

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
 6.685 Electric Machines

Problem Set 10 Solutions

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Problem 1: Permanent Magnets

Note that, if we can conspire to make the radial fields be symmetric about the angular position $\theta = 90^\circ$, a suitable form for the magnetic fields of harmonic order n will be:

$$\begin{aligned} H_{rn} &= \left(H_+^{(n)} r^{np-1} + H_-^{(n)} r^{-np-1} \right) \sin np\theta \\ H_{\theta n} &= \left(H_+^{(n)} r^{np-1} - H_-^{(n)} r^{-np-1} \right) \cos np\theta \end{aligned}$$

This combination of fields is in fact the gradient of a scalar potential that satisfies Laplace's equation, and you can check that it has zero divergence:

$$\begin{aligned} \vec{H} &= -\nabla\psi \\ \nabla^2\psi &= 0 \\ \nabla \cdot \vec{H} &= \frac{1}{r} \frac{\partial}{\partial r} r H_r + \frac{1}{r} \frac{\partial}{\partial \theta} H_\theta + \frac{\partial}{\partial z} H_z = 0 \end{aligned}$$

In this problem, we have four boundary conditions, corresponding to two regions:

$$\begin{aligned} \text{Region 1} & \quad R_i < r < R \\ \text{Region 2} & \quad R < r < R_s \end{aligned}$$

where R_i is the surface of the inner iron, R is the inner surface of the magnets and R_s is the outer surface of the magnets and the inner surface of the outer steel tube. The boundary conditions are, for each space harmonic:

$$\begin{aligned} H_\theta^{(1)} &= 0 & \text{at } r = R_i \\ H_\theta^{(2)} &= 0 & \text{at } r = R_s \\ H_\theta^{(1)} &= H_\theta^{(2)} & \text{at } r = R \\ H_r^{(1)} &= H_r^{(2)} + M_n & \text{at } r = R \end{aligned}$$

The first two of these translate to:

$$\begin{aligned} H_-^{(1)} &= R_i^{2np} H_+^{(1)} \\ H_-^{(2)} &= R_s^{2np} H_+^{(2)} \end{aligned}$$

Using these two conditions in the third and fourth conditions yields:

$$\begin{aligned} H_+^{(1)} \left(1 - \left(\frac{R_i}{r} \right)^{2np} \right) &= H_+^{(2)} \left(1 - \left(\frac{R_s}{R} \right)^{2np} \right) \\ H_+^{(1)} \left(1 + \left(\frac{R_i}{r} \right)^{2np} \right) &= H_+^{(2)} \left(1 + \left(\frac{R_s}{R} \right)^{2np} \right) + M_n R^{-(np+1)} \end{aligned}$$

A bit of algebraic tedium yields an answer for the interesting coefficient:

$$H_+(1) = \frac{M_n}{2} R^{-(np+1)} \left(\frac{R_s^{2np} - R^{2np}}{R_s^{2np} - R_i^{2np}} \right)$$

Using this amplitude and the basic expression for magnetic field, we find the magnetic fields, for which the n^{th} harmonics are:

$$\begin{aligned} H_{rn} &= \frac{M_n}{2} R^{-(np+1)} \left(\frac{R_s^{2np} - R^{2np}}{R_s^{2np} - R_i^{2np}} \right) \left(\left(\frac{r}{R} \right)^{np-1} + \left(\frac{R_i}{R} \right)^{2np} \left(\frac{r}{R} \right)^{-np-1} \right) \sin np\theta \\ H_{\theta n} &= \frac{M_n}{2} R^{-(np+1)} \left(\frac{R_s^{2np} - R^{2np}}{R_s^{2np} - R_i^{2np}} \right) \left(\left(\frac{r}{R} \right)^{np-1} - \left(\frac{R_i}{R} \right)^{2np} \left(\frac{r}{R} \right)^{-np-1} \right) \cos np\theta \end{aligned}$$

If the magnetization is uniform across the angle θ_m , the harmonic magnitudes are:

$$M_n = \frac{4}{n\pi} M_0 \sin\left(np \frac{\theta_m}{2}\right) \sin\left(np \frac{\pi}{2}\right)$$

A re-construction of the magnetization is shown in Figure 1. This shows about how good a reproduction of the actual magnetization waveform is. I have used harmonics of order up to 29.

