MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Final Exam
May 21, 2007
180 minutes
Johnson Ice Rink

1. This examination consists of five problems. Work all problems.
2. This examination is closed book.
3. Please summarize your solutions in the spaces provide in this examination packet. Draw all sketches neatly and clearly where requested. Remember to label ALL important features of any sketches.
4. All problems have equal weight.
5. Make sure that your name is on this packet and on each examination booklet.

Root Locus Rules

Rule 1 The number of branches, which are the paths of the closed-loop poles, is equal to the number of open-loop poles, P.

Rule 2 The branches start at the open-loop poles and end at the open-loop zeros. In addition to the Z explicit open-loop zeros in the transfer function, there are $P - Z$ open-loop zeros at infinity.

Rule 3 Branches of the root locus lie on the real axis to the left of an odd number of poles and zeros. Complex-conjugate pairs of poles and zeros are not counted, since they contribute no net angle to the real axis.

Rule 4 If a branch on the real axis lies between a pair of poles, the root locus must break away from the real axis somewhere between the poles. Similarly, if a branch on the real axis lies between a pair of zeros, there must be an entry point between that pair of zeros.

Rule 5 As K gets very large, $P - Z$ branches go to infinity. These branches approach asymptotes at angles to the real axis of

$$\alpha_n = \frac{(2n+1)180^\circ}{P-Z}$$

where $n = 0 \ldots (P - Z - 1)$ and the centroid of these asymptotes is on the real axis at

$$\sigma_a = \frac{\sum p_i - \sum z_j}{P-Z}.$$

Rule 6 The departure angles of the branches from an mth-order pole on the real axis are

$$\delta_n = \frac{(2n+1)180^\circ}{m}$$

if the mth-order pole is to the left of a even number of poles and zeros. If the mth-order pole is to the left of a odd number of poles and zeros, then the departure angles are

$$\delta_n = \frac{2n180^\circ}{m}.$$

Rule 7 If there are two or more excess poles than zeros ($P - Z \geq 2$), then for any gain K, the sum of the real parts of the closed-loop poles (or the average distance from the $j\omega$-axis) is constant.

Rule 8 Ignore remote poles and zeros when considering the root locus near the origin of the s-plane, and combine the poles and zeros near the origin when considering the root locus for remote poles and zeros.

Rule 9 The departure angle from a complex-conjugate pole can be found by considering the angle condition on a small circle around the pole. The result is found by summing all the angles from open-loop zeros and subtracting all the angles from all other poles

$$\delta_P = 180^\circ + \sum \angle z_i - \sum \angle\, p_j$$

The approach angle to a complex-conjugate zero follows similarly

$$\delta_Z = 180^\circ - \sum \angle z_i + \sum \angle\, p_j$$

This sum only needs to be calculated once for each complex pair, since the root-locus diagram is symmetric above and below the real axis.

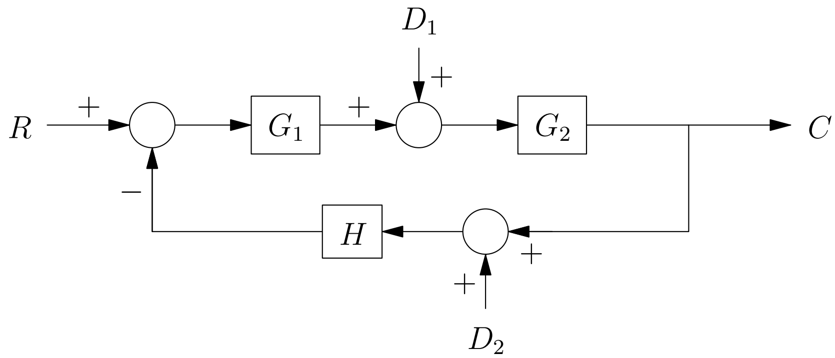
Rule 10 The break-away (entry) points from (to) the real axis between a pair of poles (zeros) can be found either by geometric construction or by finding the local maxima (minima) of the gain function $K(\sigma)$, solving

$$\frac{\partial}{\partial \sigma} K(\sigma) = \frac{\partial}{\partial \sigma} \left| \frac{1}{L(\sigma)} \right| = 0.$$

Fortunately, this level of accuracy is rarely necessary.

Problem 1

Consider the following system:



(a) Which disturbance input, D_1 or D_2, has the greater effect on the output? Assume that $G_1 H > 1$.

