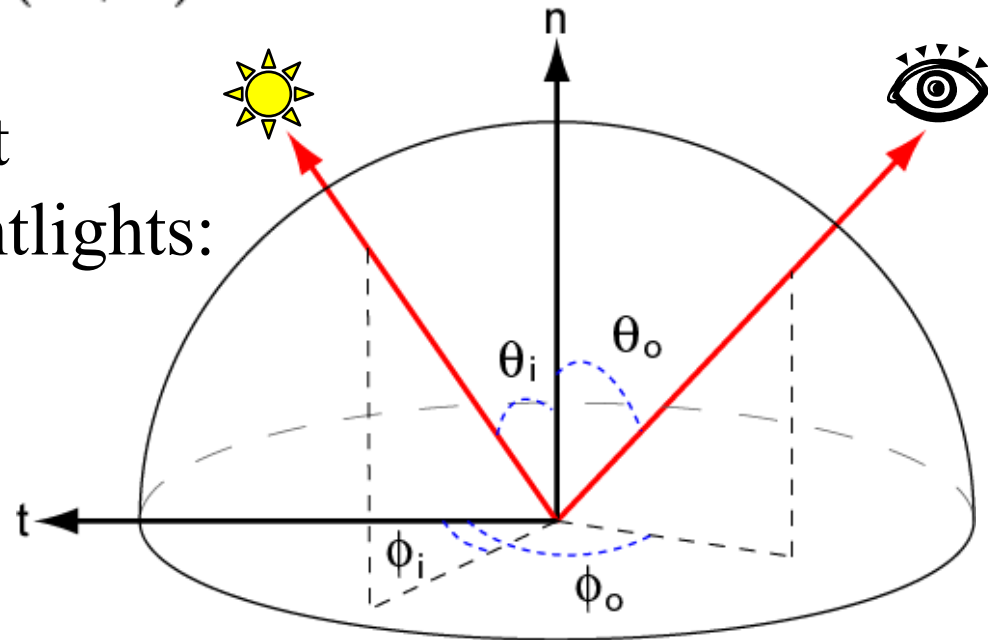
$$I_{\text{out}}(\mathbf{v}) = I_{\text{in}}(\mathbf{l}) f_r(\mathbf{v}, \mathbf{l})$$

- Let's combine with what we know already of pointlights:

$$I_{\text{out}}(\mathbf{v}) =$$

$$\frac{I_{\text{light}} \cos \theta_i}{r^2} f_r(\mathbf{v}, \mathbf{l})$$

**$\mathbf{l}$  = light direction  
(incoming)**  
 **$\mathbf{v}$  = view direction  
(outgoing)**

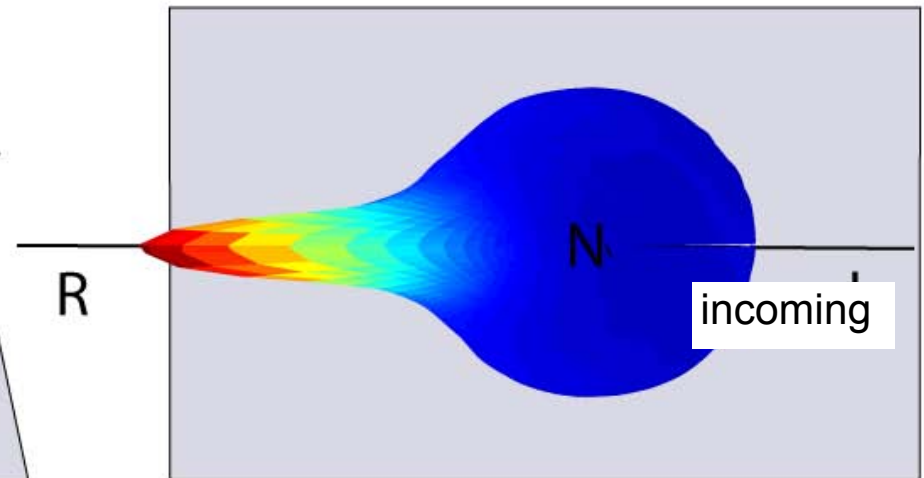
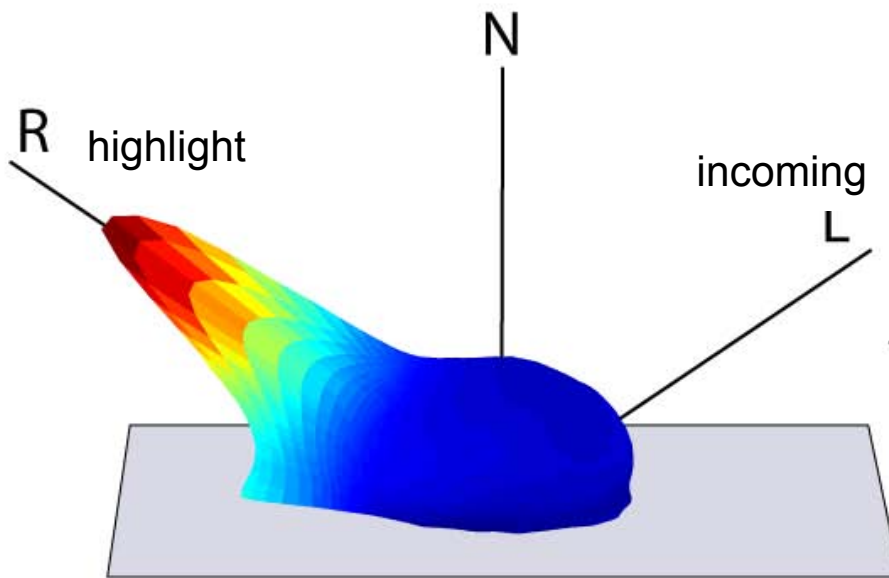


# 2D Slice at Constant Incidence

- For a fixed incoming direction, view dependence is a 2D spherical function
  - Here a moderate specular component



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Example: Plot of "PVC" BRDF at 55° incidence

Demo

# Isotropic vs. Anisotropic

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- When keeping  $\mathbf{l}$  and  $\mathbf{v}$  fixed, if rotation of surface around the normal does not change the reflection, the material is called isotropic
- Surfaces with strongly oriented microgeometry elements are anisotropic
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet

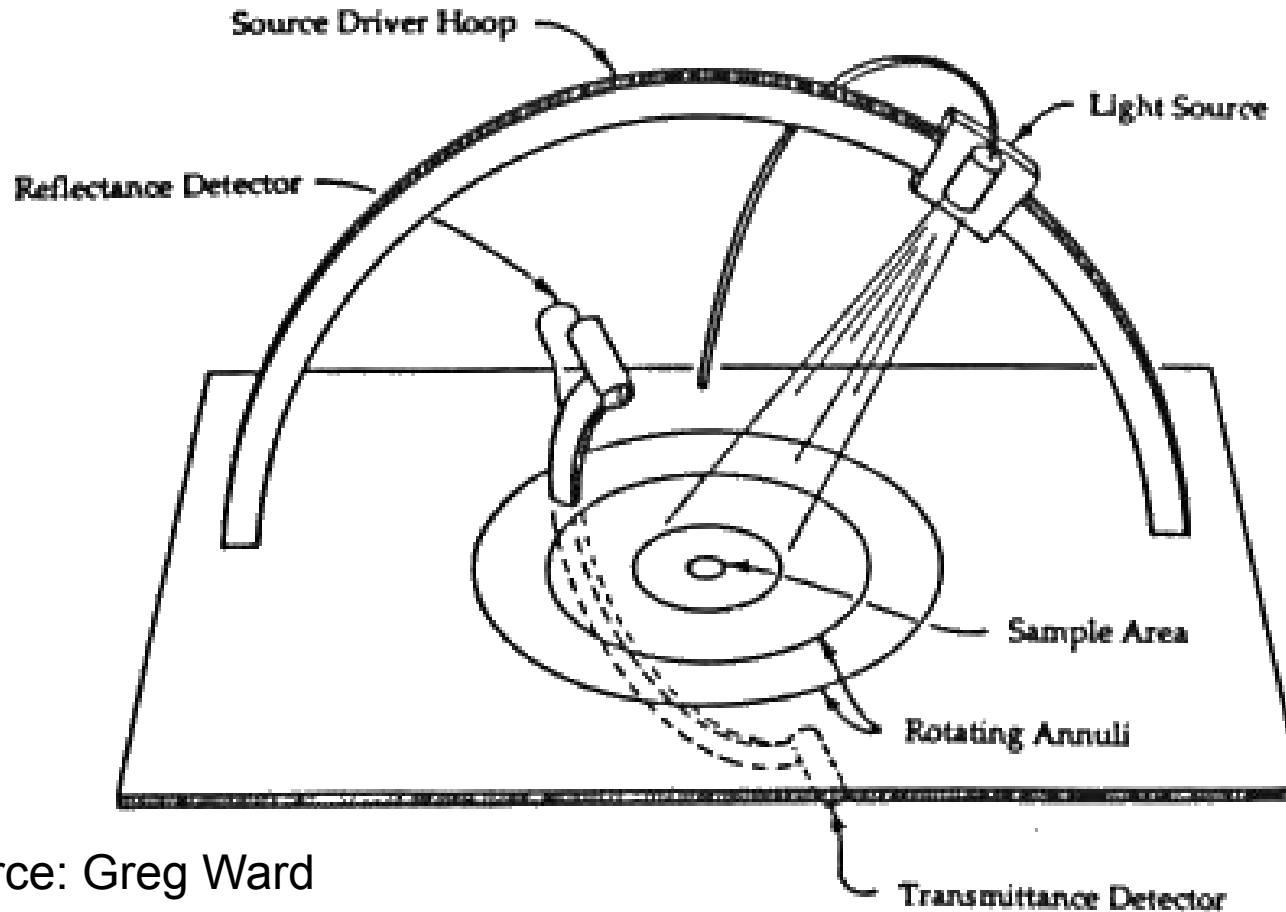


Westin et.al 92

Demo

# How do we obtain BRDFs?

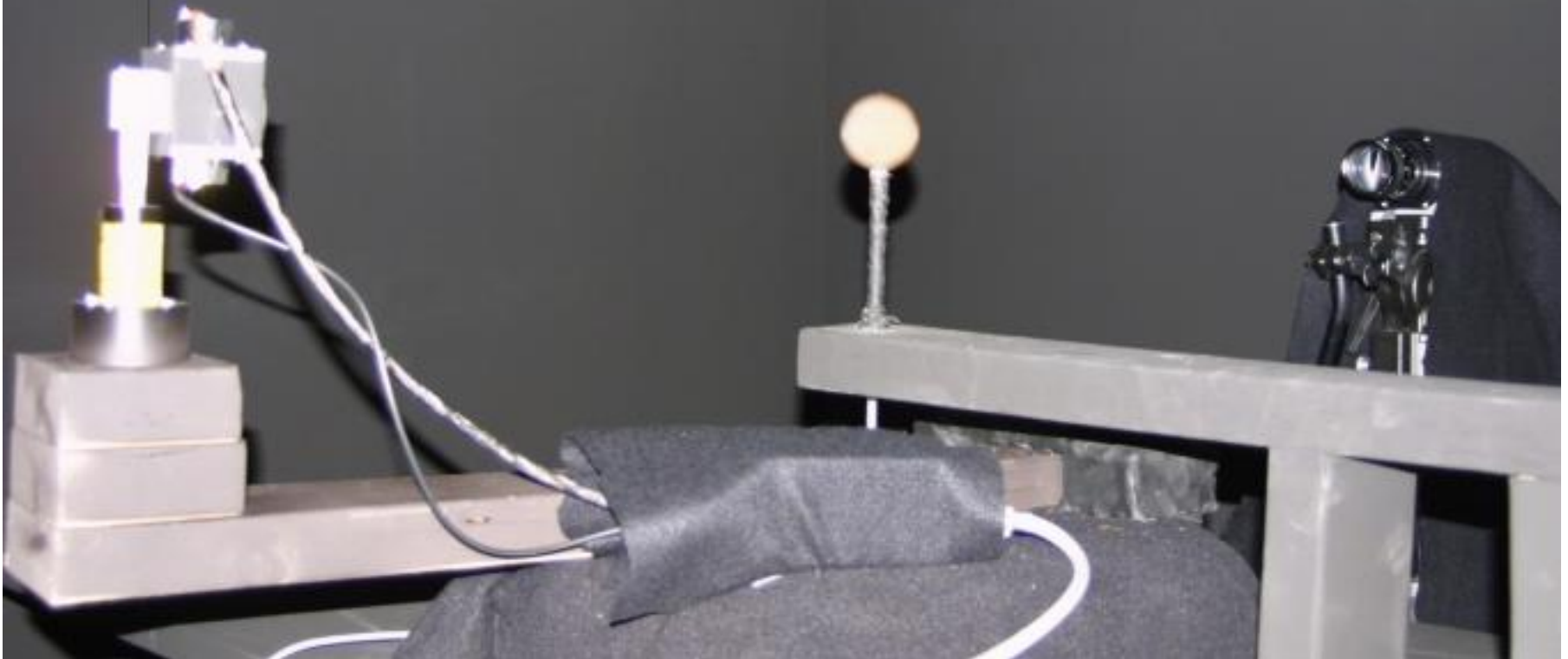
- One possibility: Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

# How Do We Obtain BRDFs?

- Another possibility: Take pictures of spheres coated with material, rotate light around a 1D arc
  - This gives 3DOF => isotropic materials only





# Parametric BRDFs

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- BRDFs can be measured from real data
  - But tabulated 4D data is too cumbersome for most uses
- Therefore, parametric BRDF models represent the relationship between incident and outgoing light by some mathematical formula
  - The appearance can then be tuned by setting parameters
    - “Shininess”, “anisotropy”, etc.
  - Physically-based or Phenomenological
  - They can model with measured data (examples later)
- Popular models: Diffuse, Blinn-Phong, Cook-Torrance, Lafortune, Ward, Oren-Nayar, etc.

