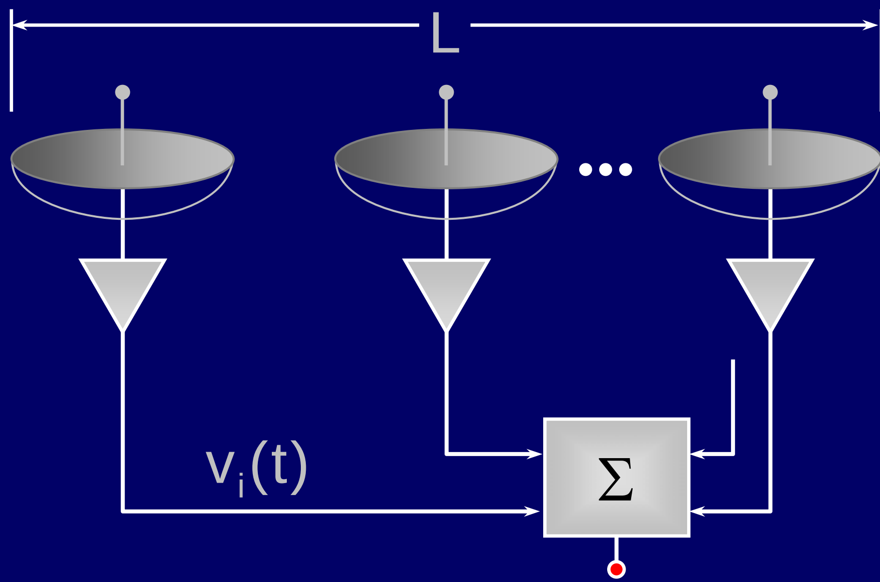
Synthetic Aperture Radar (SAR)

Basic Concept of SAR:

Consider fixed array

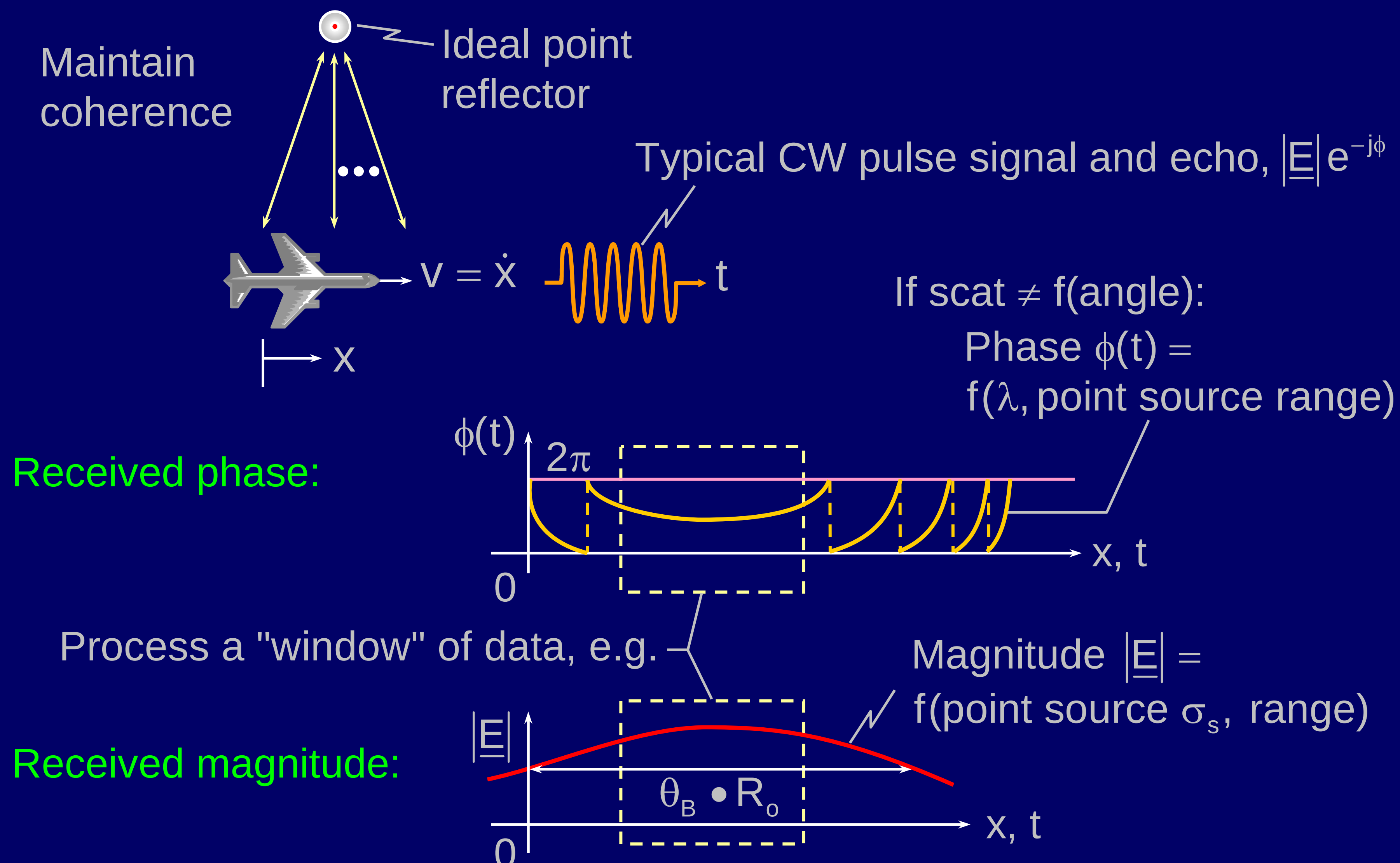


⇒ Angular resolution $\theta_B \cong \lambda / L$

Claim Equivalent to Moving Antenna:

Assumes phase coherence in oscillator during each image

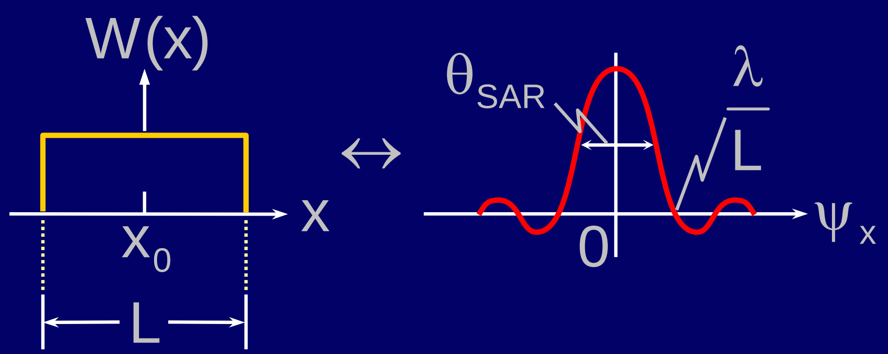
Synthetic Aperture Radar (SAR)