(b) Let $G_1 = 1$, $G_2 = \frac{10^4}{10^{-6}s+10^{-3}s+1}$, and $H = \frac{10^{-4}s+1}{10^{-5s}+1}$.

(i) Sketch the root locus for the system with input R and output C.

(ii) Sketch the root locus for input D_1 and output C.

(c) For the same values of G_1, G_2, and H as above, draw the Bode plot of the open-loop system. Use the asymptotic method for both magnitude and phase.

Match the following G_1 compensators to the phrase that best describes the resulting system. You may assume that each compensator results in a sufficiently stable system.

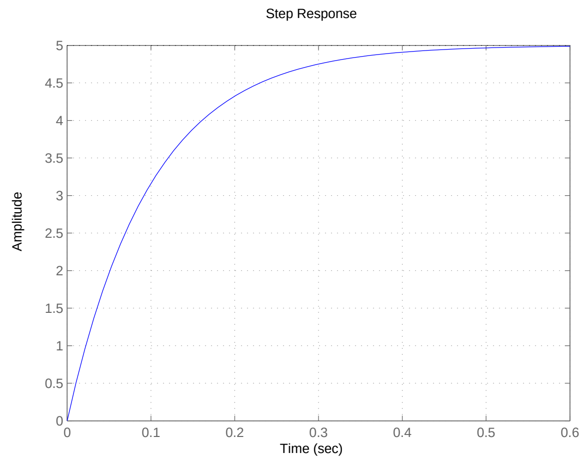
a. highest ω_c	1. $G_1 = \frac{(10^{-2}s+1)}{(10^{-1}s+1)}$ (lag)
b. highest ϕ_M	2. $G_1 = \frac{(10^{-4.5}s+1)}{10(10^{-5.5}s+1)}$ (lead)
c. lowest tracking error at 10rps.	3. $G_1 = \frac{1}{10^2 s+1}$ (dominant pole)

Problem 2

There is a plant with transfer function

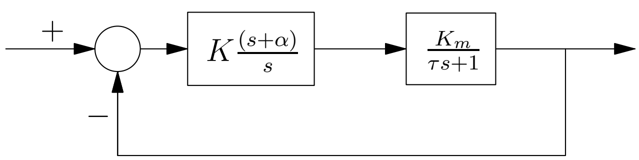
$$\frac{V_o}{V_i}(s) = \frac{K_m}{\tau_m s + 1}$$

(a) Given the measured response of the plant to a $\frac{1}{2}$V step input is:



Find K_m and τ_m.

(b) Now, we compensate the system as follows:



We want $\zeta = 0.7071$ and $\tau_{\text{env}} = \frac{1}{\zeta\omega_n} = 0.01$. Find the closed loop pole locations that allow this.

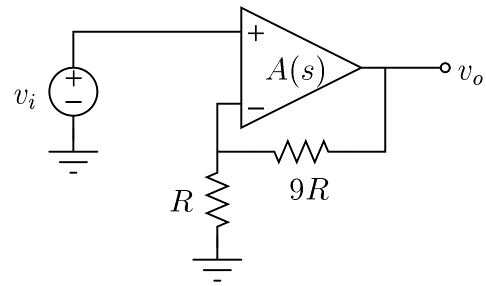
(c) Find the value of α that gives the desired closed loop pole locations. Approximations are ok. (Hint: consider the angle criterion.)

(d) Find K necessary to set closed loop poles as desired. Reasonable approximations are ok. (Hint: use magnitude condition.)

(e) Finally, sketch the root locus of the system for $K > 0$.

Problem 3

(a) It is possible to model an operation amplifier that has very high open loop gain at DC using an $A(s)$ that includes a pole at the origin. The amplifier is connected as:



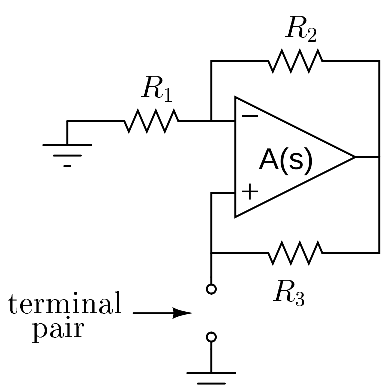
It is found that

$$\frac{V_o}{V_i} = \frac{10}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1}$$

Determine the open loop transfer function $A(s)$ in terms of ω_n and ζ.