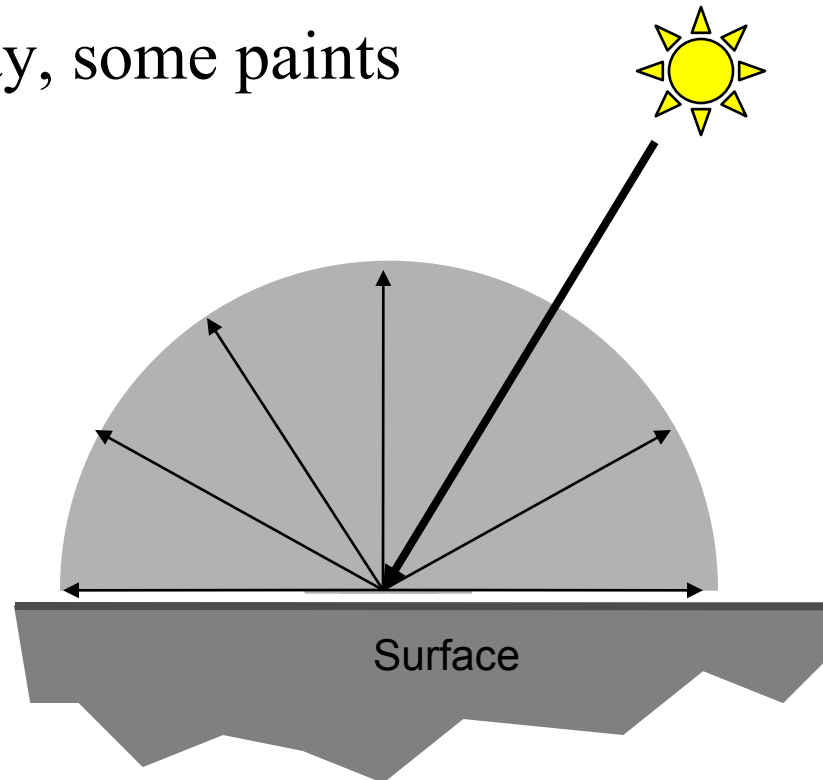
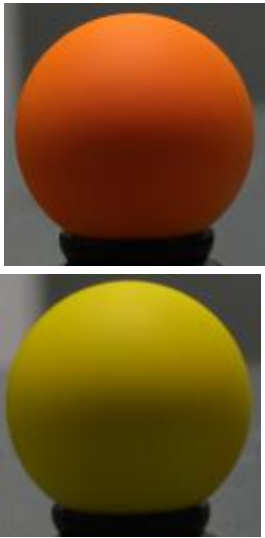
# Questions?

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# Ideal Diffuse Reflectance

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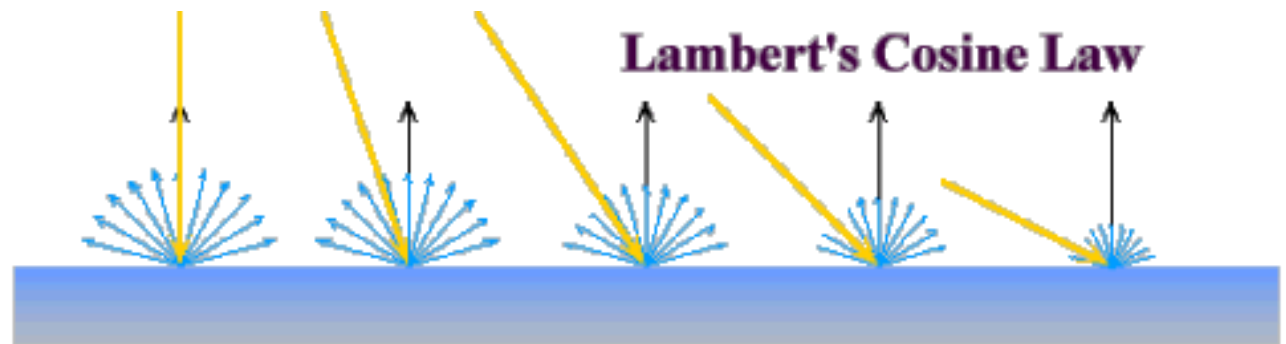
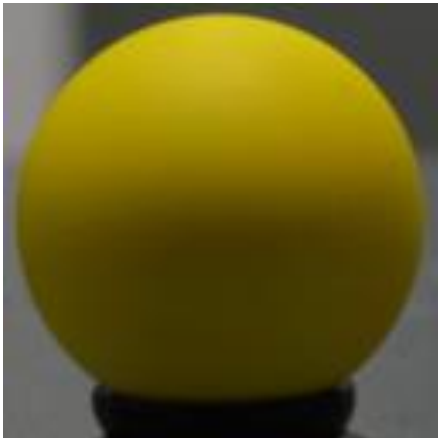
- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints



# Ideal Diffuse Reflectance

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- Ideal diffuse reflectors reflect light according to Lambert's cosine law
  - The reflected light varies with cosine even if distance to light source is kept constant

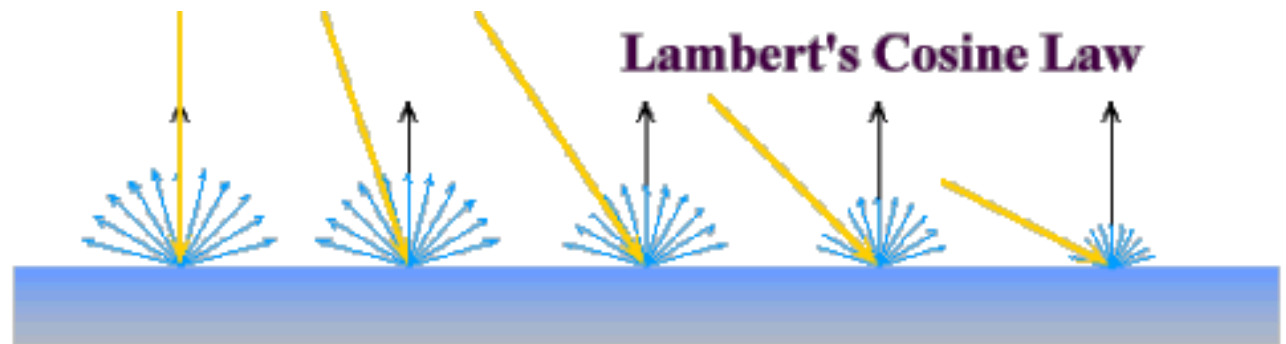
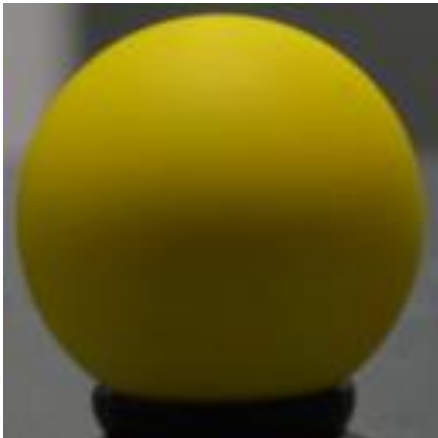


# Ideal Diffuse Reflectance

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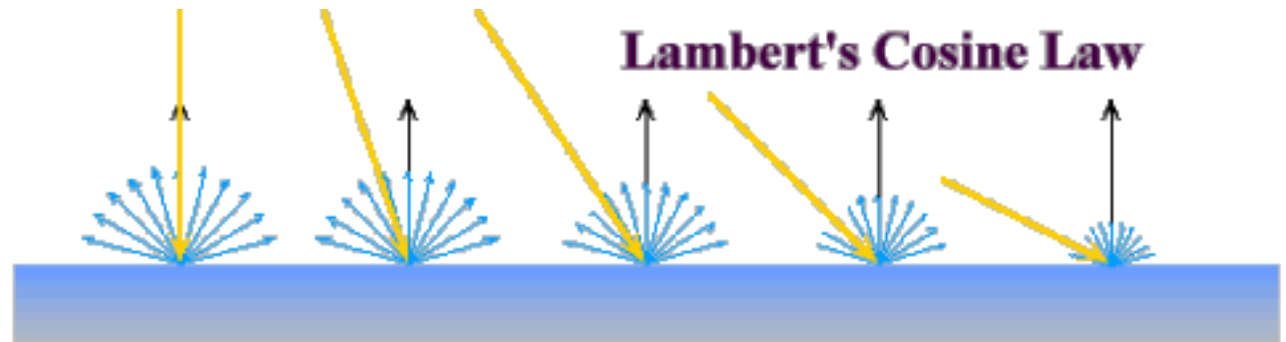
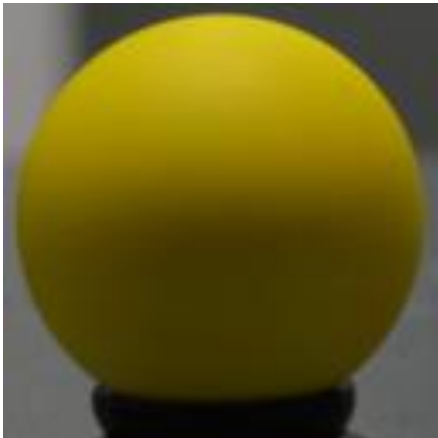
- Ideal diffuse reflectors reflect light according to Lambert's cosine law
  - The reflected light varies with cosine even if distance to light source is kept constant

**Remembering that incident irradiance depends on cosine, what is the BRDF of an ideally diffuse surface?**



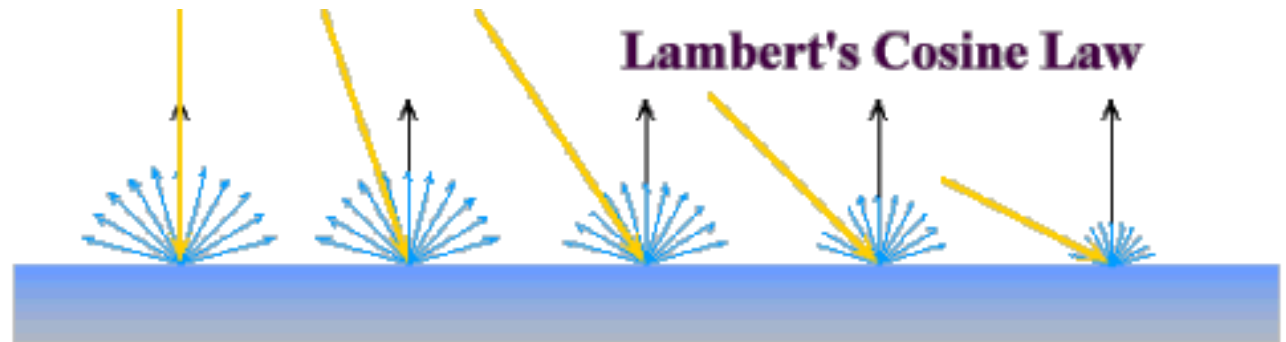
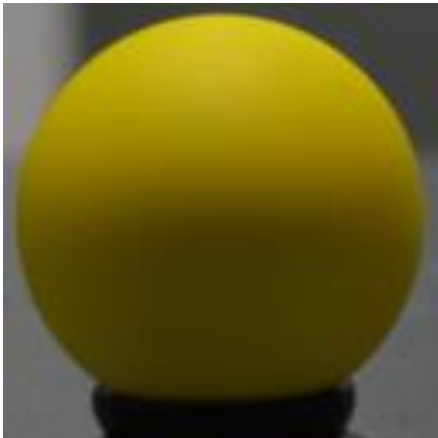
# Ideal Diffuse Reflectance

- The ideal diffuse BRDF is a constant  $f_r(\mathbf{l}, \mathbf{v}) = \text{const.}$ 
  - What constant  $\rho/\pi$ , where  $\rho$  is the *albedo*
    - Coefficient between 0 and 1 that says what fraction is reflected
  - Usually just called “diffuse color”  $k_d$
  - You have already implemented this by taking dot products with the normal and multiplying by the “color”!



# Ideal Diffuse Reflectance

- This is the simplest possible parametric BRDF
  - One parameter:  $k_d$ 
    - (One for each RGB channel)

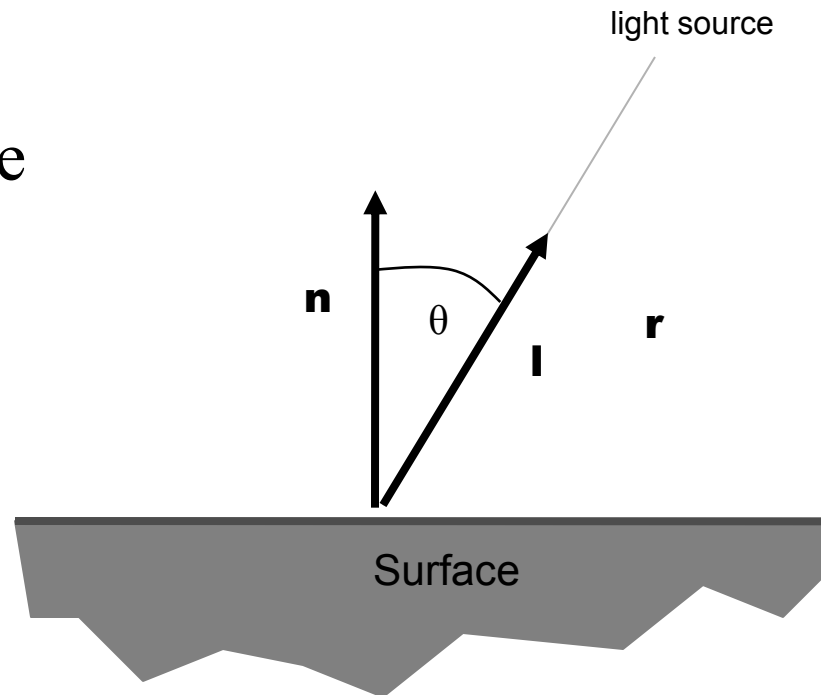




# Ideal Diffuse Reflectance Math

- Single Point Light Source
  - $k_d$ : diffuse coefficient (color)
  - $\mathbf{n}$ : Surface normal.
  - $\mathbf{l}$ : Light direction.
  - $L_i$ : Light intensity
  - $r$ : Distance to source
  - $L_o$ : Shaded color

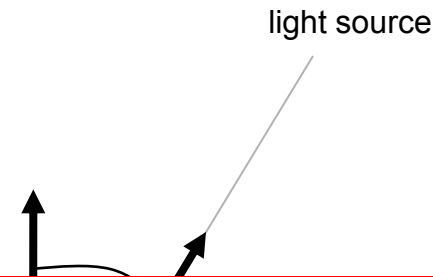
$$L_o = k_d \max(0, \mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



# Ideal Diffuse Reflectance Math

- Single Point Light Source
  - $k_d$ : diffuse coefficient (color)
  - $\mathbf{n}$ : Surface normal.
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  - $L_i$ : Light intensity
  - $r$ : Distance to source
  - $L_o$ : Shaded color

$$L_o = k_d \max(0, \mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



Do not forget  
to normalize  
your  $\mathbf{n}$  and  $\mathbf{l}$ !