We seek 2-D source characterization: σ_s = f(azimuth, range)

Resolution of "Unfocused" SAR

Reconstruction of image: first select R, x_o of interest



$$W(x) \bullet \underline{E}(x) \text{ observed}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow$$

$$W(\psi) * \underline{E}(\psi) = \hat{\underline{E}}(\psi_x)$$

SAR angular resolution: $\theta_{SAR} \cong \lambda / L_{SAR} \geq \lambda / R\theta_B = D/R$

$\theta_B \cong \lambda / D$

Resolution of "Unfocused" SAR

SAR angular resolution: $\theta_{SAR} \cong \lambda / L_{SAR} \geq \lambda / R\theta_B = D/R$

$\theta_B \cong \lambda / D$

Lateral spatial resolution $= \theta_{SAR} \bullet R \cong D$

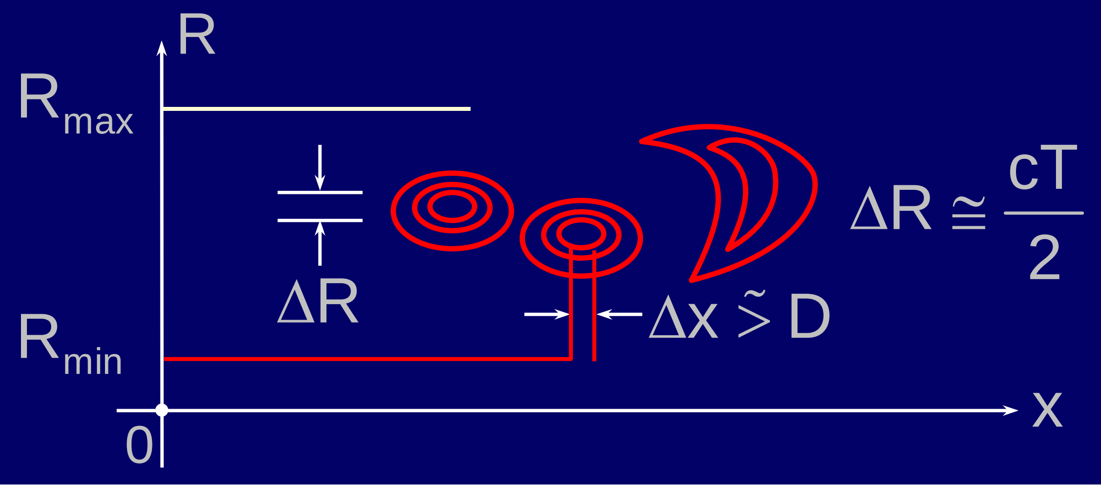
(want small D, large θ_B, large L)

Range resolution $\cong cT/2$

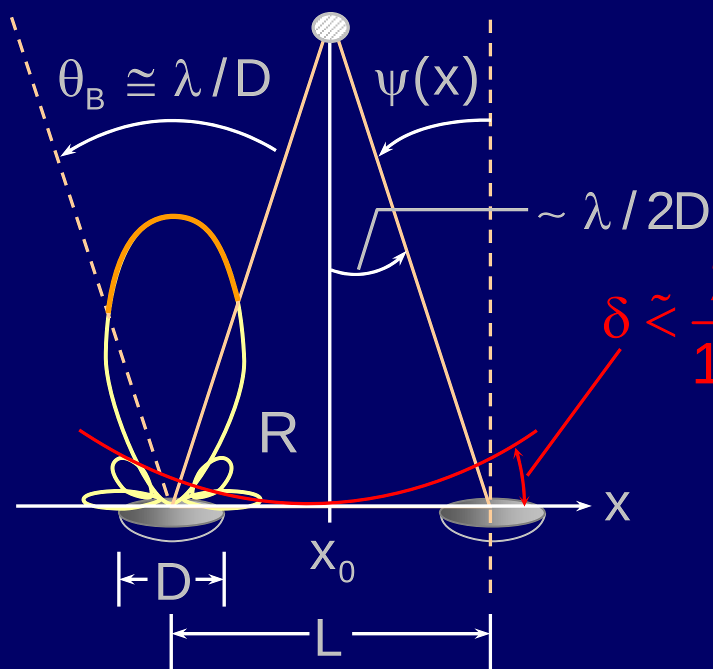
Note: If antenna is steered toward the target, increasing L, the lateral resolution can be < D.

Then repeat for all R, x_0

Yields image:



Phase-Focused SAR

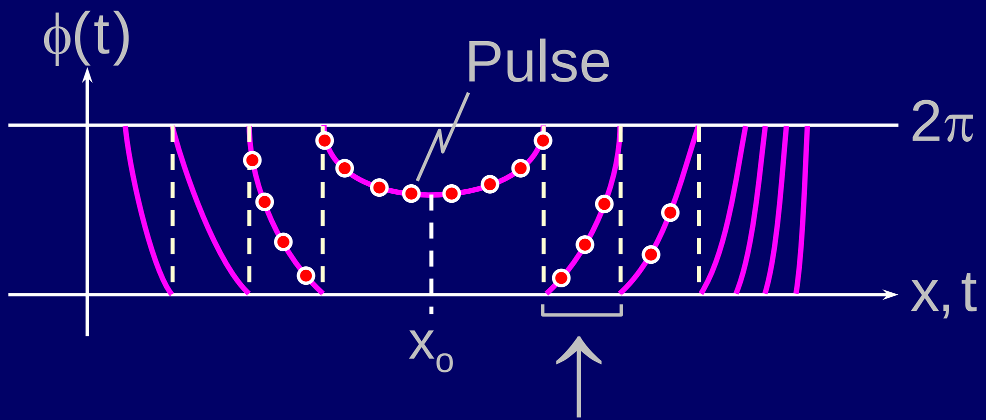


$$\delta \tilde{<} \frac{\lambda}{16} \Rightarrow \begin{cases} R^2 + \left(\frac{L}{2}\right)^2 = (R+\delta)^2 \cong R^2 + 2\delta R \\ \delta \cong \frac{L^2}{8R} \le \frac{\lambda}{16} \\ \text{therefore } R \tilde{\geq} \frac{2L^2}{\lambda} \quad \text{"farfield" and focusing is unnecessary} \end{cases}$$

Phase focusing is required if $\delta \tilde{>} \lambda/16$,
so the target is in the SAR near field,
i.e., if $R \tilde{<} 2L^2/\lambda = 2\lambda R^2/D^2$ (where $L = \lambda/\theta_{SAR} = \lambda R/D$).