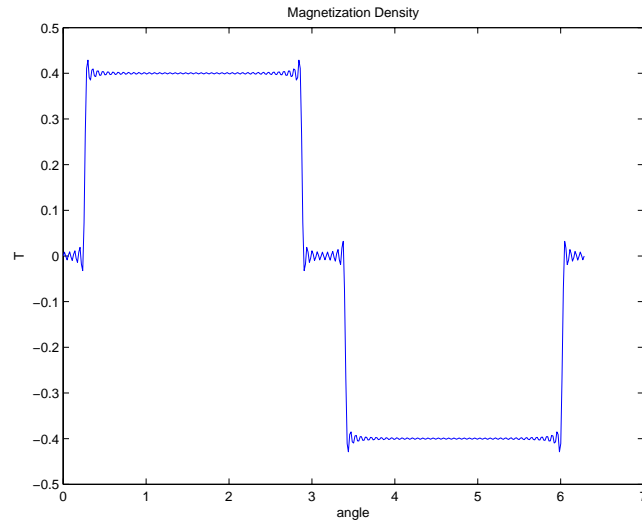


Figure 1: Reproduction of magnetization Remanent Flux Density waveform by Fourier series
 As the inner radius goes to zero, the coefficient $H_-(1)$ also must go to zero. The expressions we have derived are well behaved, however, and are correct for the case of no inner rotor. The fields are shown in Figures 2. Note I have used remanent flux density and so am plotting flux density ($B = \mu_0 H$).

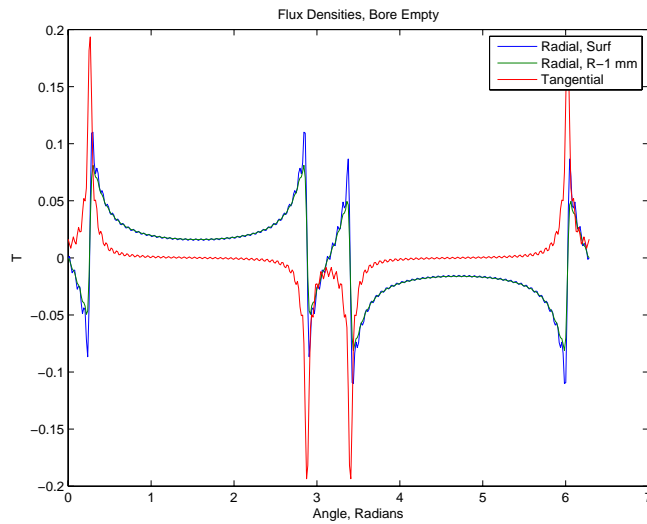


Figure 2: Fields with No Rotor

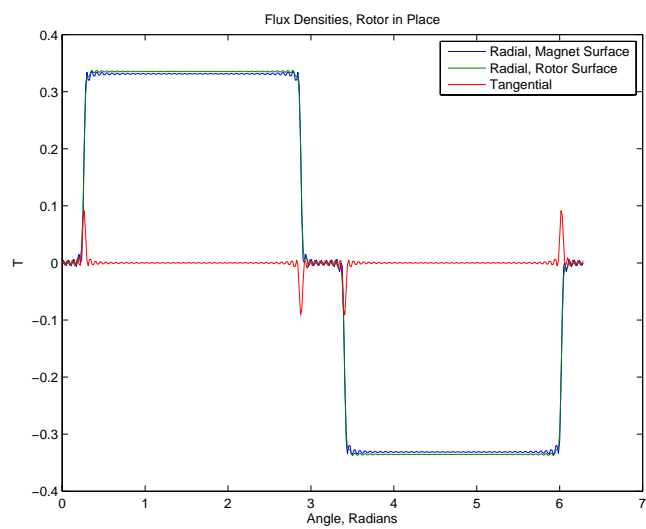


Figure 3: Fields with Rotor Present

Problem 2: Induction Motor Simulation

1. The first order model is the 'steady state' model, which assumes that the electrical states reach steady state much more quickly than the mechanical state (speed). Slip is $s = 1 - \frac{p\omega_m}{\omega_0}$, where we are using ω_m as the actual rotor speed. Then we can build up terminal impedance by:

$$\begin{aligned} Z_r &= jX_2 + \frac{R_2}{s} \\ Z_{ag} &= Z_r || jX_m \\ Z_t &= R_1 + jX_1 + Z_{ag} \end{aligned}$$

Terminal current is $I_a = \frac{V_t}{Z_t}$ and rotor branch current is found using a divider relationship:

$$I_2 = -\frac{jX_m}{jX_m + Z_r} I_a$$

If we take voltage (and consequently current) to be *peak*, torque is simply:

$$T_e = \frac{p}{\omega_0} P_{ag} = \frac{3}{2} \frac{p}{\omega_0} \frac{R_2}{s} |I_s|^2$$

We model load torque as:

$$T_\ell = T_0 \left(\frac{p\omega_m}{\omega_0} \right)^2$$

and the single state equation is:

$$\frac{d\omega_m}{dt} = \frac{1}{J} (T_e - T_\ell)$$

To get input power we simply multiply voltage by current:

$$P_{in} = \text{Re} \{V_t I_a^*\}$$

This is implemented in the first set of scripts, appended and the results for this motor are shown in Figures 4 and 5.