**We do not want light from below the surface!** From now on we always assume (on this lecture) that **dot products are clamped to zero** and skip writing out the `max()`.

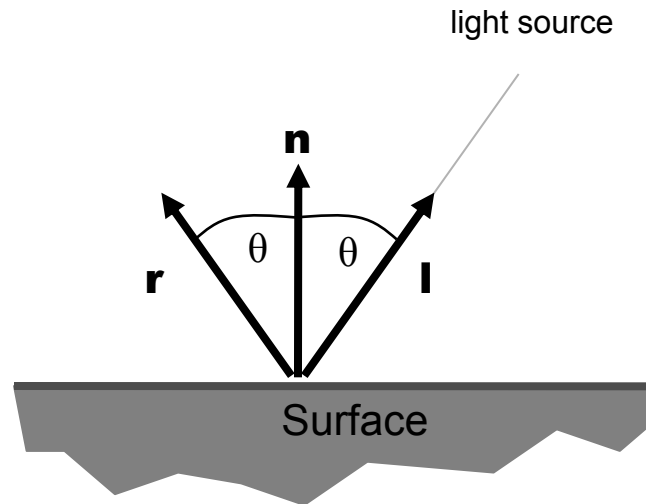
# Questions?

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# Ideal Specular Reflectance

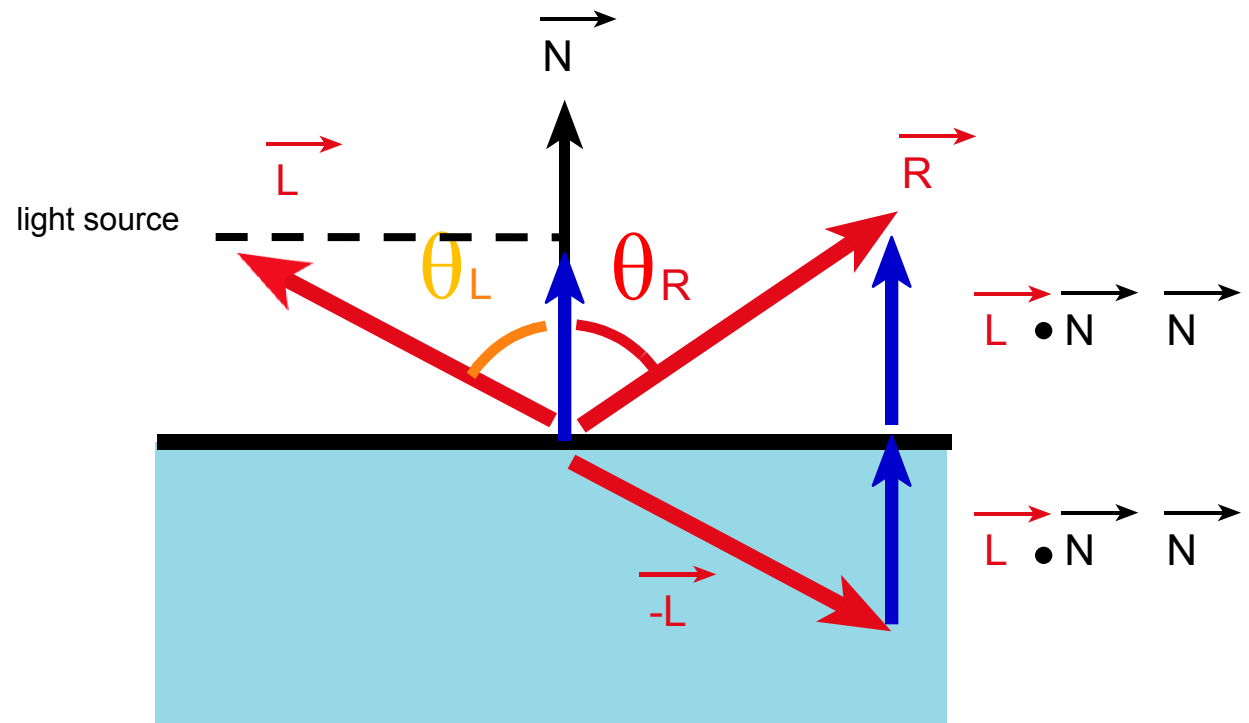
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- Reflection is only at mirror angle
- View dependent
  - Microscopic surface elements are usually oriented in the same direction as the surface itself.
  - Examples: mirrors, highly polished metals.



# Recap: How to Get Mirror Direction

- Reflection angle = light angle
  - Both  $\mathbf{R}$  &  $\mathbf{L}$  have to lie on one plane
- $\mathbf{R} = -\mathbf{L} + 2(\mathbf{L} \cdot \mathbf{N})\mathbf{N}$



# Ideal Specular BRDF

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- Light **only** reflects to the mirror direction
- A Dirac delta multiplied by a specular coefficient  $k_s$
- Not very useful for point lights, only for reflections of other surfaces
  - Why? You cannot really see a mirror reflection of an infinitely small light!

# Non-ideal Reflectors

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- Real glossy materials usually deviate significantly from ideal mirror reflectors
  - Highlight is blurry
- They are not ideal diffuse surfaces either ...

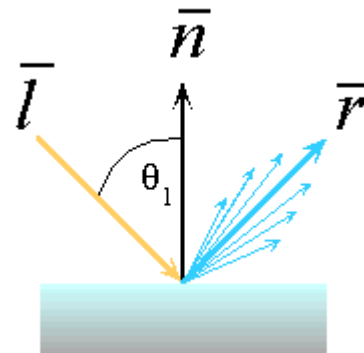
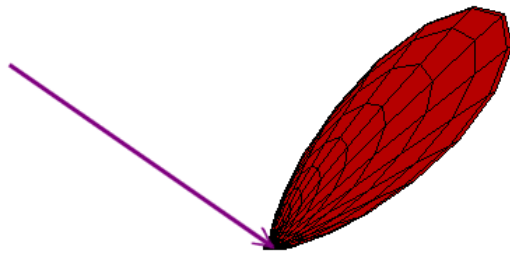




# Non-ideal Reflectors

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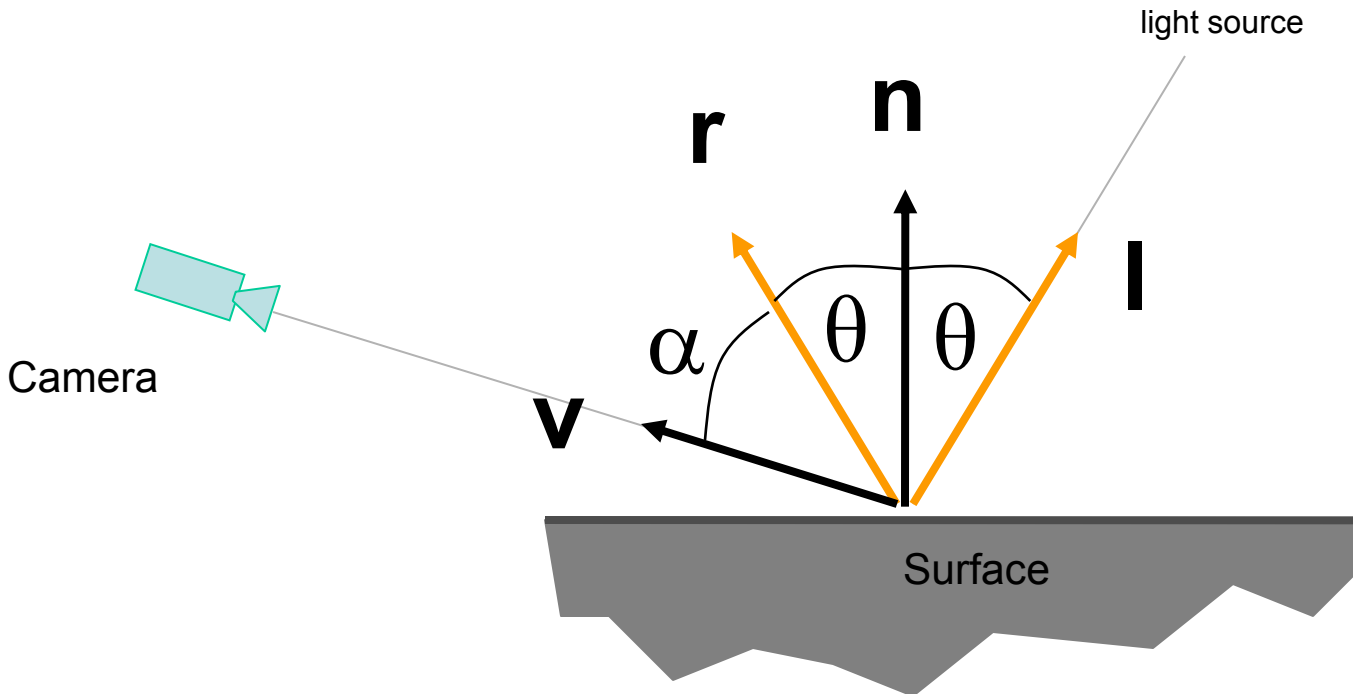
- Simple Empirical Reasoning for Glossy Materials
  - We expect most of the reflected light to travel in the direction of the ideal mirror ray.
  - However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray.
  - As we move farther and farther, in the angular sense, from the reflected ray, we expect to see less light reflected.



# The Phong Specular Model

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- How much light is reflected?
  - Depends on the angle  $\alpha$  between the ideal reflection direction  $\mathbf{r}$  and the viewer direction  $\mathbf{v}$ .

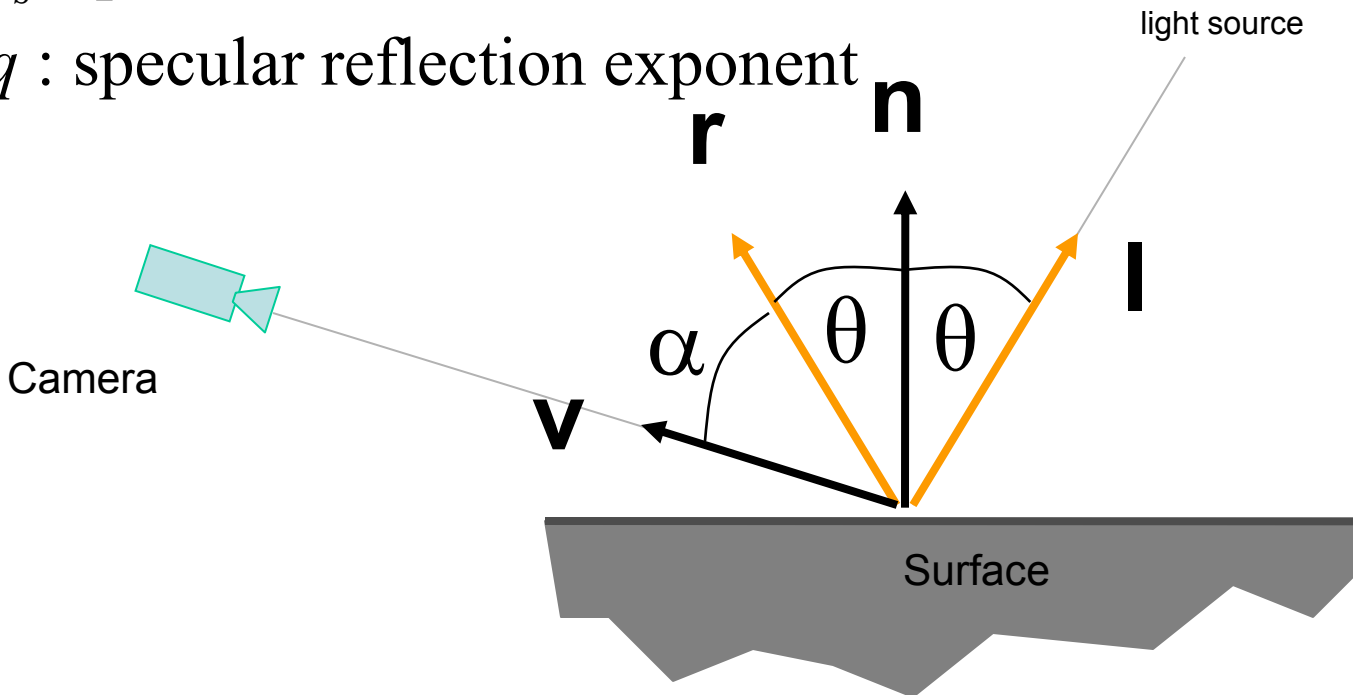


# The Phong Specular Model

$$L_o = k_s (\cos \alpha)^q \frac{L_i}{r^2} = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{L_i}{r^2}$$