Phase-Focused SAR

Solution: $W(x) \rightarrow \underline{W}'(x)$ with phase correction

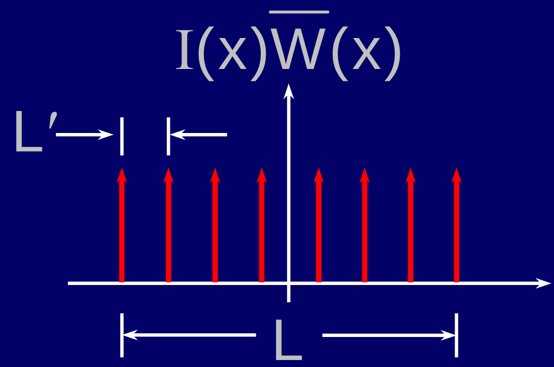


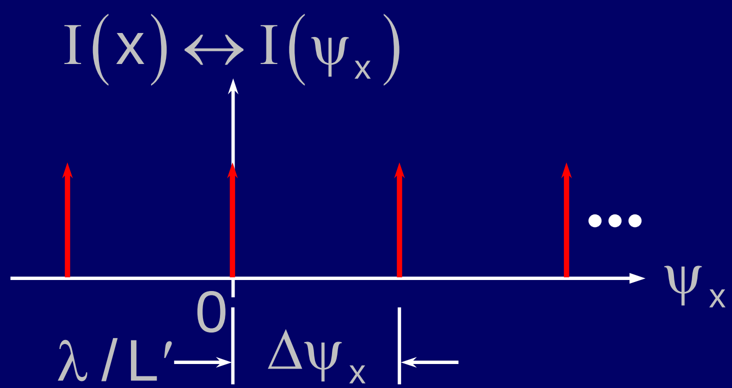
PRF must be higher; to reconstruct $\phi(t)$ we need at least 2 samples per phase wrap.

Note: With phase focusing, L increases and spatial resolution in x ca be less than D, using beamsteering (e.g. a phased array)

Also: L.O. phase drift can be corrected if bright point sources exist in scene.

Required Pulse-Repetition Frequency (PRF)





$$W(x) \bullet I(x) \bullet \underline{\hat{E}}(x) \quad \text{Observed}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

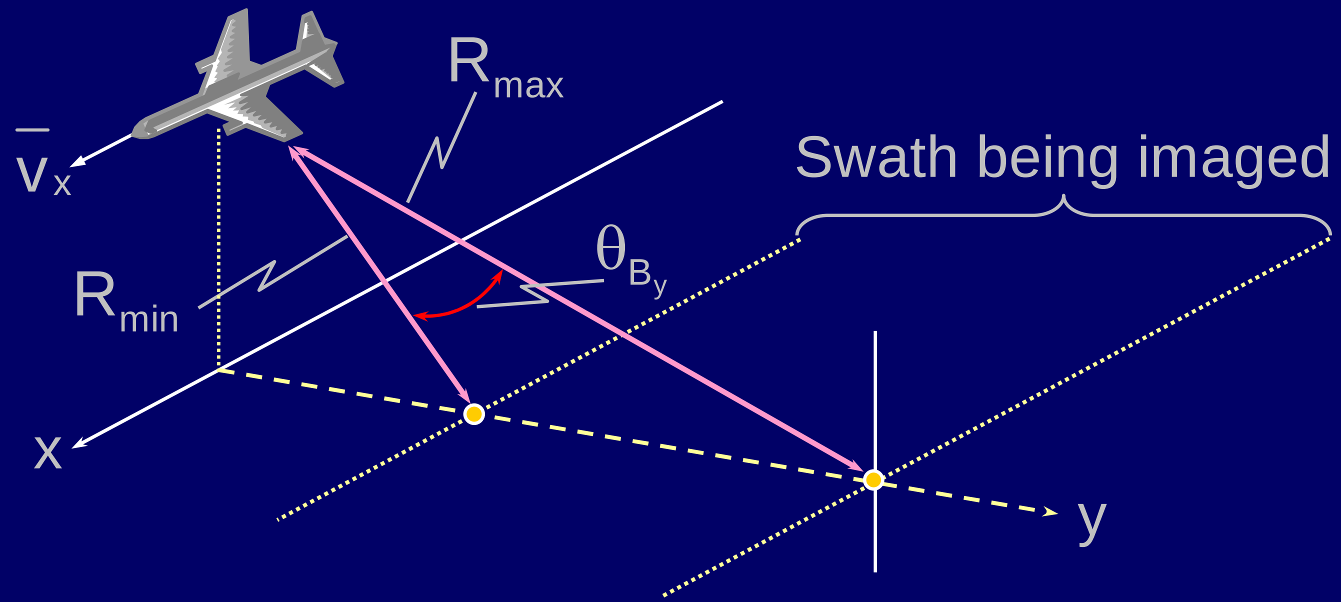
$$W(\psi_x) * I(\psi_x) * \underline{E}(\psi_x) = \underline{\hat{E}}(\psi_x)$$

$PRF = v/L'$ (e.g., $v =$ aircraft velocity)