2. For the second and third parts of this problem set we employ the induction motor model cast in terms of a coordinate system rotating at synchronous speed:

$$\begin{aligned} \frac{d\lambda_{ds}}{dt} &= V_{ds} + \omega_e \lambda_{qs} - r_s i_{ds} \\ \frac{d\lambda_{qs}}{dt} &= V_{qs} - \omega_e \lambda_{ds} - r_s i_{qs} \\ \frac{d\lambda_{dr}}{dt} &= \omega_s \lambda_{qr} - r_r i_{dr} \\ \frac{d\lambda_{qr}}{dt} &= -\omega_s \lambda_{dr} - r_r i_{qr} \\ T_e &= \frac{3}{2} p (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \\ \frac{d\omega_m}{dt} &= \frac{1}{J} (T_e - T_\ell) \end{aligned}$$

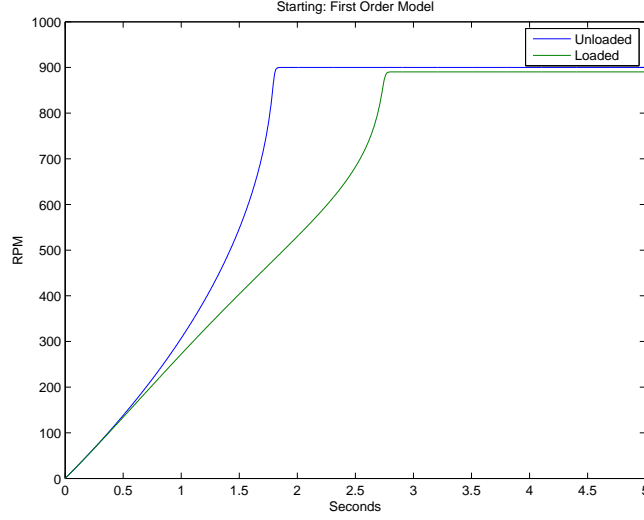


Figure 4: Starting Speed: First Order Model

Our formulation for load torque T_ℓ is the same for these parts of the problem as for the first part. Now, we need to find the currents for both rotor and stator. Since the rotor is 'round' the quadrature axis behaves exactly like the stator. The flux/current relationship is:

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{dr} \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \end{bmatrix}$$

Those inductances are:

$$\begin{aligned} L_s &= \frac{X_1 + X_m}{\omega_0} \\ L_r &= \frac{X_2 + X_m}{\omega_0} \\ M &= \frac{X_m}{\omega_0} \end{aligned}$$

Inverting that matrix yields the current/flux relationship:

$$\begin{aligned} i_{ds} &= y_{11}\lambda_{ds} + y_{12}\lambda_{dr} \\ i_{dr} &= y_{12}\lambda_{ds} + y_{22}\lambda_{dr} \end{aligned}$$

For the third order model we assume that the stator is essentially in steady state and that resistance is small enough. So that

$$\begin{aligned} \lambda_{ds} &= V_q \\ \lambda_{qs} &= -V_d \end{aligned}$$

Simulation of this case uses only the last three of the state equations. For our purposes here we assume that the terminal voltage is on the q- axis: $V_q = V_t$ and $V_d = 0$. For this

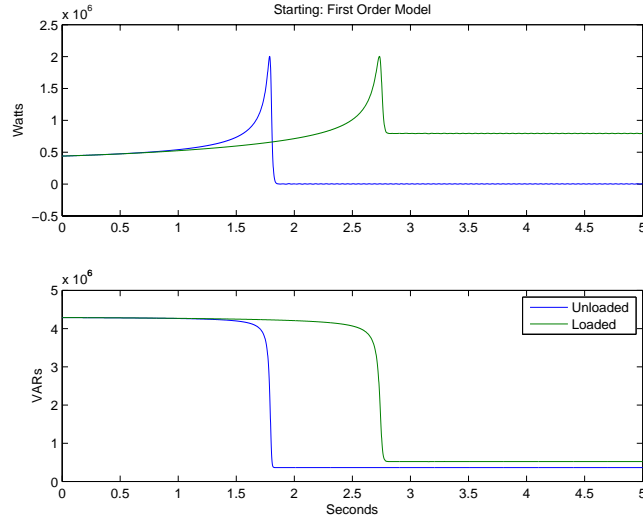


Figure 5: Starting Power: First Order Model

problem, terminal voltage was stated as 4160 volts, rms, line-line. Phase voltage, peak is then:

$$V_t = \sqrt{\frac{2}{3}} \times 4160 \approx 3397V$$

At the end we can calculate input power by multiplying terminal voltage times current, but for this case we need to add back in the resistive loss:

$$P_{in} = \frac{3}{2}V_t i_{qs} + R_1 (i_{qs}^2 + i_{ds}^2)$$

The third order model is implemented in two scripts which are appended. The results are shown in Figures 6 and 7

3. The fifth order model is even easier because we use the simulation model already stated directly. Power is simply voltage times current. The implementation of this in two scripts is appended and the results are shown in Figure 8 and 9.