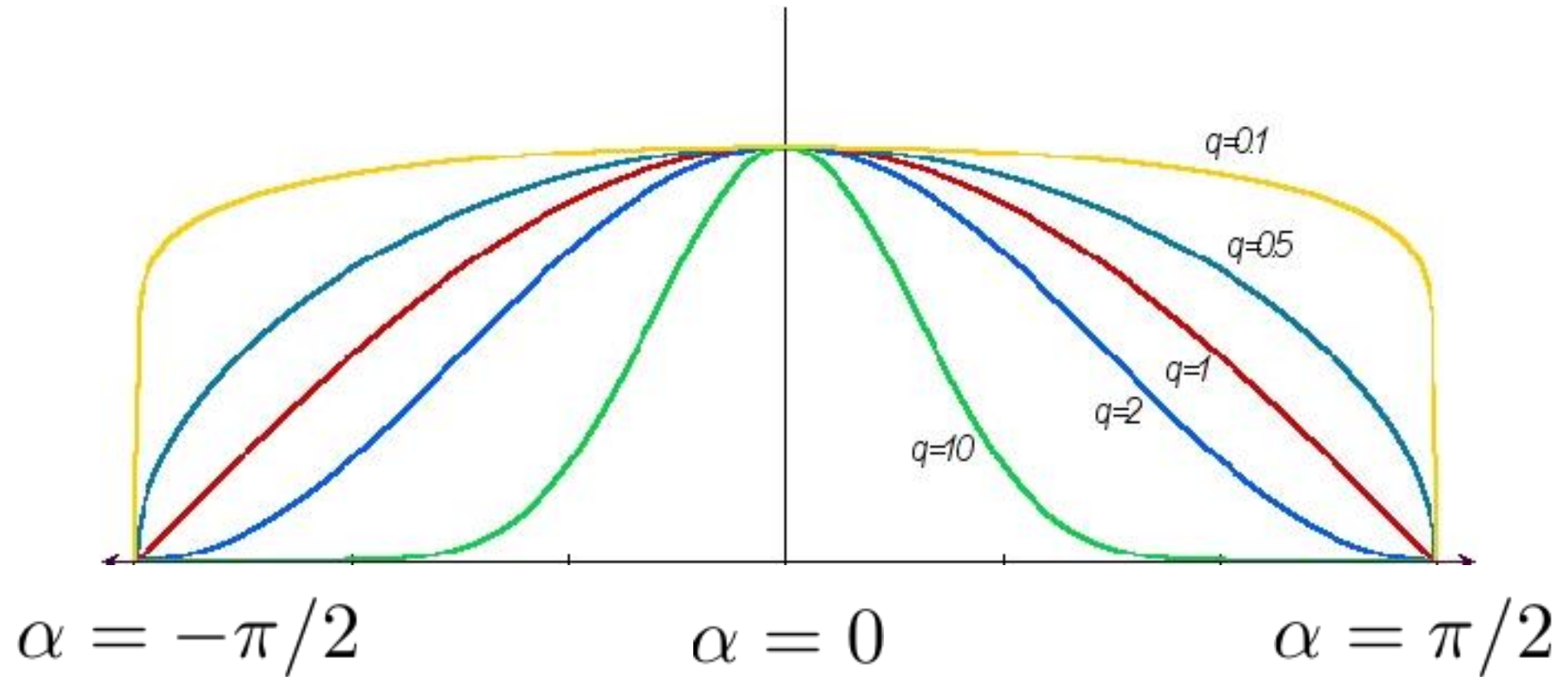
- Parameters

- $k_s$ : specular reflection coefficient
- $q$ : specular reflection exponent



# The Phong Model

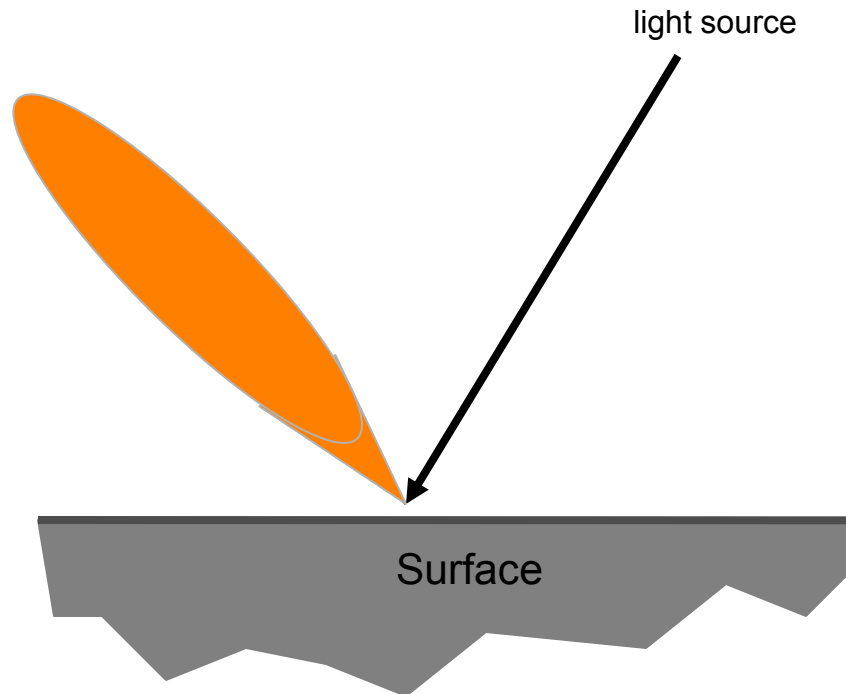
- Effect of  $q$  – the specular reflection exponent



# Terminology: Specular Lobe

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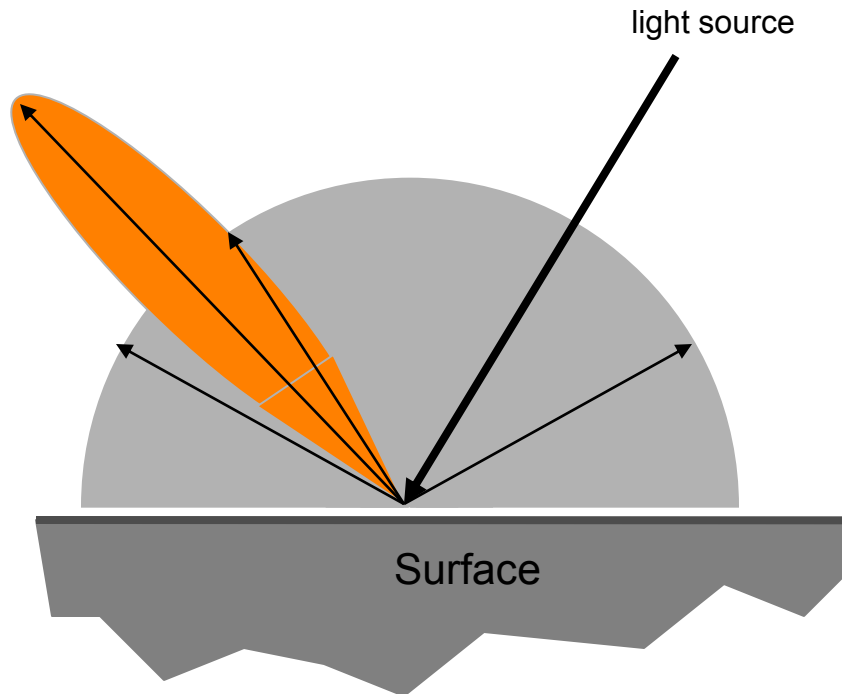
- The specular reflection distribution is usually called a “lobe”
  - For Phong, its shape is  $(\mathbf{r} \cdot \mathbf{v})^q$



# The Complete Phong Model

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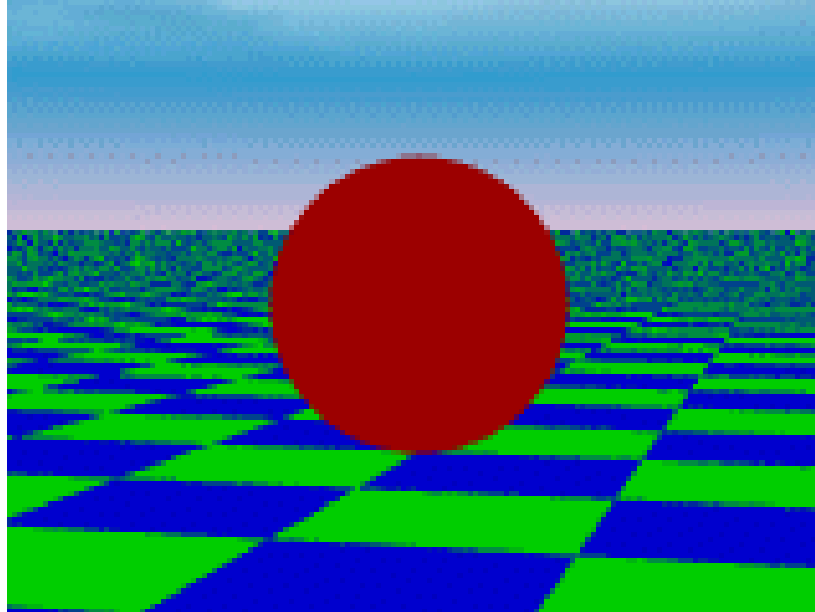
- Sum of three components:  
ideal diffuse reflection +  
specular reflection +  
“ambient”.



# Ambient Illumination

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- Represents the reflection of all indirect illumination.
- This is a total hack!
- Avoids the complexity of indirect (“global”) illumination



# Putting It All Together

- Phong Illumination Model

$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$

Phong	$\rho_{\text{ambient}}$	$\rho_{\text{diffuse}}$	$\rho_{\text{specular}}$	$\rho_{\text{total}}$
$\phi_i = 60^\circ$				
$\phi_i = 25^\circ$				
$\phi_i = 0^\circ$				



# Putting It All Together

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- Phong Illumination Model

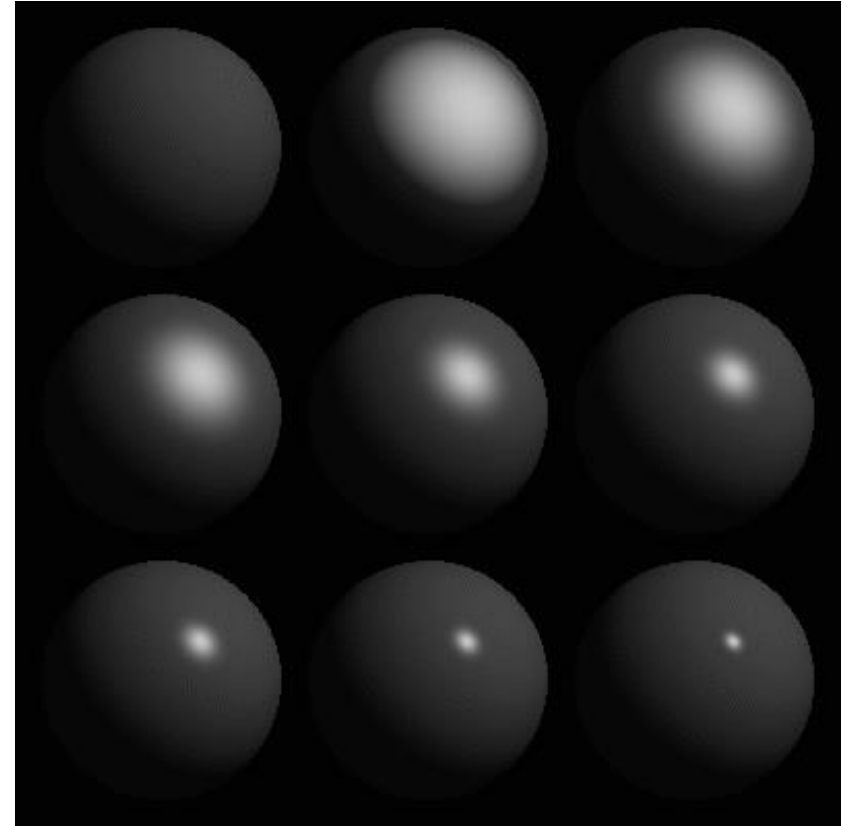
$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$

- Is it physically based?
  - No, does not even conserve energy, may well reflect more energy than what goes in
  - Furthermore, it does not even conform to the BRDF model directly (we are taking the proper cosine for diffuse, but not for specular)
  - And ambient was a total hack

# Phong Examples

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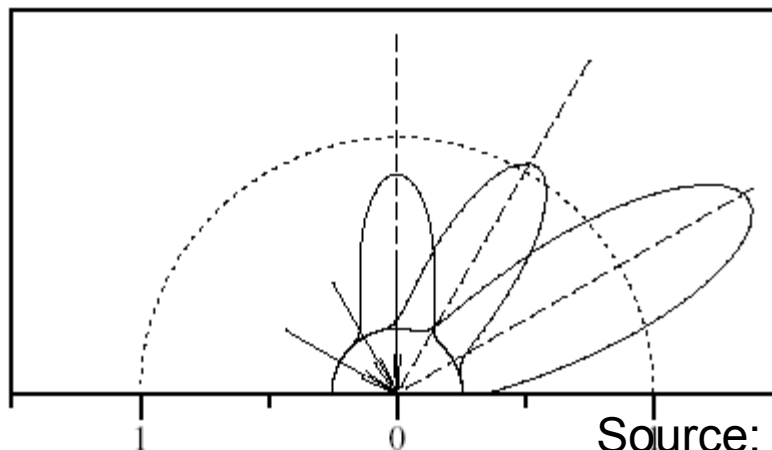
- The spheres illustrate specular reflections as the direction of the light source and the exponent  $q$  (amount of shininess) is varied.



$$L_o = \left[ k_a + k_d (\mathbf{n} \cdot \mathbf{l}) + k_s (\mathbf{v} \cdot \mathbf{r})^q \right] \frac{L_i}{r^2}$$

# Fresnel Reflection

- Increasing specularity near grazing angles.
  - Most BRDF models account for this.