Want $\Delta\psi_x = \lambda/L' >> \lambda/D$, or $D >> L'$

Therefore want PRF $>>$ v/D

PRF Impacts SAR Swath Width

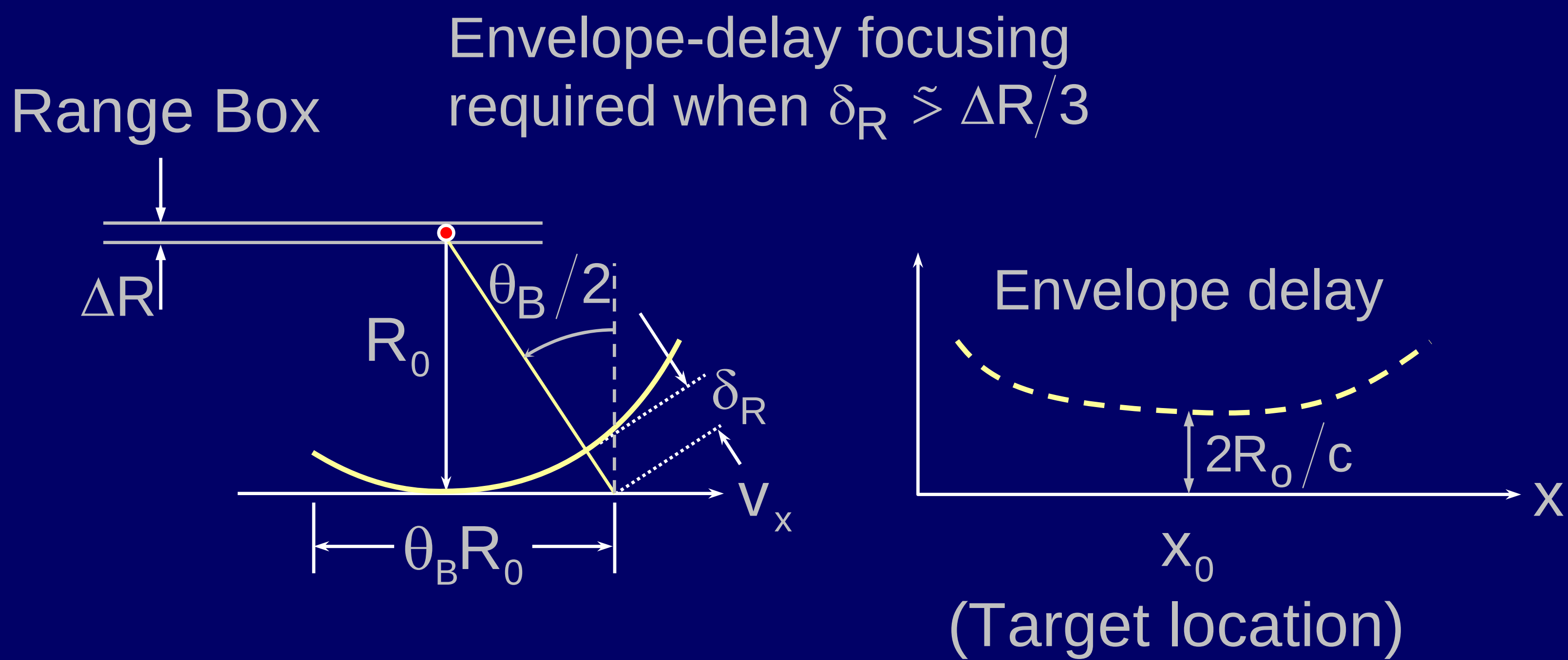


Don't want echoes to overlap for successive pulses.

$$\text{Therefore let } 2(R_{max} - R_{min})/c \lesssim \left(\frac{1}{PRF}\right) = L'/v_x << \frac{D}{v_x}$$

Implies that large swath widths require large $\frac{D}{v_x}$ and yield poorer spatial resolution.

Envelope Delay Focusing



Compensation for both envelope delay and phase delay is needed for very high spatial resolution [i.e., for small ΔR (large B)] and large $\theta_B R_0$.

SAR Intensity Estimates: Speckle

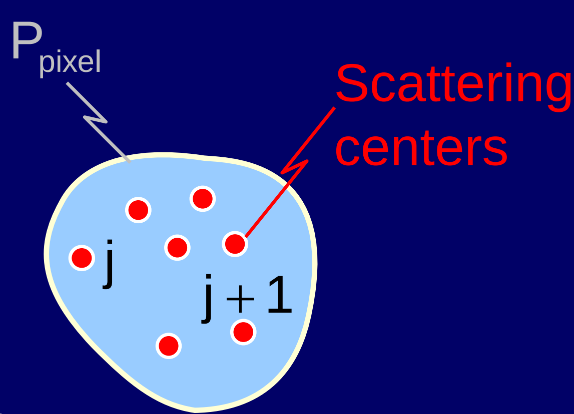
For each SAR pixel, the estimated cross-section is:

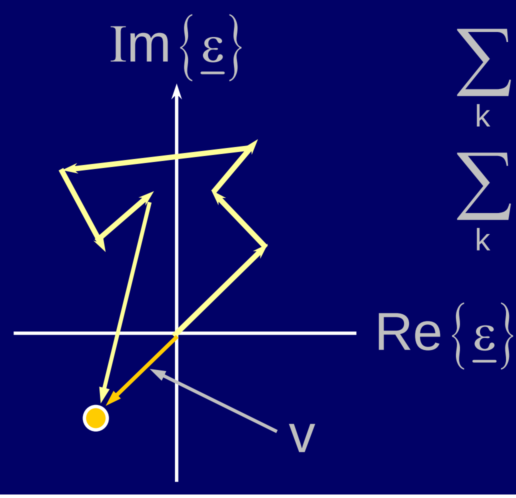
$$\hat{\sigma} = K\left|\sum_{i,j} w(x_i)\underline{E}_{ij}\right|^2 \quad \text{where} \begin{cases} K = \text{range-dependent constant} \\ i = \text{pulse number} \\ j = \text{subscattering index} \end{cases}$$

Many subscatters j per pixel

Simplifying: $$\hat{\sigma} = \left|\sum_{k=1}^{N} \underline{\varepsilon}_k\right|^2 = \left|\sum_{k=1}^{N} a_k e^{j(\omega t + \phi_k)}\right|^2 = |\underline{r}|^2$$

random variable (a_k)   uniformly over 2π typically (ϕ_k)





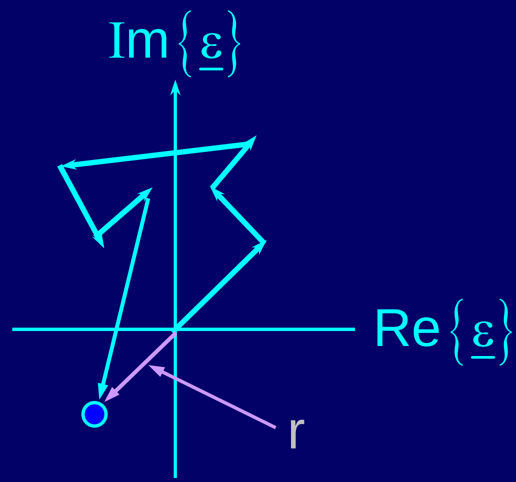
$$\sum_k \text{Re}\{\underline{\varepsilon}\} = \text{g.r.v.z.m.}$$

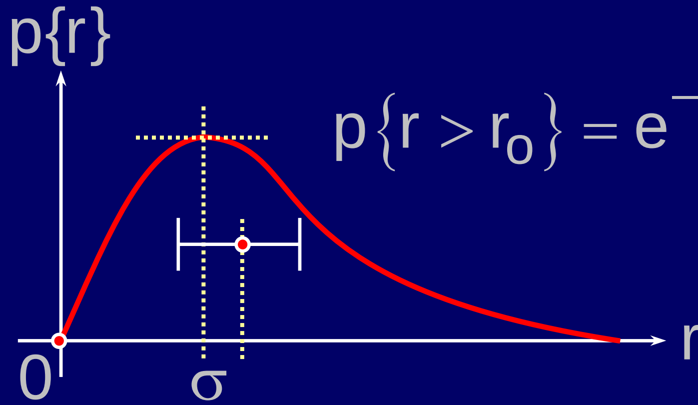
$$\sum_k \text{Im}\{\underline{\varepsilon}\} = \text{g.r.v.z.m.}$$

SAR Intensity Estimates: Speckle

$$p\left\{\left|\sum \underline{\varepsilon}_k\right|\right\} = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$$

where $\left|\sum \underline{\varepsilon}_k\right|$ = "r"





$$p\{r > r_o\} = e^{-(r_o/\sigma)^2/2}$$

$$E[r] = \sigma\sqrt{\pi/2}$$

$$E\left[r^2\right] = \sigma^2(2 - \pi/2) \propto |\underline{\varepsilon}|^2$$

Because we are adding (averaging) phasors, not scalars

$$\sqrt{E\left[r - \bar{r}\right]^2} \cong 2\sigma/3 \neq f(N)!$$

Thus raw SAR images, full spatial resolution,

have $\dfrac{\text{STD deviation}}{\text{mean intensity}} \triangleq Q \cong \dfrac{2\sigma/3}{\sigma\sqrt{\pi/2}} = 0.53 \Rightarrow$ very grainy images