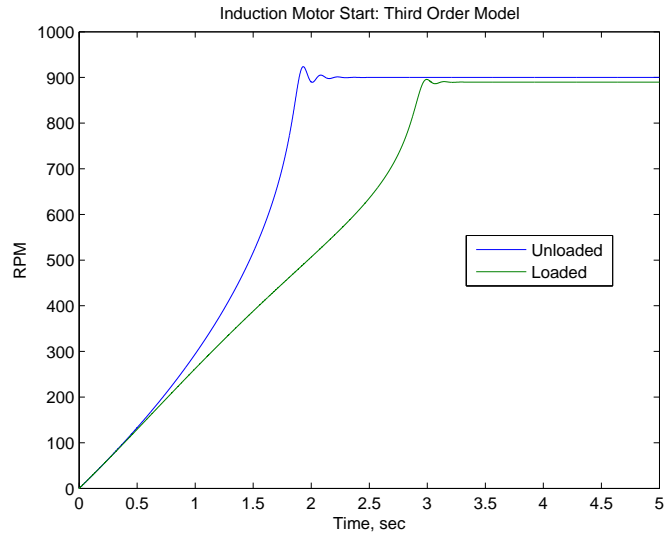


Figure 6: Starting Speed: Third Order Model

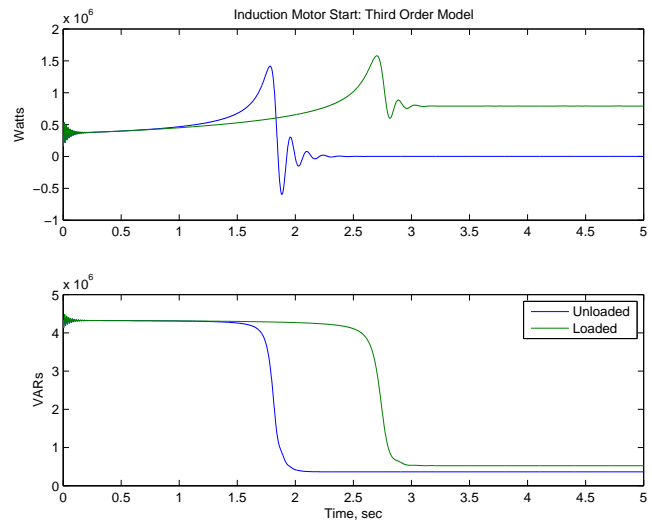


Figure 7: Starting Power: Third Order Model

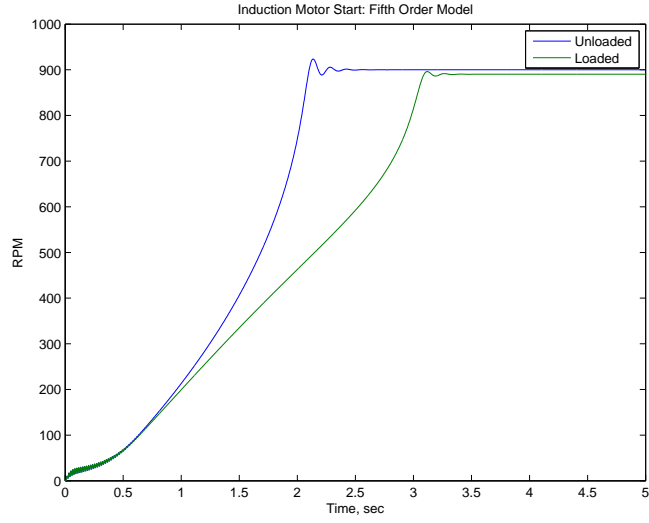


Figure 8: Starting Speed: Fifth Order Model

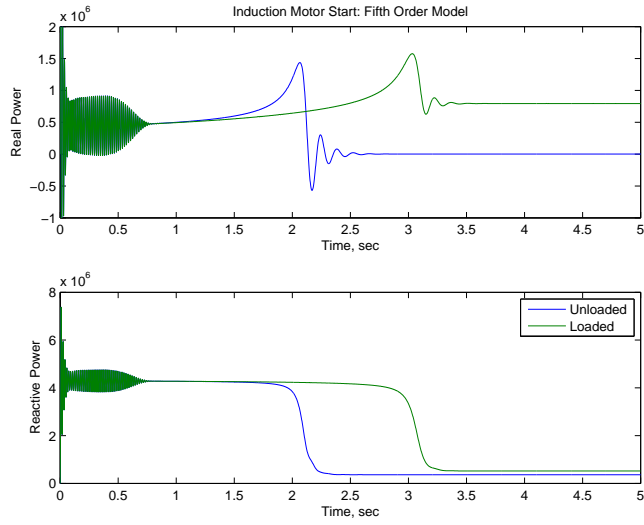


Figure 9: Starting Power: Fifth Order Model

Problem 3: Doubly Fed Induction Generator

In this one, too, the story is told by the script. Real and reactive stator power are simply:

$$\begin{aligned} P_s &= \frac{P_{out}}{1-s} \\ Q_s &= Q_{out} \end{aligned}$$

Then stator current is, since we are working this in peak amplitudes:

$$\underline{I}_s = \frac{P_s + jQ_s}{\frac{3}{2}V}$$

Magnetizing voltage is:

$$\underline{V}_\phi = V + jX_1 I_s$$

Rotor current is then just the sum of stator current and magnetizing current:

$$\underline{I}_2 = \underline{I}_s + \frac{\underline{V}_\phi}{jX_\phi}$$

Rotor voltage (in the stator frame and on the stator side of the ideal transformer) is:

$$\underline{V}_2 = \underline{V}_\phi + jX_2 \underline{I}_2$$

and rotor complex power (again, in the stator frame) is:

$$P_2 + jQ_2 = \frac{3}{2} V_2 I_2^*$$

Real power is P_2 multiplied by slip:

$$P_r = sP_2$$

And reactive power is Q_2 multiplied by the absolute value of slip (remember the phase reversal at zero slip!)

$$Q_r = |s|Q_2$$

The results of this calculation are shown in Figures 10 and 11

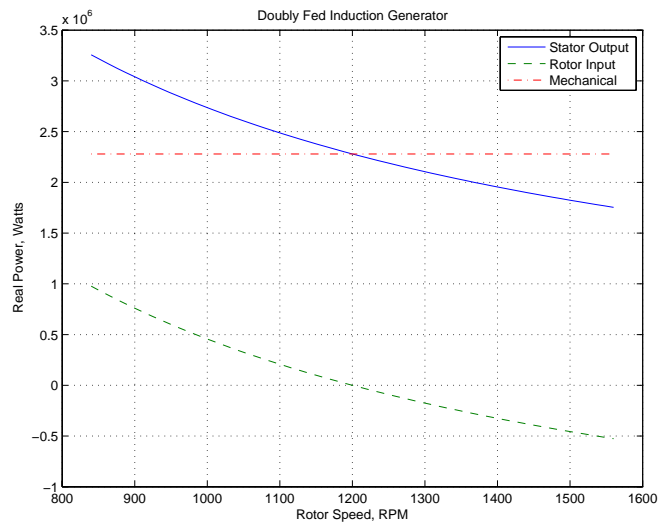


Figure 10: Doubly Fed Induction Generator Real Power

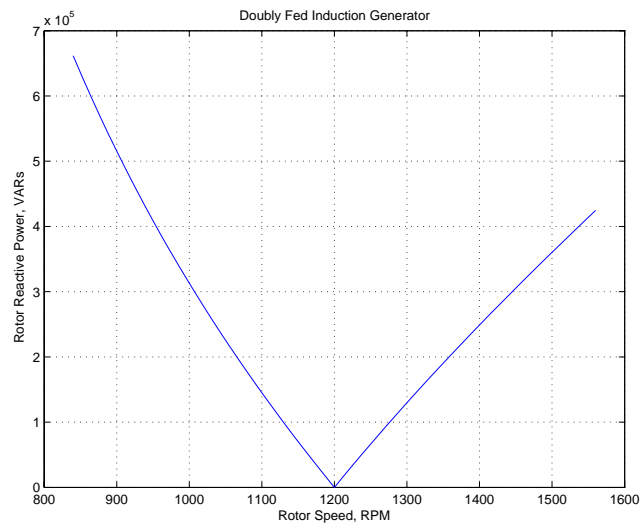


Figure 11: Doubly Fed Induction Generator Rotor Reactive Power

Script for Problem 1

% 6.685 Problem Set 10, Problem 1, 2013

% Fields from a permanent magnet array
% dimensions:

R = .04; % outer radius of magnets
h = .0025; % magnet height
thm = 5*pi/6; % magnet angular extent
g = .0005; % air-gap dimension
M0 = .4; % base magnetization (T);

Ri = R-g; % inner magnetic radius
Ro = R+h; % outer magnetic radius

n = 1:2:99; % harmonic orders to consider

th = 0:pi/200:2*pi; % range of angles for plots

M = (4/pi) * M0 .* sin(n .* pi/2) .* sin(n .* thm/2) ./ n; % magnetization harmonics

H0 = zeros(size(th));

for k = 1:length(n);
H0 = H0 + M(k) .* sin(n(k) .* th);
end

figure(1)
plot(th, H0)
title('Magnetization Density')
ylabel('T')
xlabel('angle')

% magnetic field with the bore empty

r = R; % calculate field at magnet surface
r1 = R - g; % and at a spot just inside
Ri = 0;

k1 = (Ro .^(2 .* n) - R .^(2 .* n)) ./ (Ro .^(2 .* n) - Ri .^(2 .* n));
k2 = (Ro .^(2 .* n) - R .^(2 .* n)) ./ (Ro .^(2 .* n) - Ri .^(2 .* n));