Source: Lafortune et al. 97

# Questions?

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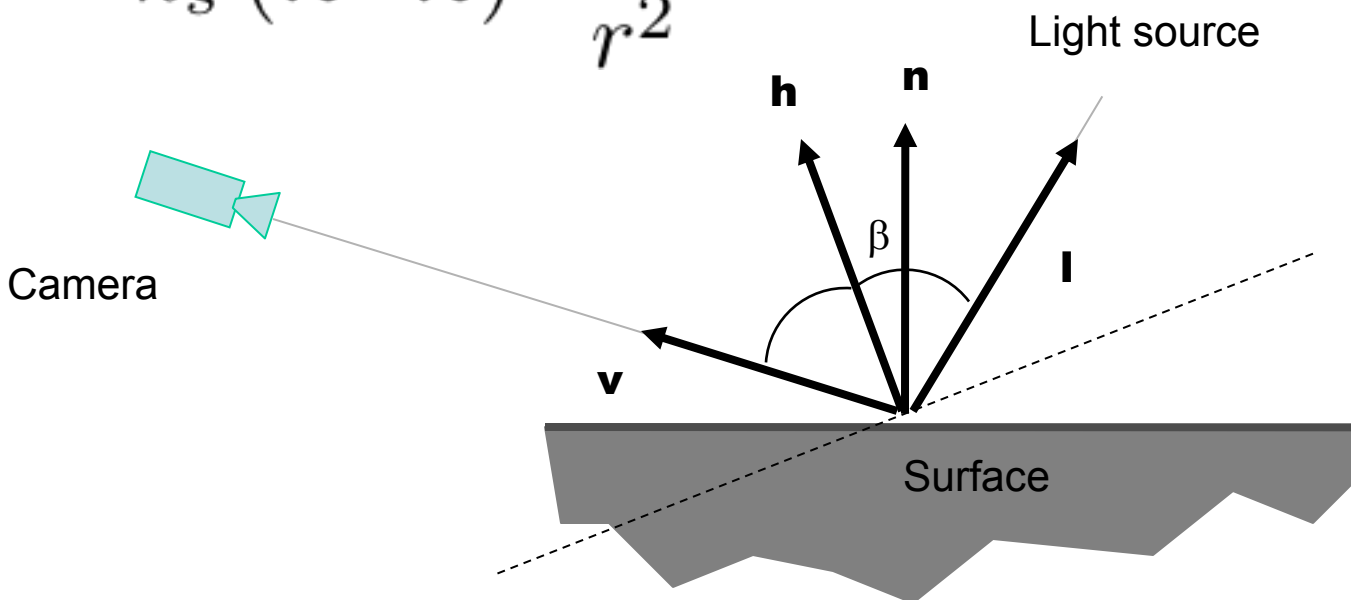
# Blinn-Torrance Variation of Phong

- Uses the “halfway vector”  $\mathbf{h}$  between  $\mathbf{l}$  and  $\mathbf{v}$ .

$$L_o = k_s \cos(\beta)^q \frac{L_i}{r^2}$$

$$= k_s (\mathbf{n} \cdot \mathbf{h})^q \frac{L_i}{r^2}$$

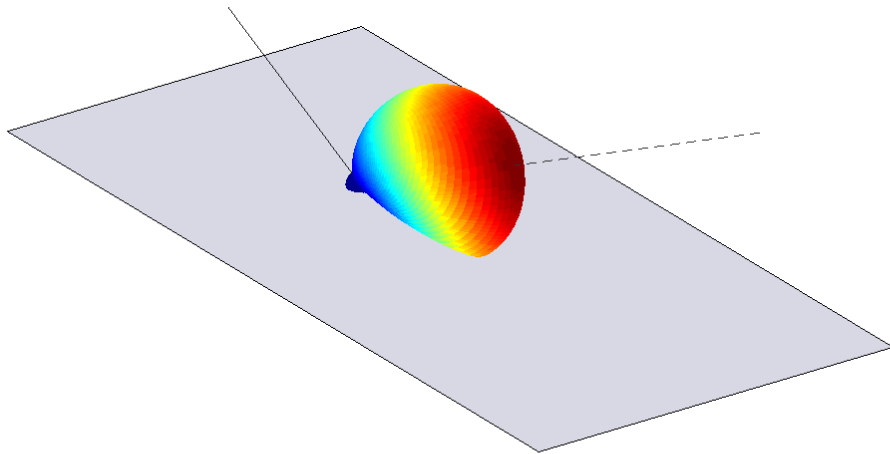
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$



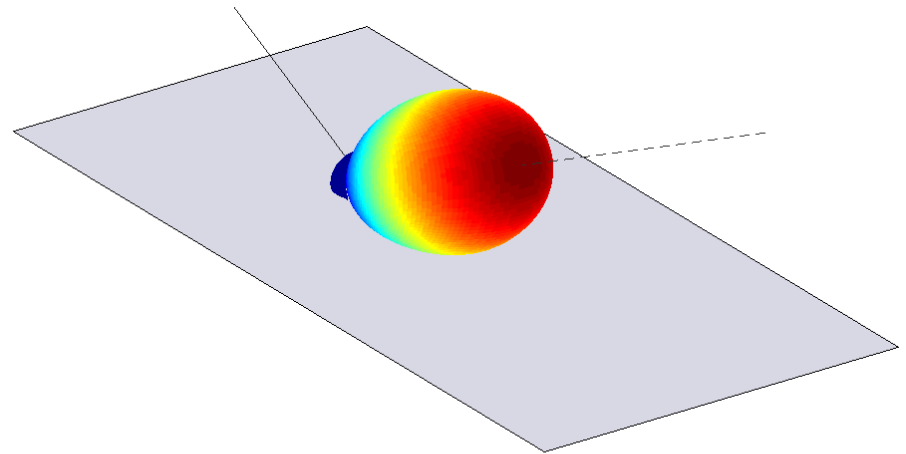
# Lobe Comparison

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- Half vector lobe
  - Gradually narrower when approaching grazing
- Mirror lobe
  - Always circular



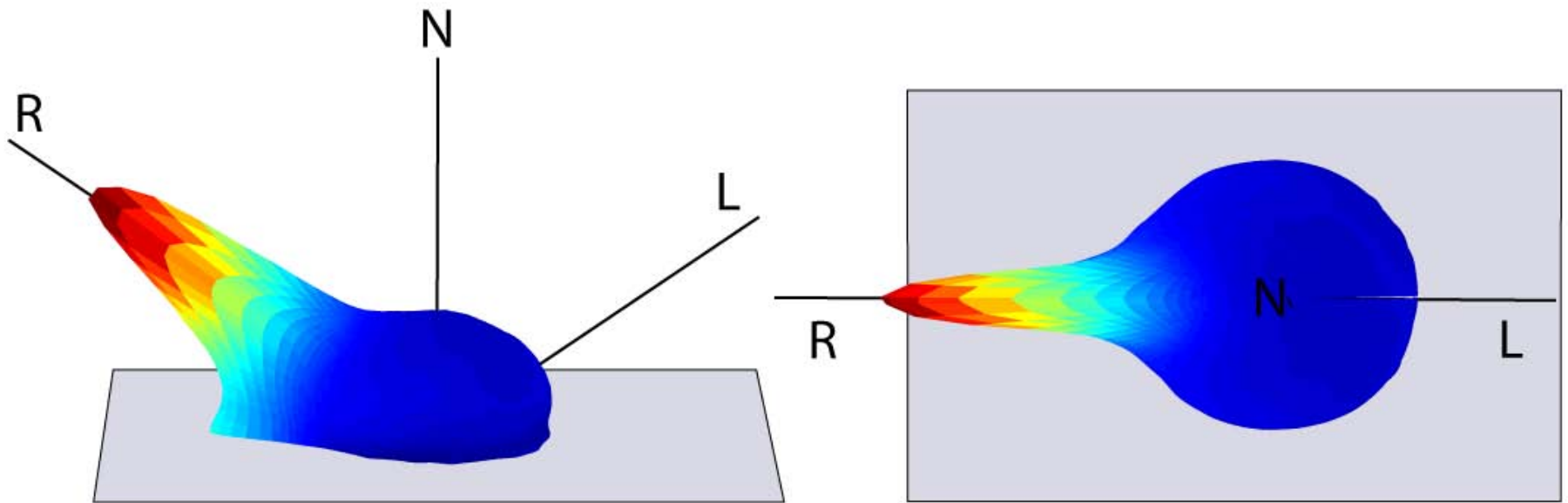
Half vector lobe



Mirror lobe

# Half Vector Lobe is Better

- More consistent with what is observed in measurements (Ngan, Matusik, Durand 2005)



Courtesy of Mitsubishi Electric Research Laboratories, Inc. Used with permission.

Example: Plot of "PVC" BRDF at 55° incidence

# Questions?

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# Microfacet Theory

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- Example
  - Think of water surface as lots of tiny mirrors (microfacets)
  - “Bright” pixels are
    - Microfacets aligned with the vector between sun and eye
    - But not the ones in shadow
    - And not the ones that are occluded

Image of sunset removed due to copyright restrictions.

# Microfacet Theory

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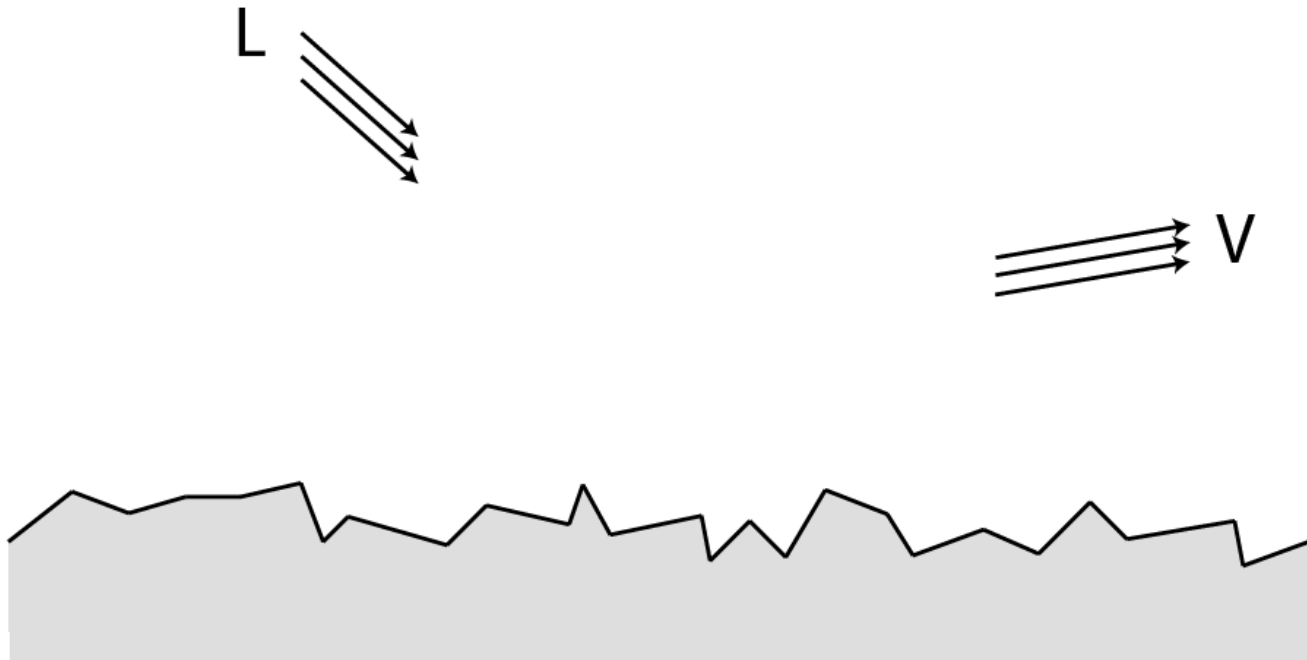
- Model surface by tiny mirrors  
[Torrance & Sparrow 1967]



# Microfacet Theory

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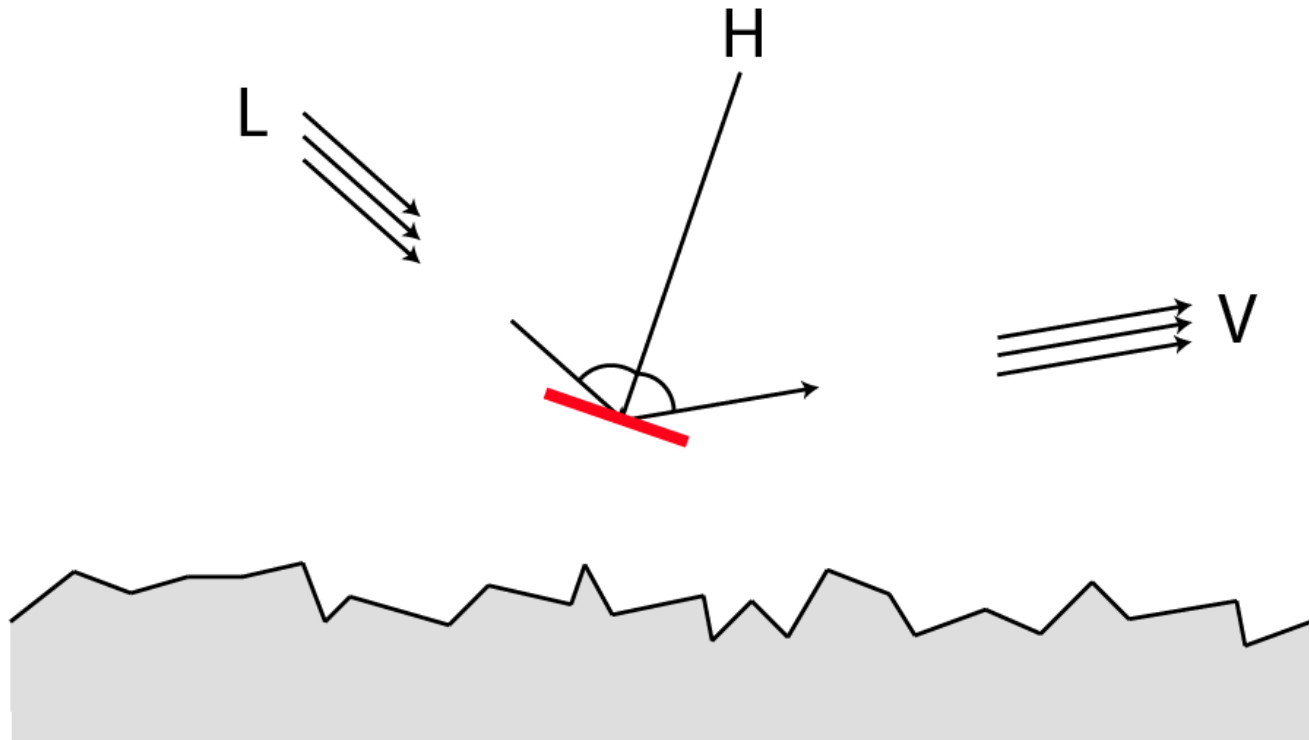
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$