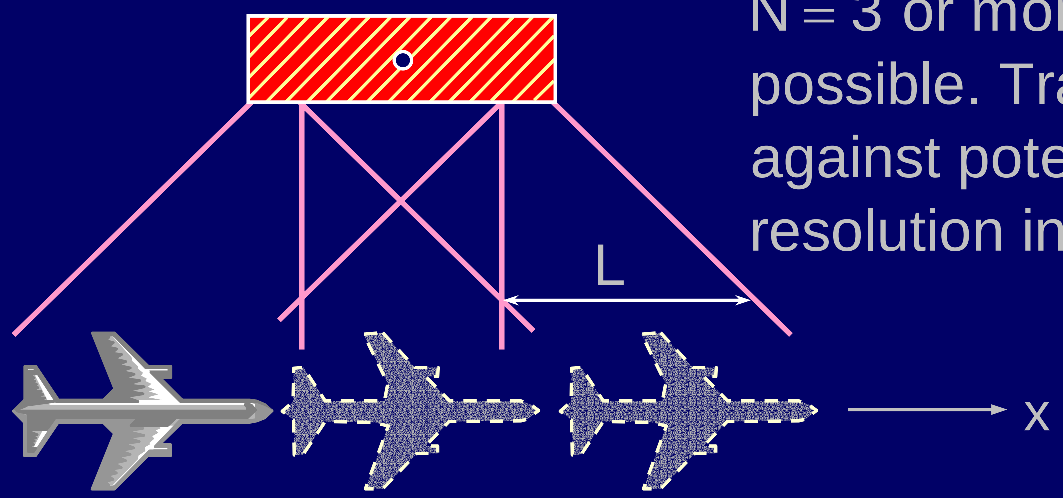
Reduction of SAR Speckle

Alternate ways to reduce "speckle" by averaging pixel intensities:

1) Blur image by averaging M^2 pixels $\Rightarrow Q = 0.53/M$
(using large $B\tau$ signals yields high resolution, can be smoothed)

2) Average using spatial diversity

$$Q = \frac{0.53}{\sqrt{N}}$$

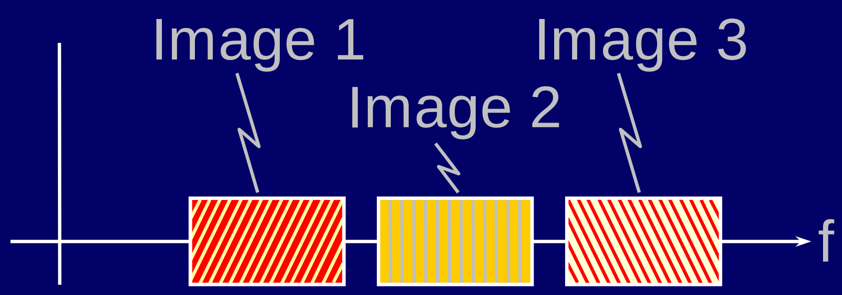


N = 3 or more looks possible. Trade against potential resolution in x.

Reduction of SAR Speckle

Alternate ways to reduce "speckle" by average pixel intensities:

3) Average using frequency diversity, where each band yields an independent image. Note that the same total bandwidth could alternatively yield more range resolution and pixels for averaging.