% now calculate and plot these fields

```

kr = (r/R) .^ (n-1) + (Ri/R) .^ (2 .* n) .* (R ./ r) .^ (n+1);
kr1 = (r1/R) .^ (n-1) + (Ri/R) .^ (2 .* n) .* (R ./ r1) .^ (n+1);

kt = (r/R) .^ (n-1) - (Ri/R) .^ (2 .* n) .* (R ./ r) .^ (n+1);

Br = zeros(size(th));
Bt = zeros(size(th));
Br1 = zeros(size(th));
Bt1 = zeros(size(th));

for k = 1:length(n)
    Br = Br + .5 .*M(k) .* k1(k) .* kr(k) .* sin(n(k) .* th);
    Br1 = Br1 + .5 .*M(k) .* k2(k) .* kr1(k) .* sin(n(k) .* th);
    Bt = Bt + .5 .*M(k) .* k1(k) .* kt(k) .* cos(n(k) .* th);
end

figure(2)
plot(th, Br, th, Br1, th, Bt)
title('Flux Densities, Bore Empty')
legend('Radial, Surf', 'Radial, R-1 mm', 'Tangential')
ylabel('T')
xlabel('Angle, Radians')

% now with a rotor

r = R; % calculate field at magnet surface
r1 = R - g; % and at a spot just inside
Ri = R - g; % here is the rotor surface

k1 = (Ro .^(2 .* n) - R .^ (2 .* n)) ./ (Ro .^ (2 .* n) - Ri .^ (2 .* n));
k2 = (Ro .^(2 .* n) - R .^ (2 .* n)) ./ (Ro .^ (2 .* n) - Ri .^ (2 .* n));

% now calculate and plot these fields

kr = (r/R) .^ (n-1) + (Ri/R) .^ (2 .* n) .* (R ./ r) .^ (n+1);
kr1 = (r1/R) .^ (n-1) + (Ri/R) .^ (2 .* n) .* (R ./ r1) .^ (n+1);

kt = (r/R) .^ (n-1) - (Ri/R) .^ (2 .* n) .* (R ./ r) .^ (n+1);

Br = zeros(size(th));
Bt = zeros(size(th));
Br1 = zeros(size(th));
Bt1 = zeros(size(th));

for k = 1:length(n)

```

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    Br = Br + .5 .*M(k) .* k1(k) .* kr(k) .* sin(n(k) .* th);
    Br1 = Br1 + .5 .*M(k) .* k2(k) .* kr1(k) .* sin(n(k) .* th);
    Bt = Bt + .5 .*M(k) .* k1(k) .* kt(k) .* cos(n(k) .* th);
end

figure(3)
plot(th, Br, th, Br1, th, Bt)
title('Flux Densities, Rotor in Place')
legend('Radial, Magnet Surface', 'Radial, Rotor Surface', 'Tangential')
ylabel('T')
xlabel('Angle, Radians')

```

Scripts for Problem 2

```

% Sample Parameters for Induction Motor Starting Scripts
% Representative Parameters of a large 2400 v motor
global vt omz y11 y12 y22 r1 r2 p J T_1

p = 4;
omz = 120*pi;
vt = sqrt(2/3)*6000;
r1 = .460;
r2 = .433;
x1 = 3.51;
x2 = 5.05;
xad = 95.6;
J = 80;
T_0 = (p/omz)*8e5;
-----
% 6.685 Problem Set 10, Problem 2, First Order Model
% First Order Model: Induction Motor Acceleration

global x1 x2 xad r1 r2 J p omz vt T_1
bparms % file with relevant parameters

x0=[0]; % start from rest
T_1 = 0; % unloaded start
tspan = 0:.001:5;
options = odeset('reltol',3e-5,'abstol',3e-6);

[t,x] = ode23('im1',tspan,x0, options); % this is the simulator
T_1 =T_0; % start under load
[t1, x1] = ode23('im1', tspan, x0, options);
rpm = (60/(2*pi)) .* x; % just translates units
rpml = (60/(2*pi)) .*x1;

```

```

figure(1)
clf
plot(t, rpm, t1, rpml); % draw picture
xlabel('Seconds');
ylabel('RPM');
title('Starting: First Order Model');
legend('Unloaded', 'Loaded')