# Microfacet Theory

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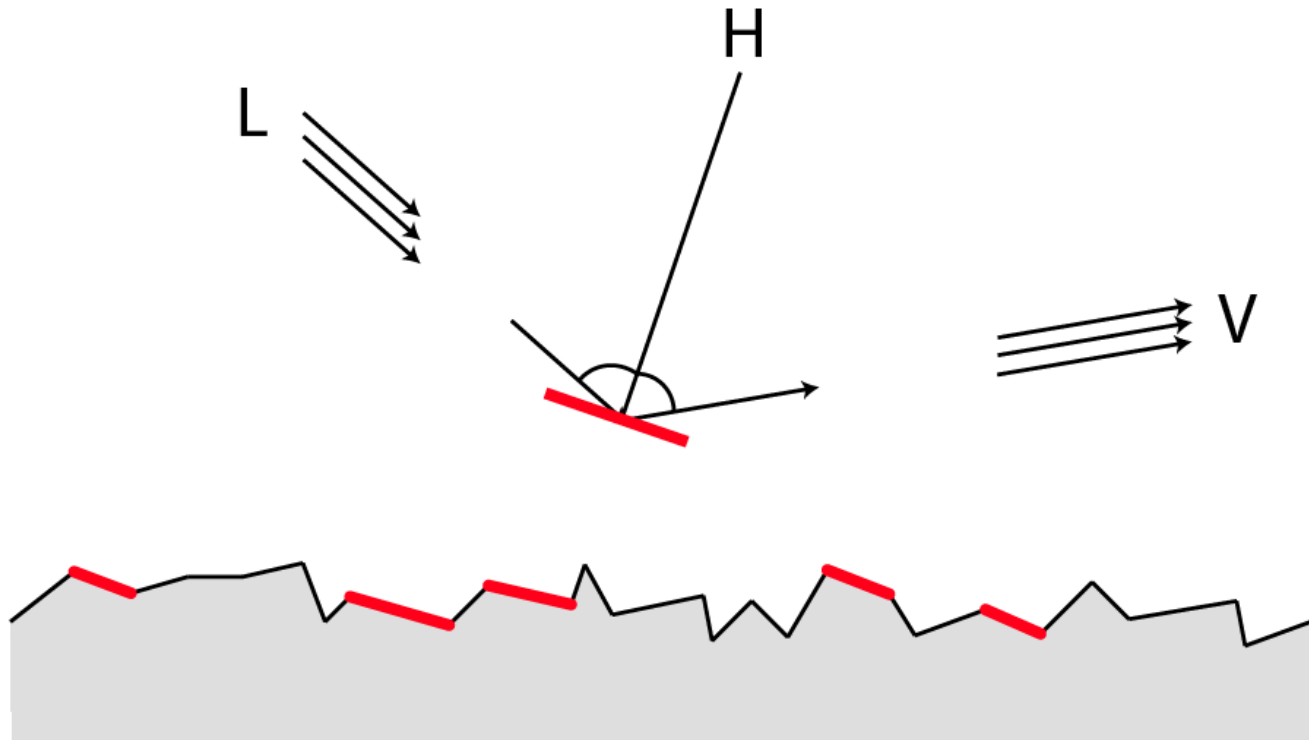
- Value of BRDF at  $(L, V)$  is a product of
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# Microfacet Theory

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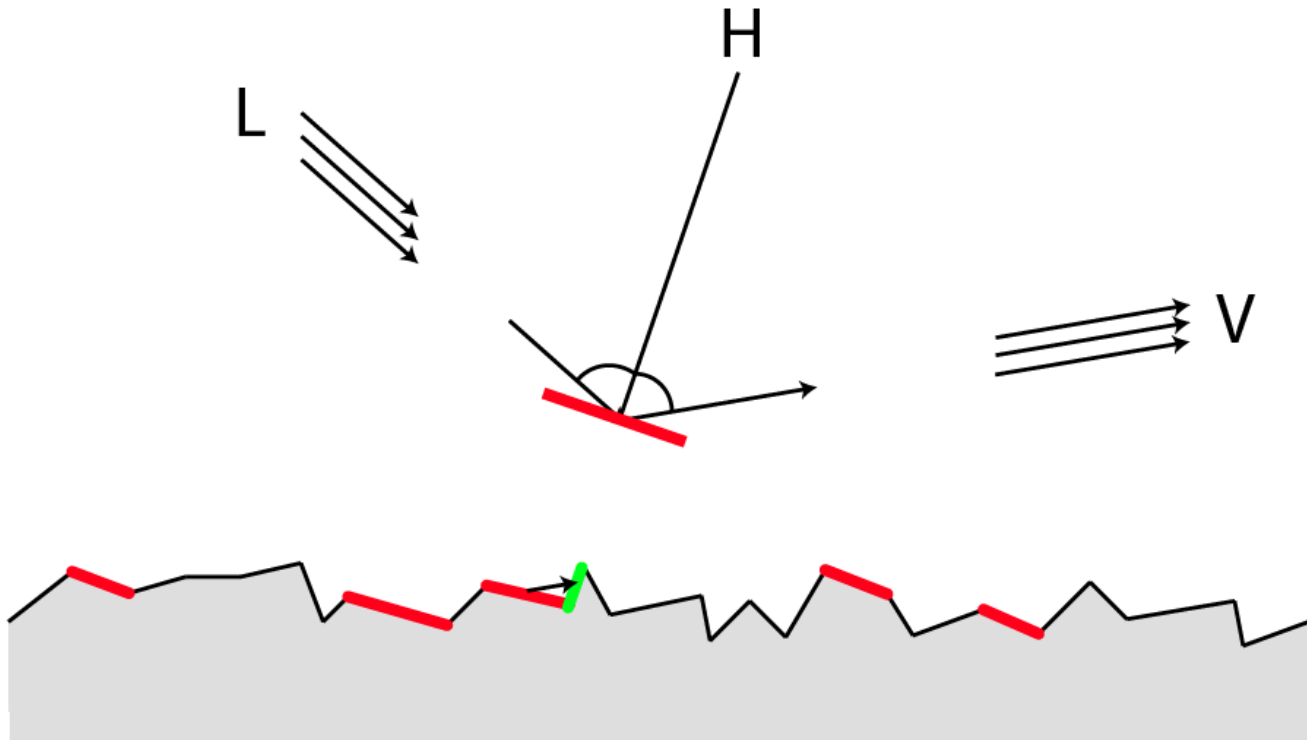
- Value of BRDF at  $(L, V)$  is a product of
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# Microfacet Theory

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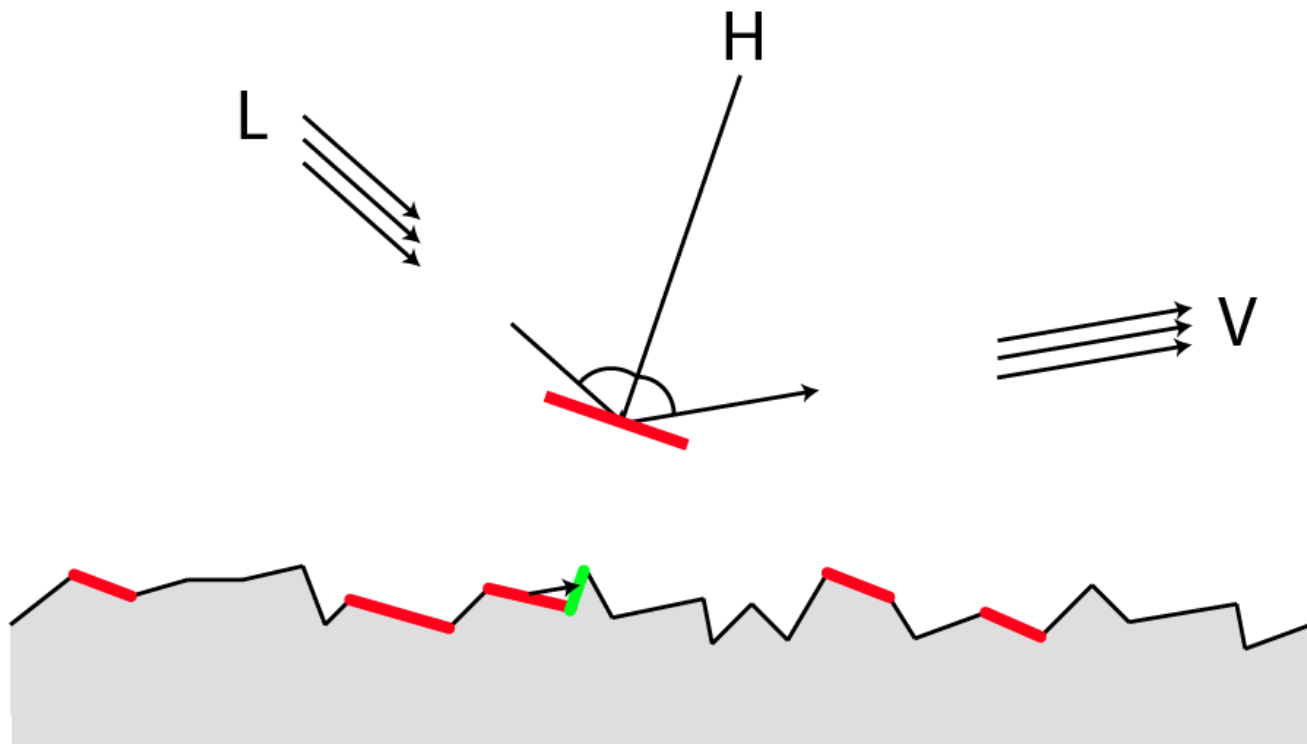
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$
  - ratio of the un(shadowed/masked) mirrors



# Microfacet Theory

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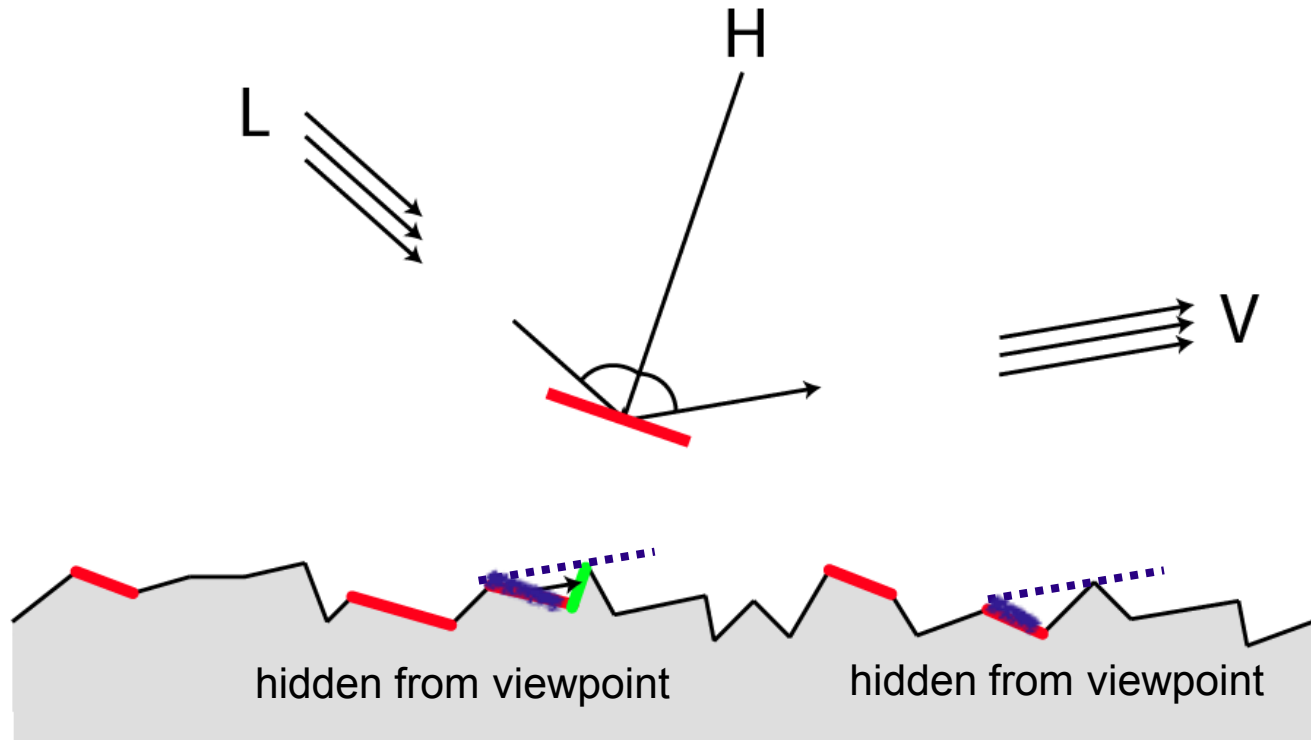
- Value of BRDF at  $(L, V)$  is a product of
  - number of mirrors oriented halfway between  $L$  and  $V$
  - ratio of the un(shadowed/masked) mirrors
  - Fresnel coefficient