(b) Assume for this part of the problem that the amplifier connection shown above has $\zeta = 0.707$ and thus the closed-loop $\omega_h = \omega_n$. What is the value of the system crossover frequency in this case?

(c) The amplifier is now connected as:

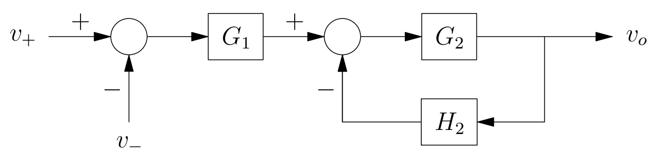


Draw a block diagram for the connection that will let you find the input admittance at the indicated terminal pair. To do this the input to your block diagram should be the voltage V_i applied to the terminal pair and the reponse should be the resultant input current I_i.

In the limiting case of low frequency operation such that $|A(j\omega)| \approx \infty$, what is the input conductance $\frac{I_i}{V_i}$?

Problem 4

A block diagram for an operational amplifier that uses minor-loop compensation is:



We see from this block diagram that the open loop gain of the operational amplifier is

$$A(s) = \frac{V_o}{V_+ - V_-} = \frac{G_1 G_2}{1 + G_2 H_2}$$

(a) Assume $G_1 = \frac{1.4\times10^{-4}}{10^{-7}s+1}$ and that the G_2H_2 product is large at frequencies of interest. The amplifier is connected as a voltage follower, with $V_- = V_o$. Find H_2 such that the system has a single pole loop transmission over a wide range of frequencies, and crosses over with 45° of phase margin in the follower connection.

If H_2 is realized with a single passive component around a high gain second stage, what component type and value should be used?

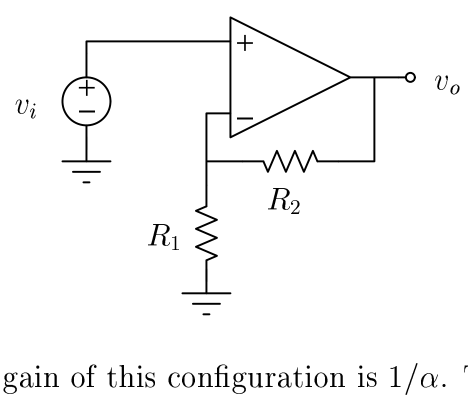
(b) Another operational amplifier (same block diagram) has

$$G_1 = \frac{10^{-4}}{10^{-5}s+1} \quad , \quad G_2 = \frac{10^9}{(10^{-5}s+1)^2}$$

Determine H_2 so that when this amplifier is connected as a voltage follower, the major loop has a single pole loop transmission over a wide range of frequencies and crosses over at 10^6 radians/second with nearly 90° of phase margin.

(c) There is a problem with the circuit described in part (b). Describe it.

(d) The amplifier with G_1 and G_2 as in part (b) is connected as a non-inverting amplifier:

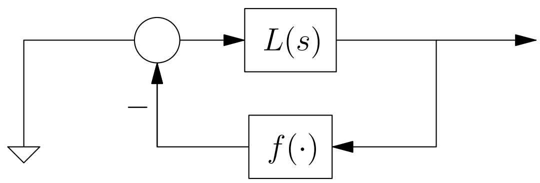


$$\frac{R_1}{R_1+R_2} = \alpha$$

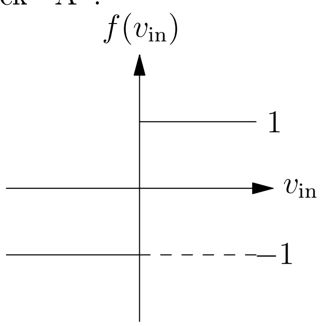
The ideal closed loop gain of this configuration is $1/\alpha$. The operational amplifier is decompensated ($H_2 = 0$). How small must α be for this configuration to be stable?

Problem 5: "Oscillations"

Our goal is to make a constant-amplitude oscillator, and we have a few different blocks available to us.