% OK: now we compute terminal current
omm = p .* x;
omm1 = p .* x1;
s = 1 - omm ./ omz;
s1 = 1 - omm1 ./ omz;
Zr = j*x2 + r2 ./ s;
Zr1 = j*x2 + r2 ./ s1;
Zt = r1 + j*x1 + j*xad .* Zr ./ (j*xad + Zr);
Zt1 = r1 + j*x1 + j*xad .* Zr1 ./ (j*xad + Zr1);
P = 1.5 .* real(vt^2 ./ Zt);
P1 = 1.5 .* real(vt^2 ./ Zt1);
Q = 1.5 * imag(vt^2 ./ conj(Zt));
Q1 = 1.5 * imag(vt^2 ./ conj(Zt1));

figure(2)
clf
subplot 211
plot(t, P, t1, P1)
ylabel('Watts')
title('Starting: First Order Model')
subplot 212
plot(t, Q, t, Q1)
ylabel('VARs')
xlabel('Seconds')
legend('Unloaded', 'Loaded')
-----
function xdot = im1(t, x)          % First Order Acceleration

global x1 x2 xad r1 r2 J p omz vt T_1

omm = x;          % rotational frequency
s = 1-p*omm/omz;  % per-unit slip
zr = r2/s + j*x2; % rotor impedance
zm = j*xad;      % magnetizing impedance
zag = zm*zr/(zm+zr); % looking across the air-gap
zt = zag + j*x1 + r1; % terminal impedance
ia = vt / zt;    % terminal current

```

```

ir = ia * zm / (zm+zr);      % rotor current
pag = 1.5*abs(ir)^2 * r2/s;  % air-gap power
Tl = T_1*(p*omm/omz)^2;     % load torque
xdot = ((pag/omz) * p -Tl)/ J; % torque over inertia
-----
% 6.685 Problem Set 10, Problem 3
% Implementation of third order induction motor model
% simulation of across-the-line starting
global vt omz y11 y12 y22 r1 r2 p J T_1

bparms          % go get parameters

% first, generate admittances
Ls = (x1+xad)/omz;
Lr = (x2+xad)/omz;
M = xad/omz;

y11 = Lr/(Ls*Lr-M^2);
y22 = Ls/(Ls*Lr-M^2);
y12 = -M/(Ls*Lr-M^2);

% first simulated unloaded start
T_1 = 0;
tspan = 0:.001:5;
X0 = [0 0 0]';
[t, X] = ode23('im3', tspan, X0);
omm = X(:,3);
rpm = (30/pi) .* omm;
iq = y12 .* X(:,2);          % quadrature axis current
id = y11 * vt/omz + y12 .* X(:,1); % direct axis current
Pin = 1.5*vt .* iq + r1 .* (id.^2 + iq.^2);
Qin = 1.5*vt .* id;

% now simulate loaded
T_1 = T_0;
[t1, X] = ode23('im3', tspan, X0);
omml = X(:,3);
rpml = (30/pi) .* omml;
iq = y12 .* X(:,2);          % quadrature axis current
id = y11 * vt/omz + y12 .* X(:,1); % direct axis current
Pinl = 1.5*vt .* iq + r1 .* (id.^2 + iq.^2);
Qinl = 1.5*vt .* id;

figure(3)
plot(t, rpm, t1, rpml)

```



```

title('Induction Motor Start: Third Order Model')
ylabel('RPM')
xlabel('Time, sec')
legend('Unloaded', 'Loaded')

figure(4)
clf
subplot 211
plot(t, Pin, t1, Pin1)
title('Induction Motor Start: Third Order Model')
ylabel('Watts')
subplot 212
plot(t, Qin, t, Qin1)
ylabel('VARs')
xlabel('Time, sec')
legend('Unloaded', 'Loaded')
-----
function xdot = im3(t, x)

global vt omz y11 y12 y22 r1 r2 p J T_1

lamdad = vt/omz;
lamdaq = 0;
lamdadr = x(1);
lamdaqr = x(2);
omm = x(3);

omme = p*omm;
oms = omz-omme;

Tl = T_1*(omme/omz)^2;

ids = y11*lamdad + y12*lamdadr;
iqs = y11*lamdaq + y12*lamdaqr;
idr = y12*lamdad + y22*lamdadr;
iqr = y12*lamdaq + y22*lamdaqr;

dlamdadr = oms*lamdaqr - r2*idr;
dlamdaqr = -oms*lamdadr - r2*iqr;
domm = (1.5*p/J)*(lamdad * iqs - lamdaq * ids)-Tl/J;

xdot = [dlamdadr dlamdaqr domm]';
-----
% 6.685 Problem Set 10, Problem 3
% Implementation of fifth order induction motor model

```

```

% simulation of across-the-line starting
global vt omz y11 y12 y22 r1 r2 p J T_l

bparms          % go get parameters

% first, generate admittances
Ls = (x1+xad)/omz;
Lr = (x2+xad)/omz;
M = xad/omz;

y11 = Lr/(Ls*Lr-M^2);
y22 = Ls/(Ls*Lr-M^2);
y12 = -M/(Ls*Lr-M^2);