# Shadowing and Masking

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- Some facets are hidden from viewpoint
- Some are hidden from the light

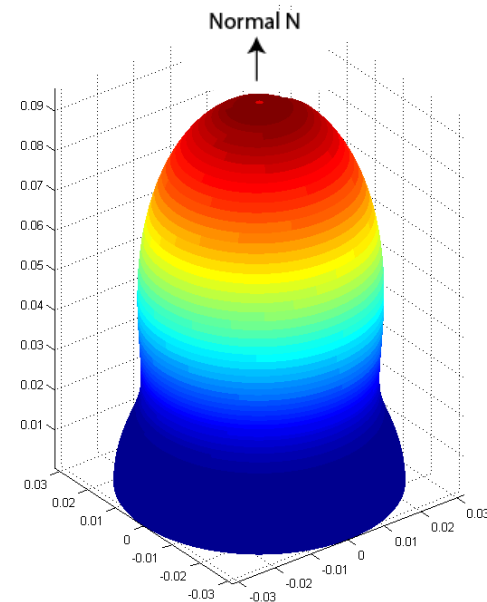




# Microfacet Theory-based Models

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- Develop BRDF models by imposing simplifications [Torrance-Sparrow 67], [Blinn 77], [Cook-Torrance 81], [Ashikhmin et al. 2000]
- Model the distribution  $p(H)$  of microfacet normals
  - Also, statistical models for shadows and masking



spherical plot of a Gaussian-like  $p(H)$

# Full Cook-Torrance Lobe

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- $\rho_s$  is the specular coefficient (3 numbers RGB)
- $D$  is the microfacet distribution
  - $\delta$  is the angle between the half vector  $H$  and the normal  $N$
  - $m$  defines the roughness (width of lobe)
- $G$  is the shadowing and masking term
- Need to add a diffuse term

$$K = \frac{\rho_s}{\pi} \frac{DG}{(N \cdot L)(N \cdot V)} \text{Fresnel}(F_0, V \cdot H)$$

where  $G = \min\left\{1, \frac{2(N \cdot H)(N \cdot V)}{(V \cdot H)}, \frac{2(N \cdot H)(N \cdot L)}{(V \cdot H)}\right\}$  and  $D = \frac{1}{m^2 \cos^4 \delta} e^{-[(\tan \delta)/m]^2}$

# Questions?

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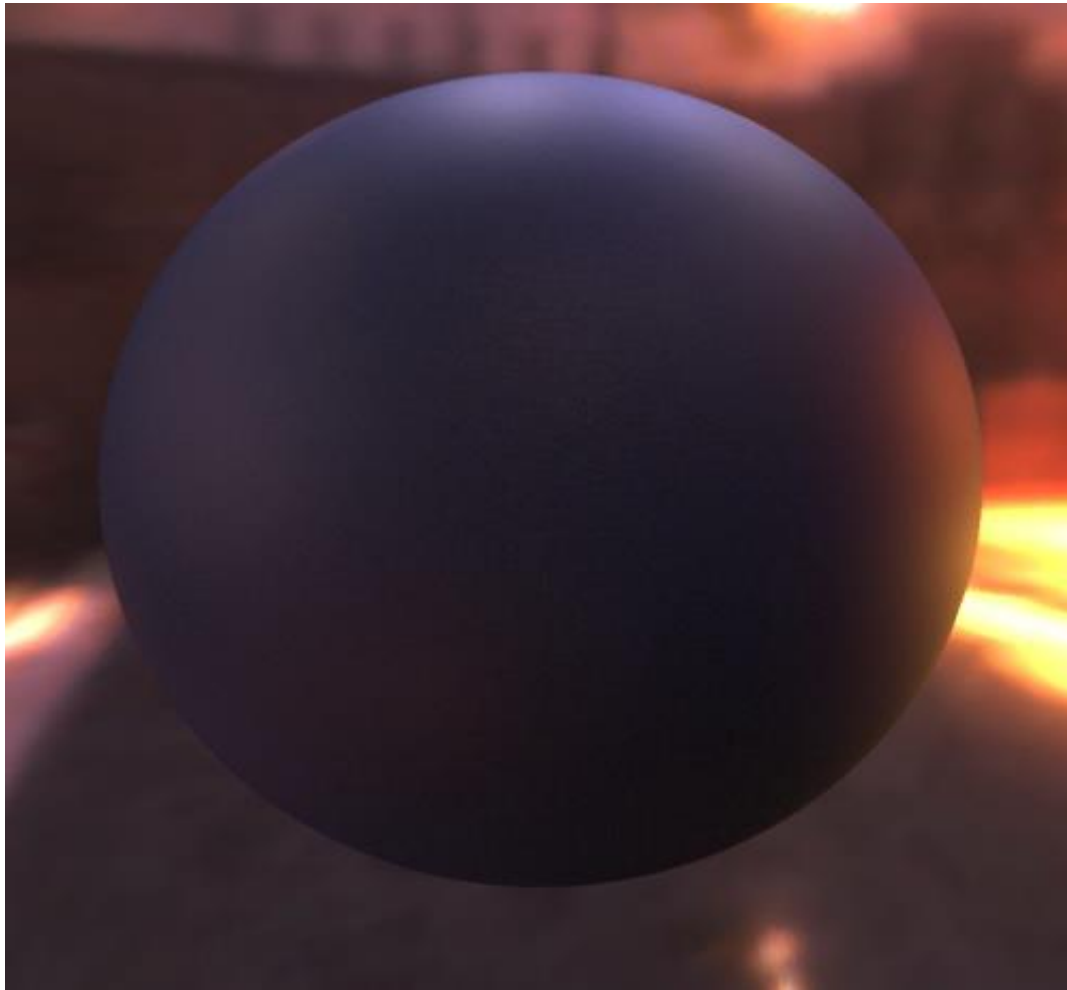
- “Designer BRDFs” by [Ashikhmin et al.](#)



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# BRDF Examples from Ngan et al.

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**Material – Dark blue paint**

Lighting

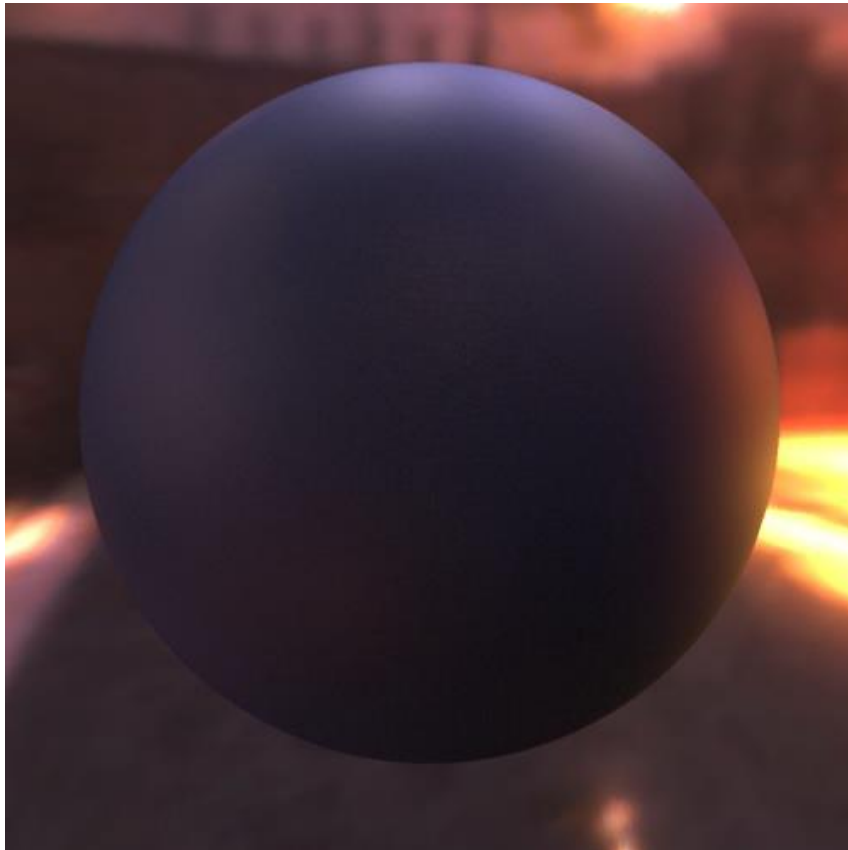


Courtesy of Mitsubishi Electric Research Laboratories, Inc. Used with permission.

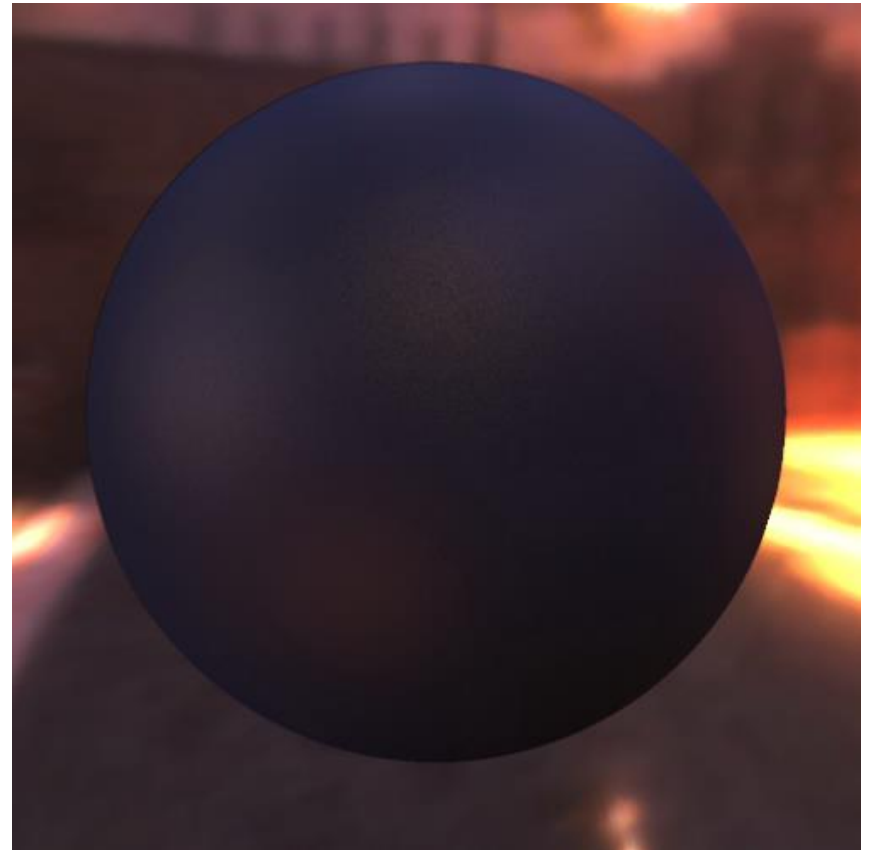
# Dark Blue Paint

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Acquired data



Blinn-Phong



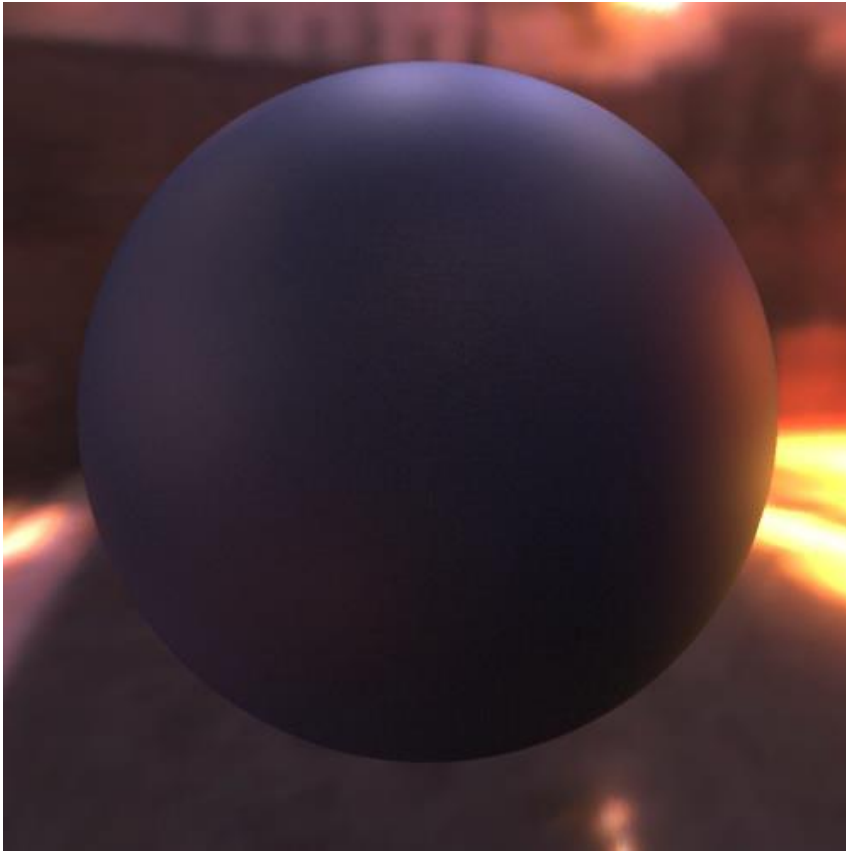
Courtesy of Mitsubishi Electric Research Laboratories, Inc. Used with permission.