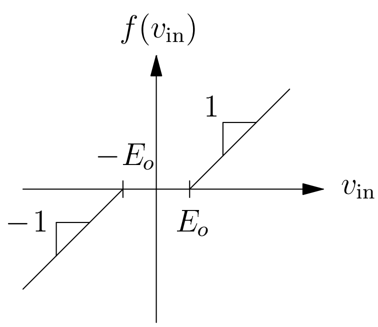


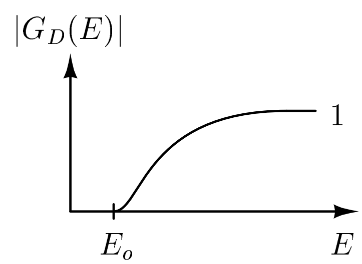
Nonlinear block "A":



$G_D(E) = \frac{4}{\pi E}, \angle = 0°$

Nonlinear block "B":





$\angle G_D(E) = 0°$

For each of the following choices of $L(s)$, choose a nonlinear block that could be used to build an oscillator. It is not necessary to do a formal describing function analysis on a gain-phase plane. If you wish, you can justify your answers with root locus plots and qualitative discussions of the behavior of the relevant describing functions.

(a) $L(s) = \frac{K}{(\tau s+1)}^3$

(b) $L(s) = \frac{K(\tau s+1)^2}{s^3}$

(c) $L(s) = \frac{K(\tau s+1)^2}{(\alpha\tau s+1)^3}$, $\alpha > 1$

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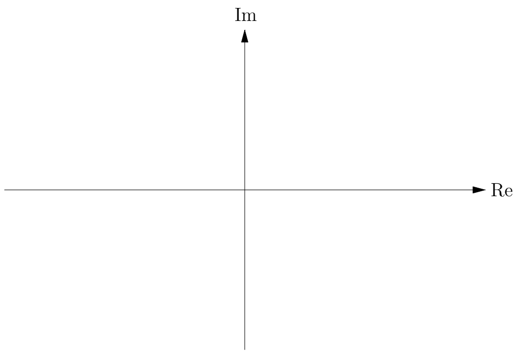
Final Exam Answer Sheet

Problem	Score	Grader
1		
2		
3		
4		
5		
Total		

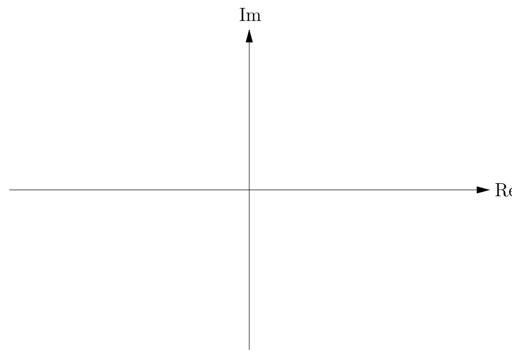
Problem 1

a. Which disturbance has the greater effect? ______

b. Root Locus for:



input R, output C



input D_1, output C

c. Match a G_1 to each of the statements.

a. ________________

b. ________________

c. ________________

Problem 2

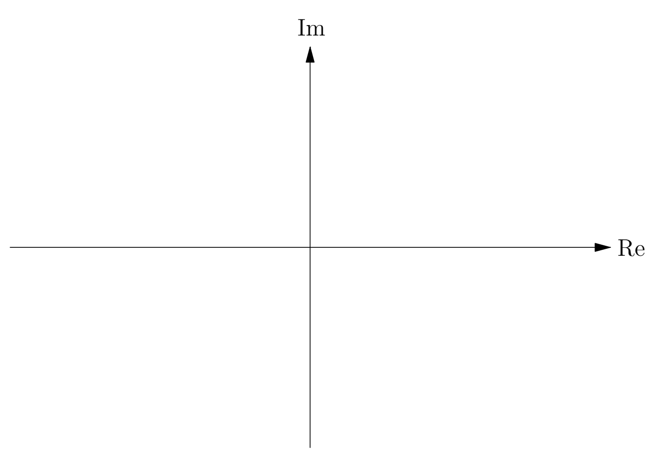
a. $K_m =$______, $\tau_m =$______

b. Closed loop poles at ______$\pm j$______

c. $\alpha \approx$______

d. $K \approx$______

e. Root locus (indicate values for initial pole locations.)



 Name:______________________________

Problem 3

a. $A(s) =$______

b. $\omega_c =$______

c. Bock diagram:

For $A(j\omega) \approx \infty$, $\frac{I_i}{V_i} =$______

Problem 4

a. The system crosses over with 45° of phase margin with $H_2 =$______

The required compensating component is ______

b. $H_2 =$______

c. The problem is:

d. The connection is stable for $\alpha <$______

Problem 5

a. Oscillations with block _______

Because

b. Oscillations with block _______

Because

c. Oscillations with block _______

Because