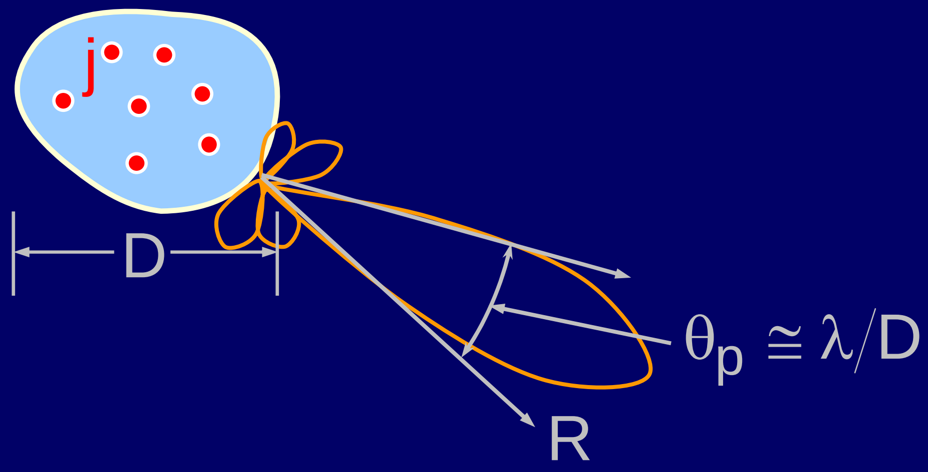


$$Q = \frac{0.53}{\sqrt{N}}$$

N Bands

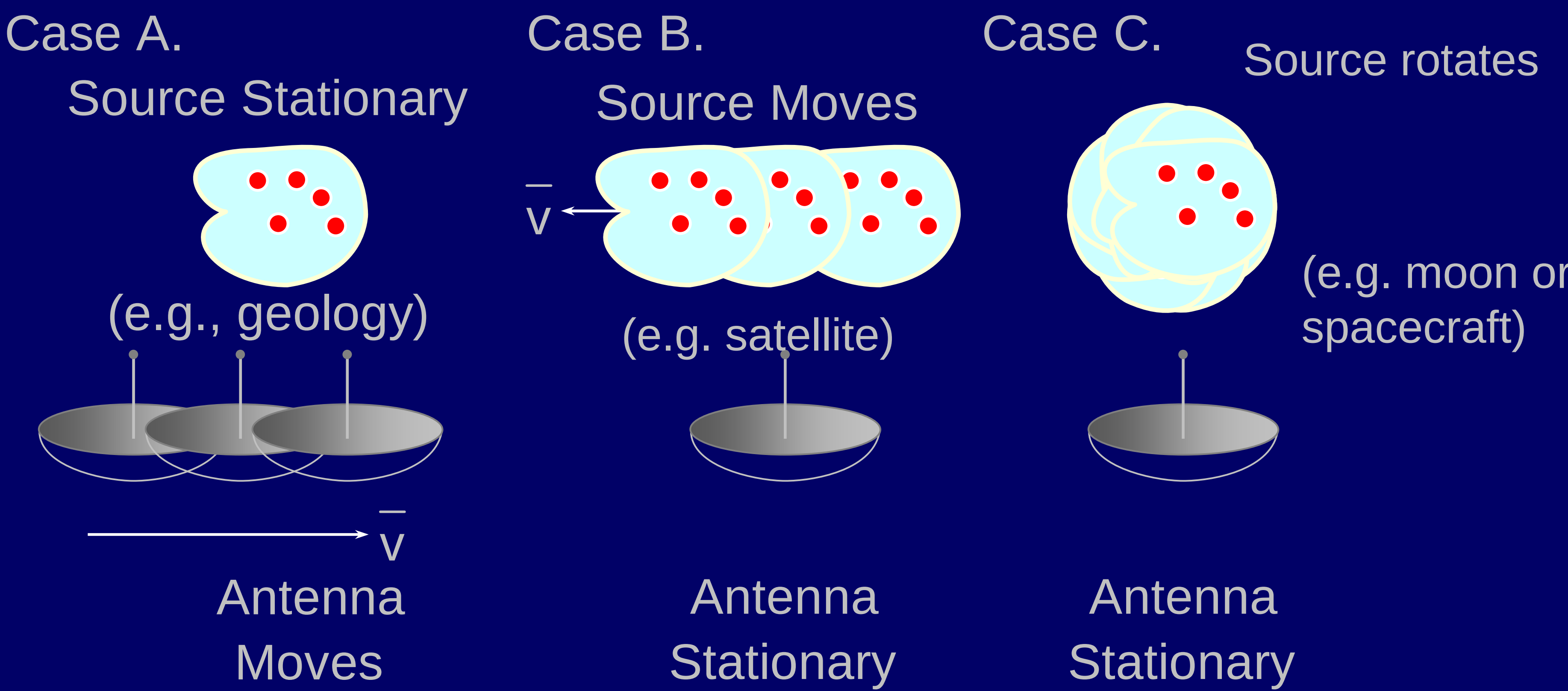
Reduction of SAR Speckle

4) Time averaging works only if source varies. For example a narrow swath permits high PRF and an increase in N, but unless the antenna translates more than ~λR/D between pulses [R = range, D = pixel width (m)], then adjacent pulse returns are correlated.



In general, reasonably smooth images need > 8 levels, so $Q' = S.D./M \cong (1/3)/4$ levels = 1/12, versus $Q \cong 1/2$. To reduce standard deviation by 6, need $N \gtrsim 6^2 = 36$ looks.

Alternative SAR Geometries and Applications



Velocity dispersion within inverse SAR (ISAR) source (e.g., a moving car) can displace its apparent position laterally.

ESTIMATION

Examples taken from Remote Sensing
Principles are Nonetheless Widely Applicable

Linear and Non-Linear Problems
Gaussian and Non-Gaussian Statistics

Professor David H. Staelin
Massachusetts Institute of Technology

Linear Estimation: Smoothing and Sharpening

Smoothing reduces image speckle and other noise; sharpening or "deconvolution" increases it.

Sharpening can "de-blur" images, compensating for diffraction or motion induced blurring.

Audio smoothing and sharpening are similar.

Consider antenna response example; we observe:

$$T_A\left(\overline{\psi}_A\right) = \frac{1}{4\pi}\int_{4\pi} G\left(\overline{\psi}_A - \overline{\psi}_s\right) T_B\left(\overline{\psi}_s\right) d\Omega_s$$

Linear Estimation: Smoothing and Sharpening

Consider antenna response example; we observe:

$$T_A\left(\overline{\psi}_A\right) = \frac{1}{4\pi}\int_{4\pi} G\left(\overline{\psi}_A - \overline{\psi}_s\right) T_B\left(\overline{\psi}_s\right) d\Omega_s$$

$$T_A\left(\overline{\psi}_A\right) \cong \frac{1}{4\pi} G\left(\overline{\psi}\right) * T_B\left(\overline{\psi}\right) \quad \text{(small solid angles)}$$

$$\updownarrow \qquad \updownarrow \quad \updownarrow \; \updownarrow$$

$$\underline{T}_A\left(\overline{f}_\psi\right) = \frac{1}{4\pi}\underline{G}\left(\overline{f}_\psi\right) \bullet \underline{T}_B\left(\overline{f}_\psi\right)$$

↑ Cycles/Radian ↑ Antenna Spectral Response

$$\left[\text{i.e., } \underline{G}\left(f_{\psi x}, f_{\psi y}\right) = \iint G\left(\psi_x, \psi_y\right) e^{-j2\pi\left(\psi_x f_{\psi x} + \psi_y f_{\psi y}\right)} d\psi_x d\psi_y\right]$$

Example: Square Uniformly-Illuminated Aperture

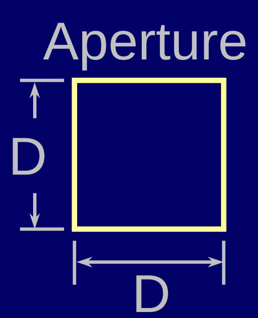
$$T_A(\overline{\psi}_A) \cong \frac{1}{4\pi} G(\overline{\psi}) * T_B(\overline{\psi}) \quad \text{(small solid angles)}$$

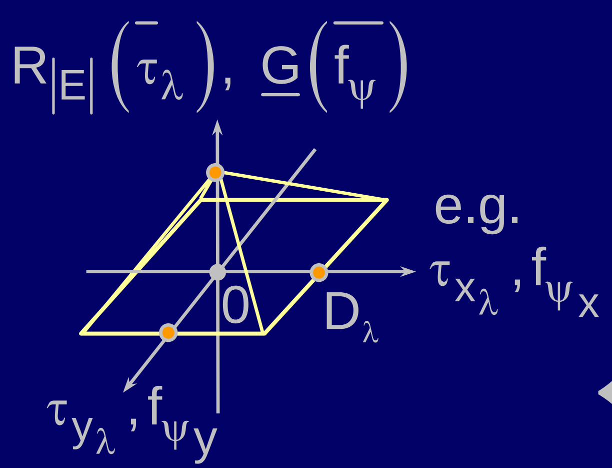
$$\updownarrow \qquad\qquad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$\underline{T}_A(\overline{f}_\psi) = \frac{1}{4\pi} \underline{G}(\overline{f}_\psi) \bullet \underline{T}_B(\overline{f}_\psi)$$

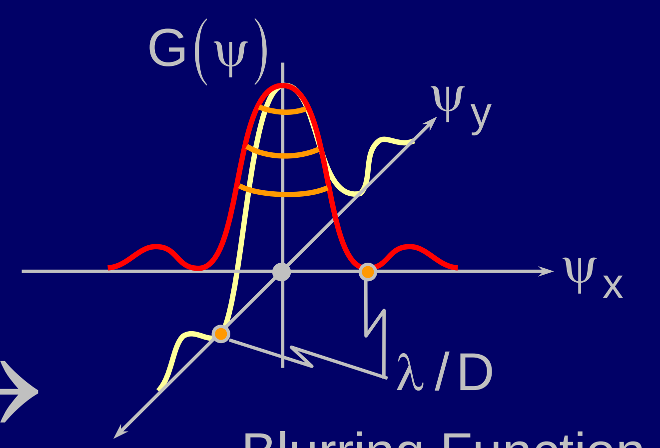
$$\downarrow \qquad\qquad \downarrow$$

Cycles/Radian Antenna Spectral Response





$\longleftrightarrow$



Blurring Function

Try:

$$\hat{T}_B(\overline{f}_\psi) = \frac{4\pi \underline{T}_A(\overline{f}_\psi)}{\underline{G}(\overline{f}_\psi)}$$

