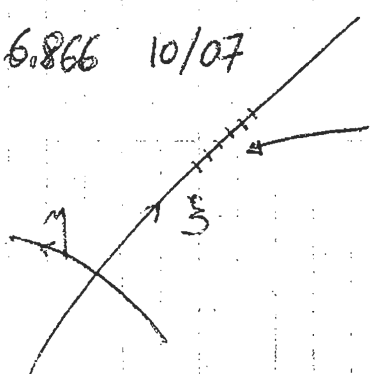
$$E_x = (a^2+b^2)x + (a+c)b\,y$$
$$E_y = (a+c)b\,y + (c^2+b^2)y$$

$$E_x = 0 \;\&\; E_y = 0 \quad \Rightarrow \quad x = y = 0$$

$$\left.\begin{aligned} E_{xx} &= a^2+b^2 \\ E_{xy} &= (a+c)b \\ E_{yy} &= b^2+c^2 \end{aligned}\right\} \quad 2\times 2\times 2 = 8 \text{ max solutions (in fact 4)}$$

6.866 10/07



So how do we solve the ODEs

We take steps of same length which gives a constraint on $\dot{\xi}$ (either on the image or in the world). We prefer to take same length in the world but of course, it's harder.

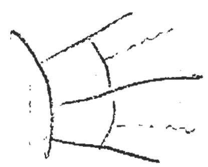
Another idea is to take equal steps in height ($\frac{dz}{ds} = 1$) or equal change in brightness on the image ($\frac{dE}{ds} = 1$) so we step from isophote to isophote (instead of contour to contour).

<u>Consistency of solution</u>

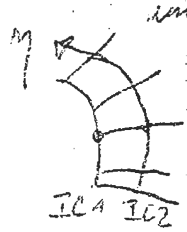
We might end up with crossing strips which happens if the image is noisy. A solution is to solve the strips in parallel and check consistency.

Also strips may spread out and we want to create new ones,

so again, it is important <u>not to solve independently</u>.



Now: from a new step, we have a new initial curve so you can improve the solution as you go. And this can help solve ambiguities.



Binary Image Processing (Chapter)

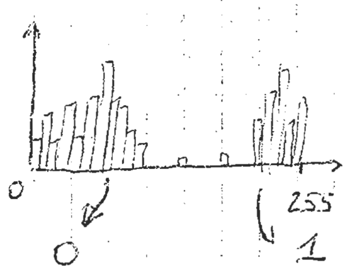
Binary images: (0,1) Easy to handle. Less information.

The "characteristic function" is $b(x,y) = \begin{cases} 0 \\ 1 \end{cases}$

We suggest boolean operators (point-by-point) ex: AND, OR

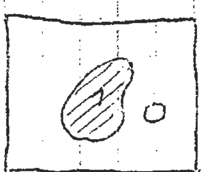
We can think about set of points, which suggest set operators ex UNION

From 8 bits → 1 bit: histogram



It's very rare to work...

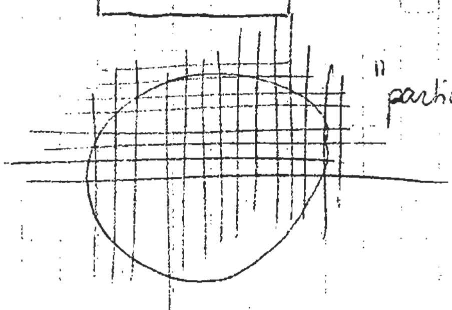
You can have more sophisticated algorithms to get binary images.
Bin img. may be extracted from sensors: depth, contact.



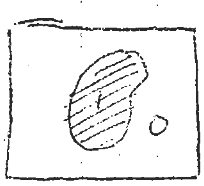
Size ? → area

Position ? → centroid, center of the bounding box
find minimum intertia axis

$$I = \sum_{i}^{N} m_i r_i^{2}$$



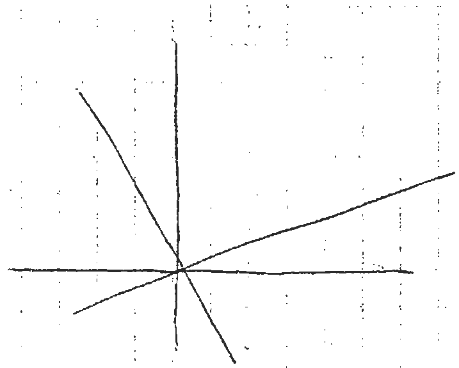
"partial pixel" effect



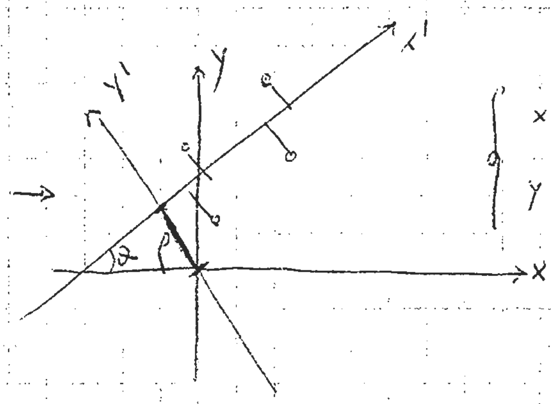
Orientation ? Bounding box doesn't work well.

but: least square line fit $\iint r^2 b(x,y)\,dx\,dy$

Line : $x \sin\theta - y \cos\theta + \rho = 0$



→



$$\begin{cases} x' = x\cos\theta + y\sin\theta \\ y' = -x\sin\theta + y\cos\theta - \rho \end{cases}$$

$$\min_{\rho,\theta} \iint y'^2\,dx\,dy \Rightarrow \boxed{x_0 \sin\theta - y_0 \cos\theta + \rho = 0} \quad \left(\frac{d}{d\rho} = 0\right)$$

$$\min_{\theta} \iint (x'' \sin\theta - y'' \cos\theta)\, b(x,y)\, dx\, dy \quad \text{where } \begin{aligned} x'' &= x - x_0 \\ y'' &= y - y_0 \end{aligned}$$

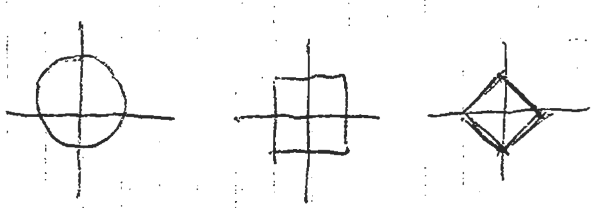
$$\frac{d}{d\theta}(\,) = 0 \quad : \quad \sin^2\theta \cdot a - \sin 2\theta \cdot b + \cos^2\theta\, c = 0$$

$$I = \frac{1}{2}(a+c) + \frac{1}{2}(c-a)\cos 2\theta - b \sin 2\theta$$

$$\boxed{\tan 2\theta = \frac{2b}{a-c}} \iff \text{eigenvalues, eigenvectors.}$$

Pb when $b = 0$ and $c = a$.

$$\iint x' y' \, b(x,y) = 0$$



Projection makes computation faster