Finding the BRDF model parameters that best reproduce the real material  
**Material – Dark blue paint**

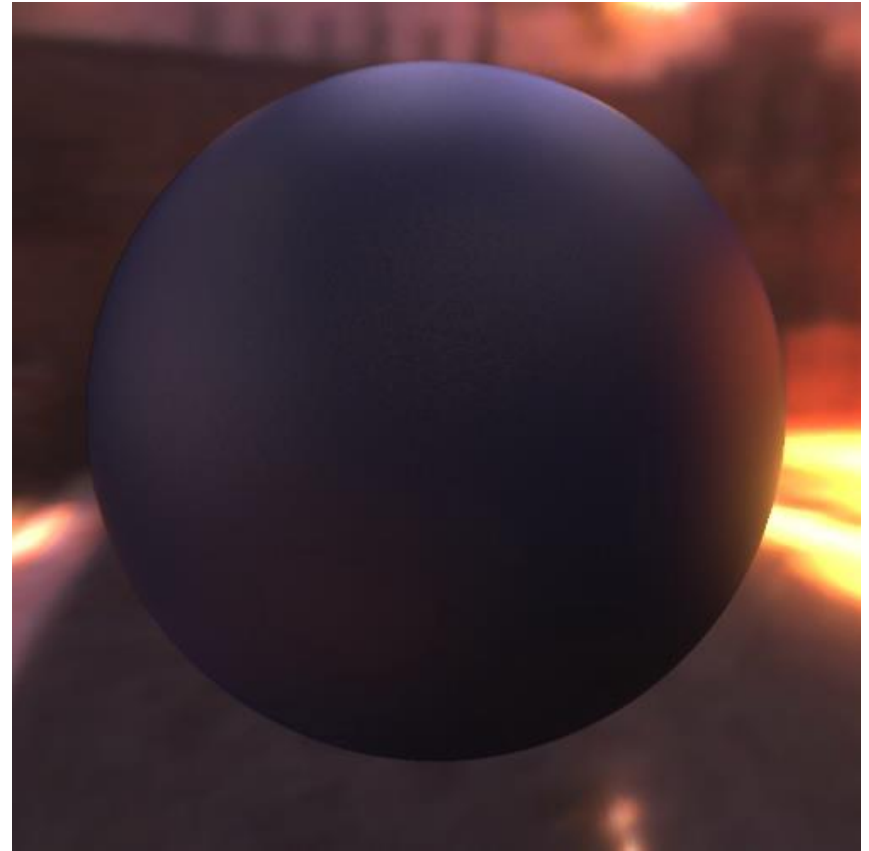
# Dark Blue Paint

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Acquired data



Cook-Torrance



Courtesy of Mitsubishi Electric Research Laboratories, Inc. Used with permission.

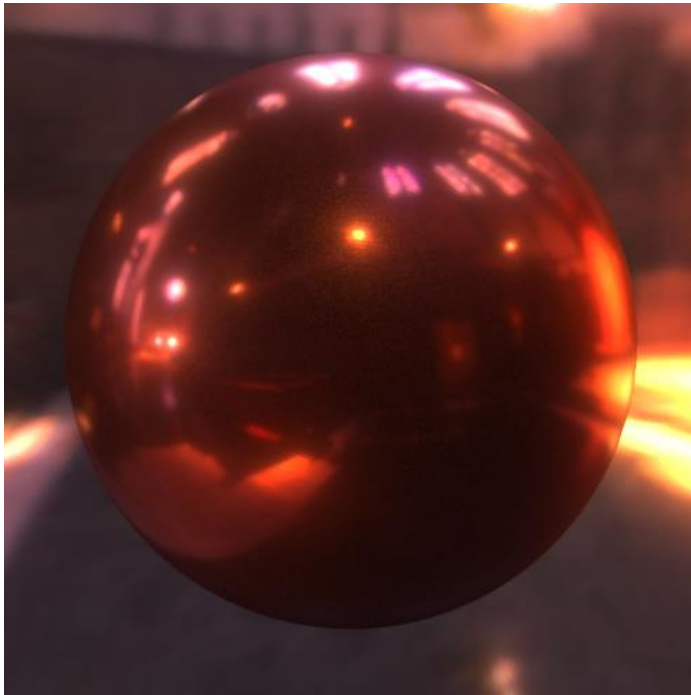
Finding the BRDF model parameters that best reproduce the real material  
**Material – Dark blue paint**

# Observations

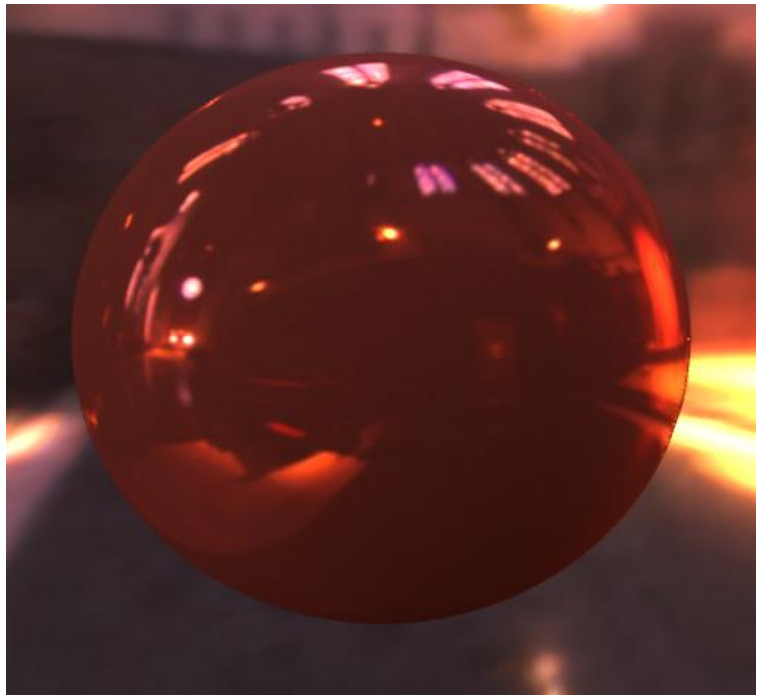
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- Some materials impossible to represent with a single lobe

Acquired data



Cook-Torrance



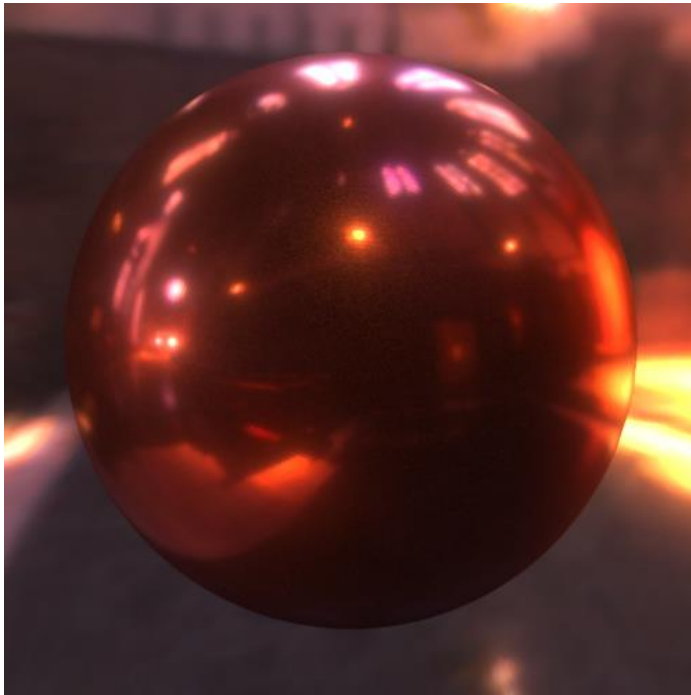
**Material – Red Christmas Ball**

# Adding a Second Lobe

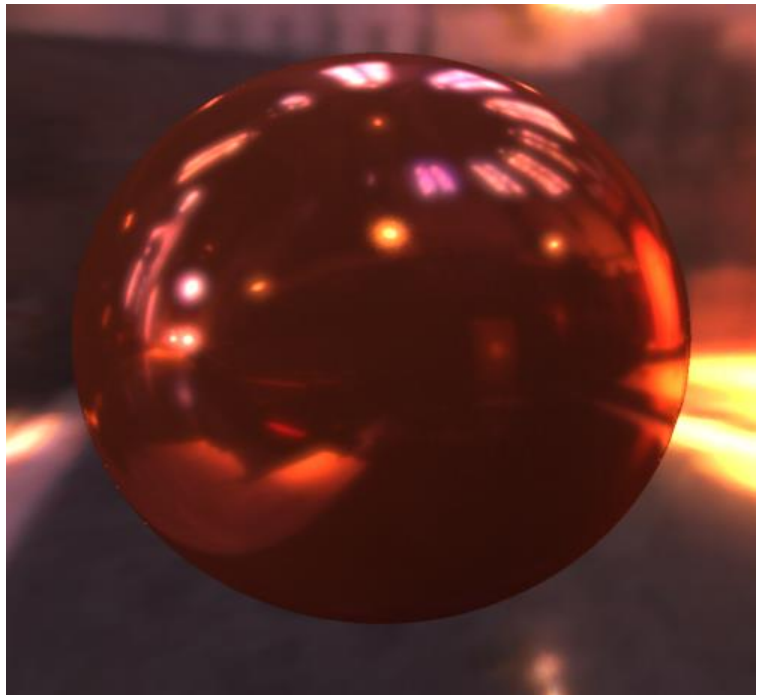
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- Some materials impossible to represent with a single lobe

Acquired data



Cook-Torrance 2 lobes



**Material – Red Christmas Ball**

Courtesy of Mitsubishi Electric Research Laboratories, Inc. Used with permission.



# Image-Based Acquisition

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- A Data-Driven Reflectance Model, SIGGRAPH 2003
  - The data is available  
<http://people.csail.mit.edu/wojciech/BRDFDatabase/>



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# Questions?

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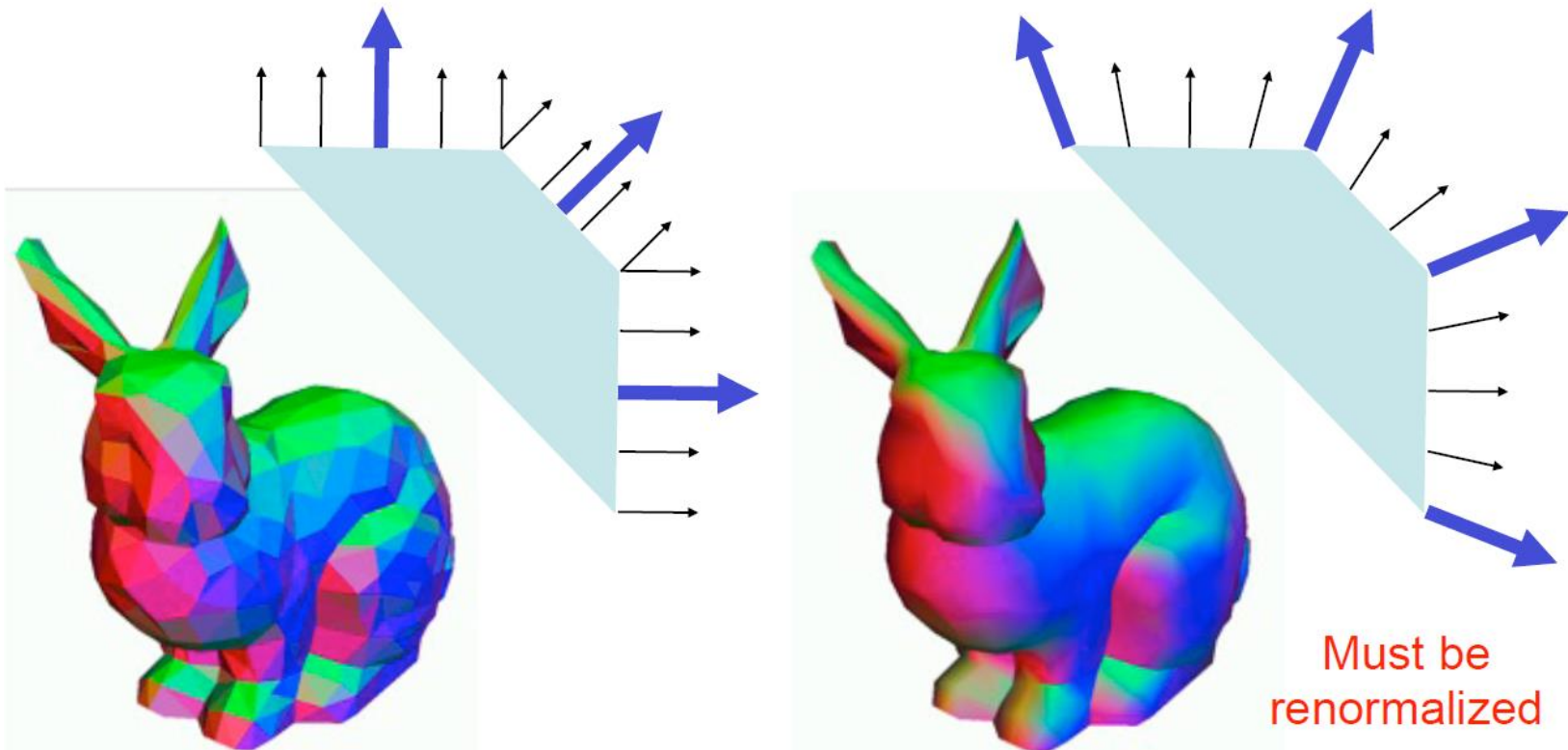
T. Weyrich et al., Fabricating  
Microgeometry for Custom Surface  
Reflectance, SIGGRAPH 2009

Images of Fig. 1 and Fig. 6 in Weyrich T. et al, "Fabricating Microgeometry for Custom Surface Reflectance."  
SIGGRAPH '09 ACM SIGGRAPH 2009 papers; Article No. 32 --removed due to copyright restrictions.

# Phong Normal Interpolation

(Not Phong  
Shading)

- Interpolate the average vertex normals across the face and use this in shading computations
  - Again, use barycentric interpolation!



# That's All for Today!

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Images from the movie, "The Matrix," removed due to copyright restrictions.

# Spatial Variation

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- All materials seen so far are the same everywhere
  - In other words, we are assuming the BRDF is independent of the surface point  $x$
  - No real reason to make that assumption
  - More next time



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