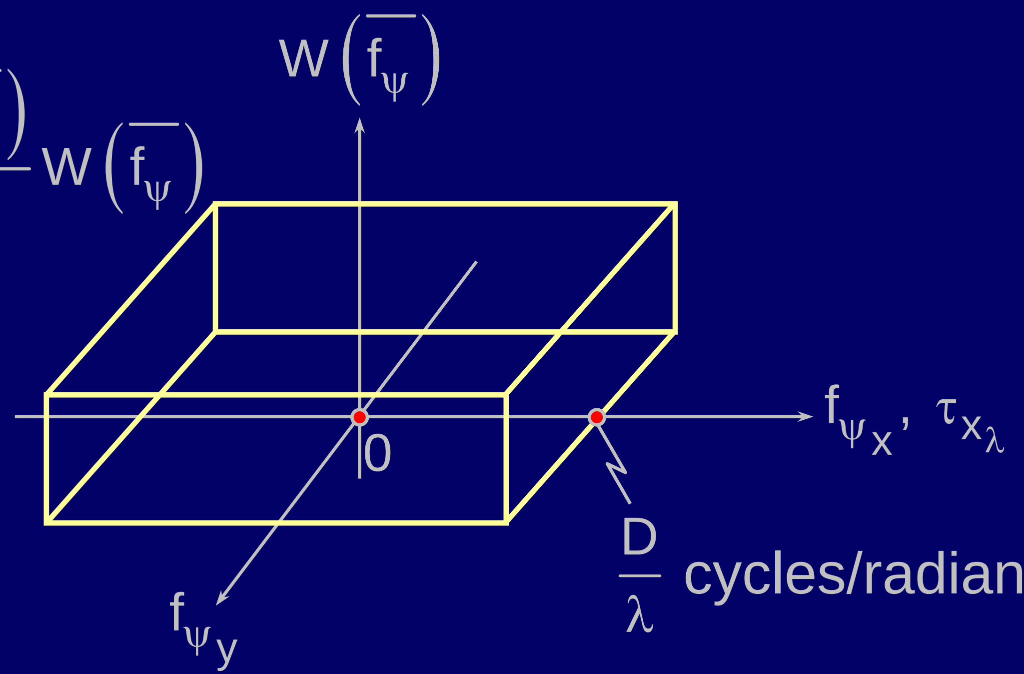
This restores high-frequency components, but $\leftarrow$ goes to zero for $\left|\overline{f}_{\psi x}\right| > D/\lambda$!

Example: Square Uniformly-Illuminated Aperture

$$\hat{T}_B\left(\overline{f_\psi}\right) = \frac{4\pi \underline{T}_A\left(\overline{f_\psi}\right)}{\underline{G}\left(\overline{f_\psi}\right)}$$

This restores high-frequency components, but $\underline{G}\left(\overline{f_\psi}\right) \leftarrow$ goes to zero for $\left|\overline{f_{\psi x}}\right| > D/\lambda$!

Try: $$\hat{T}_B\left(\overline{f_\psi}\right) = \frac{4\pi \underline{T}_A\left(\overline{f_\psi}\right)}{\underline{G}\left(\overline{f_\psi}\right)} W\left(\overline{f_\psi}\right)$$



The window function $W\left(\overline{f_\psi}\right)$ avoids the singularity.

The "principal solution" uses a boxcar W(s).

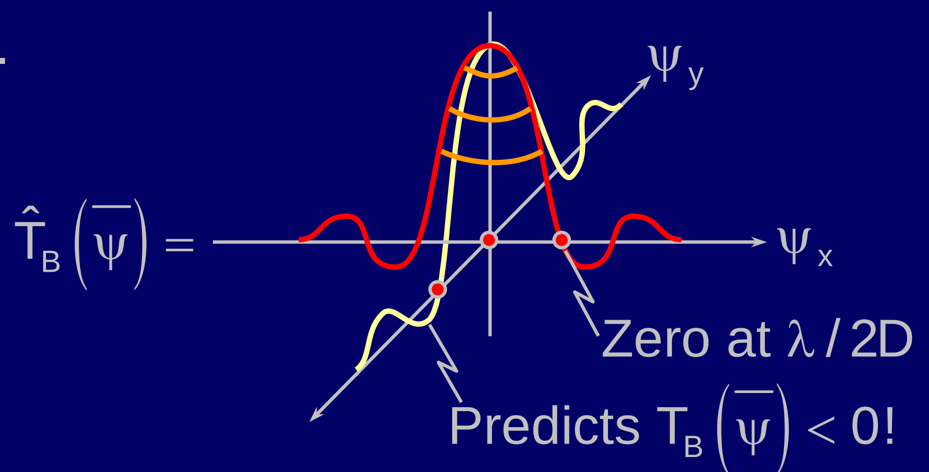
Example: Square Uniformly-Illuminated Aperture

Consider the restored (sharpened) image of a point source, $T_B(\overline{\psi}) = \delta(\overline{\psi})$:

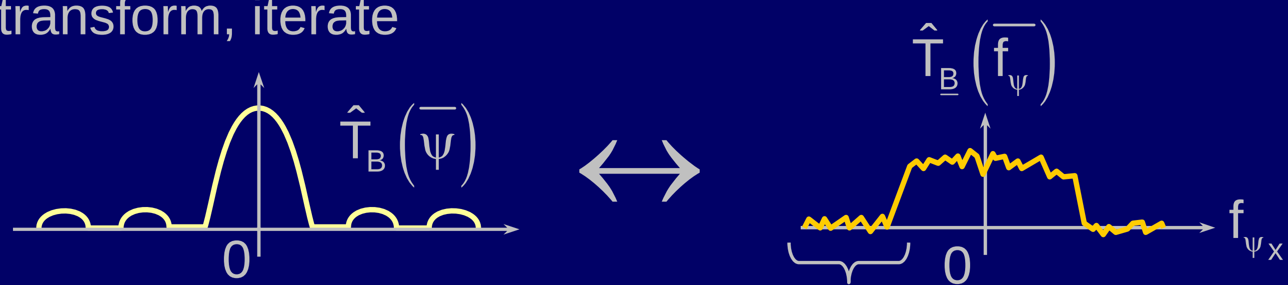
Then $T_{\underline{B}}(\bar{f}_\psi) = 1$, so $\hat{T}_{\underline{B}}(\overline{f_\psi}) = 4\pi \underline{T}_A(\bar{f}_\psi)\, W(\bar{f}_\psi) / G(\bar{f}_\psi) = W(\overline{f_\psi})$,

since $T_A(\bar{f}_\psi) = G(\bar{f}_\psi)\, T_B(\bar{f}_\psi) / 4\pi$

$\hat{T}_B(\overline{\psi})$ = 2-D sinc function for a uniformly illuminated rectangular antenna aperture.



