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Recursive function on M

Def. tree-depth(s) for s \in M

td(\lambda) ::= 0

td([s]t) ::= 1 + max{td(s), td(t)}
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Recursive Functions

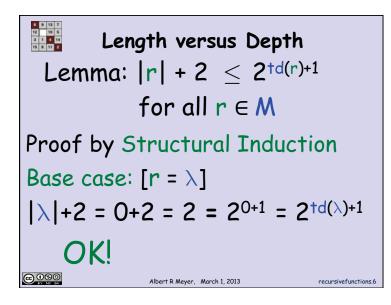
summary:

f: Data → Values
f(b) def'd directly for base b
f(cnstr(x)) def'd using f(x), x



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ecursivefunct





Size versus Depth

Constructor case: [r = [s]t] by ind. hypothesis:

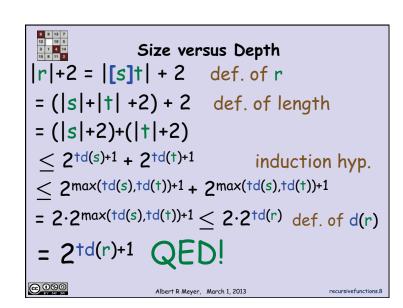
$$|s| + 2 \leq 2^{\mathsf{td}(s)+1}$$

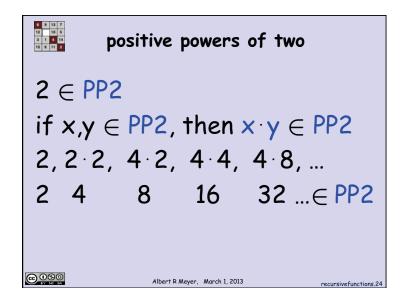
$$|+| + 2 < 2^{+d(+)+1}$$

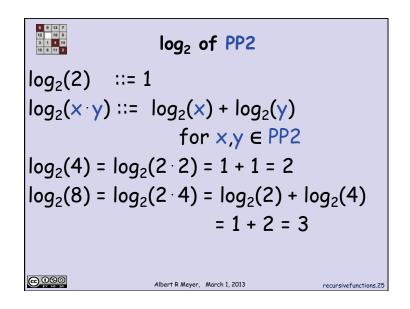
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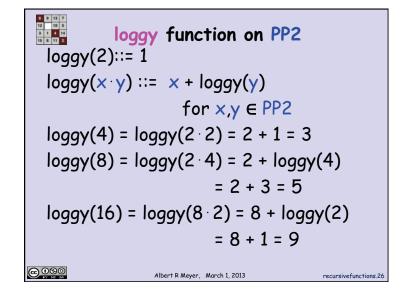
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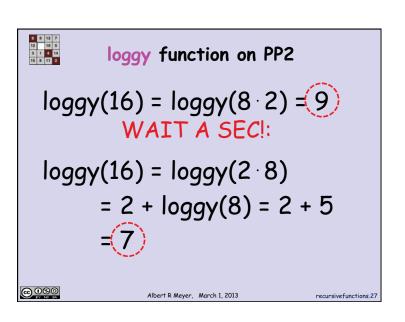
recursivefunctions.7











9 9 13 7 12 10 5 3 1 4 14 15 8 11 2

ambiguous constructors

The Problem: more than one way to construct elements of PP2 from $cnstrct(x,y) = x \cdot y$

16 = cnstrct(8,2) but also

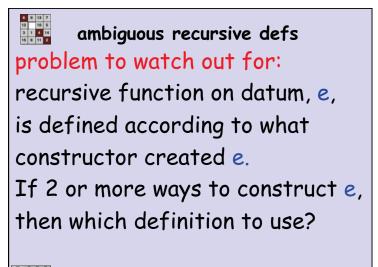
16 = cnstrct(2,8)

ambiguous



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