% first simulated unloaded start
T_l = 0;
tspan = 0:.001:5;
X0 = [0 0 0 0 0]';
[t, X] = ode23('im5', tspan, X0);
omm = X(:,5);
rpm = (30/pi) .* omm;
iq = y11 .* X(:,2) + y12 .* X(:,4);
id = y11 .* X(:,1) + y12 .* X(:,3);
Pin = 1.5*vt .* iq;
Qin = 1.5*vt .* id;
% now simulate loaded
T_l = T_0;
[t1, X] = ode23('im5', tspan, X0);
omml = X(:,5);
rpml = (30/pi) .* omml;
iq = y11 .* X(:,2) + y12 .* X(:,4);
id = y11 .* X(:,1) + y12 .* X(:,3);
Pinl = 1.5*vt .* iq;
Qinl = 1.5*vt .* id;

figure(5)
plot(t, rpm, t1, rpml)
title('Induction Motor Start: Fifth Order Model')
ylabel('RPM')
xlabel('Time, sec')
legend('Unloaded', 'Loaded')

figure(6)
subplot 211
plot(t, Pin, t1, Pinl)

```

```

title('Induction Motor Start: Fifth Order Model')
ylabel('Real Power')
xlabel('Time, sec')
axis([0 5 -1e6 2e6])
subplot 212
plot(t, Qin, t1, Qin1)
ylabel('Reactive Power')
xlabel('Time, sec')
legend('Unloaded', 'Loaded')
-----
function xdot = im5(t, x)

global vt omz y11 y12 y22 r1 r2 p J T_1

lamdad = x(1);
lamdaq = x(2);
lamdadr = x(3);
lamdaqr = x(4);
omm = x(5);

omme = p*omm;
oms = omz-omme;

Tl = T_1*(omme/omz)^2;

ids = y11*lamdad + y12*lamdadr;
iqs = y11*lamdaq + y12*lamdaqr;
idr = y12*lamdad + y22*lamdadr;
iqr = y12*lamdaq + y22*lamdaqr;

dlamdad = omz*lamdaq - r1*ids;
dlamdaq = vt - omz*lamdad - r1*iqs;
dlamdadr = oms*lamdaqr - r2*idr;
dlamdaqr = -oms*lamdadr - r2*iqr;
domm = (1.5*p/J)*(lamdad * iqs - lamdaq * ids)-Tl/J;

xdot = [dlamdad dlamdaq dlamdadr dlamdaqr domm]';

```

Script for Problem 4

```
% 6.685 Fall 2011 Problem set 10, Problem 4
```

```

om = 2*pi*60;
LaA = 10.37e-3;           % rotor to stator peak mutual inductance
N_t = 3;                 % transformer turns ratio
La = 3.5e-3;             % stator phase inductance

```

```

Lab = -1.75e-3;           % stator phase to phase mutual inductance
Lm = 1.5*LaA/N_t;       % magnetizing inductance: referred to stator side
LA = 31.5e-3;           % rotor phase A inductance
LAB = -15.75e-3;        % rotor phase a-b inductance
Ll = La-Lab-Lm;         % leakage inductance
Lrl = (LA-LAB)/N_t^2 - Lm; % this is rotor side leakage referred to the stator side
Xm = om*Lm;
Xl = om*Ll;
Xrl = om*Lrl;
V = 690*sqrt(2/3);      % working in peak amplitudes
VA = 2.4e6;             % to be running at this output
pf = .95;               % and this power factor
omm = om .* (.7:.01:1.3); % range of working speeds
p = 3;                  % number of pole pairs
N = 60/(2*pi*p) .* omm;
s = 1 - omm ./ om;      % and resulting slips
Pout = pf * VA;         % real power at system terminals
Qout = sqrt(1-pf^2)*VA; % reactive power at system terminals
P = Pout ./ (1 - s);    % real part at machine terminals
Q = Qout;               % reactive part
Ir = P ./ (1.5*V);      % real part of current
Ii = Q ./ (1.5*V);      % reactive part of current
Is = Ir-j .* Ii;        % complex stator current
Vm = V + j*Xl .* Is;    % voltage at magnetizing branch
Im = Vm ./ (j*Xm);      % magnetizing branch current
Ir = Is+Im;             % current into the rotor
Vx = Vm+j*Xrl .* Ir;    % rotor voltage in stator frame
Vrs = s .* Vx;         % rotor voltage at low speed
Pcr = 1.5 .*Vrs .* conj(Ir); % complex power into rotor
Pr = real(Pcr);
Qr = imag(Pcr) .* sign(s);
Pw = P-Pr;
figure(1)
clf
%subplot 211
plot(N, P, N, Pr, '--', N, Pw, '-.')
title('Doubly Fed Induction Generator')
ylabel('Real Power, Watts')
xlabel('Rotor Speed, RPM')
legend('Stator Output', 'Rotor Input', 'Mechanical')
grid on
%subplot 212
figure(2)
plot(N, Qr)
title('Doubly Fed Induction Generator')

```

```
ylabel('Rotor Reactive Power, VARs')  
xlabel('Rotor Speed, RPM')  
grid on
```

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6.685 Electric Machines
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