Can clip, transform, iterate



Can set to zero (*apriori* information)

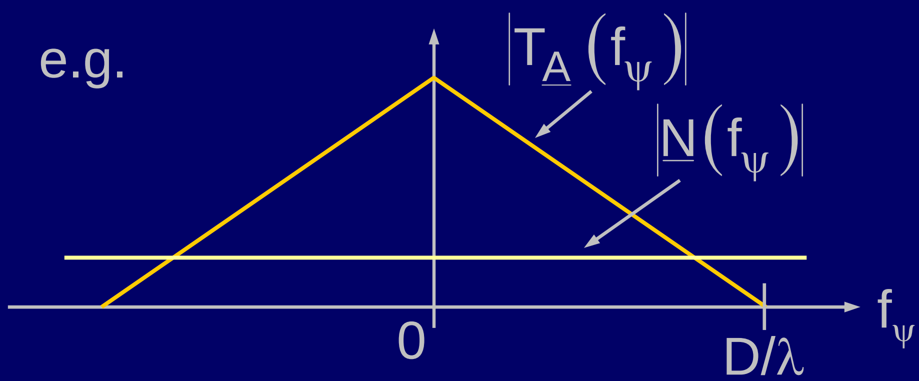
Sharpening Noisy Images

$$\hat{\underline{T}}_A(\overline{f}_\psi) = \underline{T}_A(\overline{f}_\psi) + \underline{N}(\overline{f}_\psi)$$

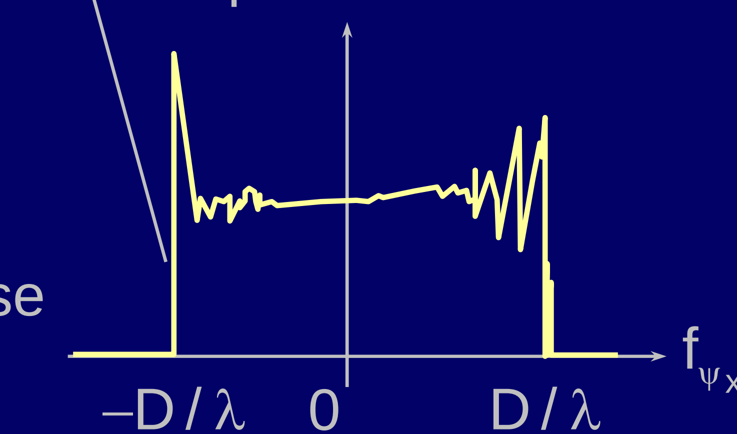
↑ Truth ↑ ΔT_{rms}

Amplifies white noise $\underline{N}(\overline{f}_\psi)$ for small $\underline{G}(\overline{f}_\psi)$

e.g.



$\hat{\underline{T}}_B(\overline{f}_\psi)$ for point source



→ 0! Amplifier Noise

$$\hat{\underline{T}}_B(\overline{f}_\psi) = \left[4\pi \hat{\underline{T}}_A(\overline{f}_\psi) / \underline{G}(\overline{f}_\psi)\right] W(\overline{f}_\psi)$$

Therefore optimize $W(\overline{f}_\psi)$

Sharpening Noisy Images

Therefore optimize $W\left(\overline{f_\psi}\right)$:

$$\text{Minimize } E\left[\left|\underline{T}_B\left(\overline{f_\psi}\right)-\frac{4\pi \underline{W}\left(\overline{f_\psi}\right)}{G\left(\overline{f_\psi}\right)}\left(\underline{T}_{A_0}\left(\overline{f_\psi}\right)+\underline{N}\left(\overline{f_\psi}\right)\right)\right|^2\right]\overset{\Delta}{=}Q$$

$\partial Q/\partial W=0$ yields:

$$\underline{W}\left(\overline{f_\psi}\right)_{optimum}=\frac{E\left[\left|\underline{T}_{A_o}\left(\overline{f_\psi}\right)\right|^2+\frac{1}{2}\underline{T}_{A_o}\left(\overline{f_\psi}\right)\underline{N}\left(\overline{f_\psi}\right)^*+\frac{1}{2}\underline{T}^*_{A_o}\left(\overline{f_\psi}\right)\underline{N}\left(\overline{f_\psi}\right)\right]}{E\left[\left|\underline{T}_{A_o}\left(\overline{f_\psi}\right)+\underline{N}\left(\overline{f_\psi}\right)\right|^2\right]}$$

If $E\left[\underline{T}_A\underline{N}\right]=0$, then

$$\underline{W}_{optimum}\left(\overline{f_\psi}\right)=\frac{1}{1+\dfrac{E\left[\left|\underline{N}\left(\overline{f_\psi}\right)\right|^2\right]}{E\left[\left|\underline{T}_{A_o}\left(\overline{f_\psi}\right)\right|^2\right]}}\left(\cong\frac{1}{1+\frac{N}{S}}\right)$$

Sharpening Noisy Images

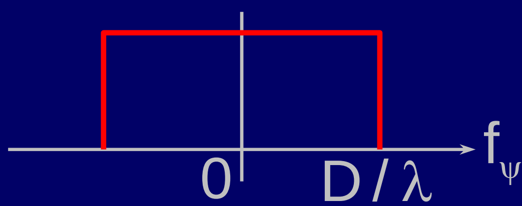
If $E[\underline{T}_A \underline{N}] = 0$, then

The weighting $(1+N/S)^{-1}$ is widely used

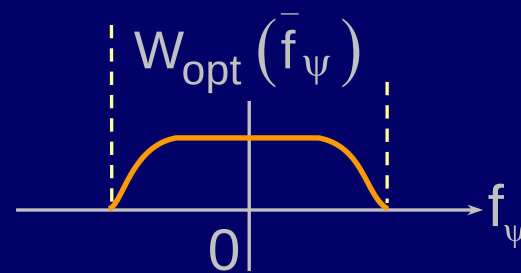
$$\underline{W}_{optimum}\left(\overline{f}_\psi\right) = \frac{1}{1+\dfrac{E\left[\left|\underline{N}\left(\overline{f}_\psi\right)\right|^2\right]}{E\left[\left|\underline{T}_{A_o}\left(\overline{f}_\psi\right)\right|^2\right]}} \quad \left(\cong \frac{1}{1+\dfrac{N}{S}}\right)$$

$\underline{W}_{opt}\left(\overline{f}_\psi\right)$

for $\underline{N} = 0$:



$\underline{N} \neq 0$



$\underline{W}_{opt}\left(\overline{f}_\psi\right) \Rightarrow$ wider beam (lower spatial resolution), lower sidelobes.

Can be used for restoration of blurred images of all types: photographs TV, $T_A\left(\overline{\psi}_A\right)$ maps, SAR images, filtered speech, etc.