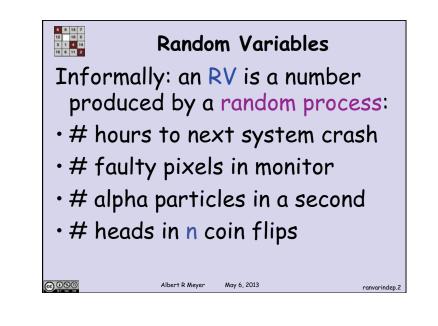


Intro to Random Variables
Example: Flip three fair coins

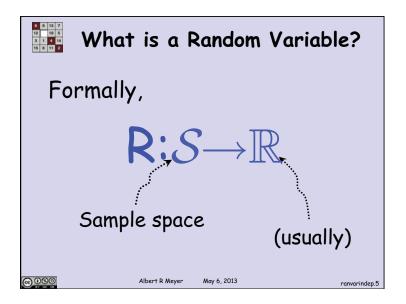
$$C ::= \#$$
 heads (Count)
 $A ::= \begin{cases} 1 & \text{if all Match,} \\ 0 & \text{otherwise.} \end{cases}$



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ranvarindep.4



Mutally Independent Variables Def: $R_1, R_2, ..., R_n$ are mutually indep RV's iff $[R_1=a_1], [R_2=a_2], ..., [R_n=a_n]$ are mutually indep events for all a_1, a_2, \dots, a_n 0000 Albert R Meyer May 6, 2013

ranvar-mutual.7

What is a Random Variable?
R packages together the
events
$$[R = a]$$
 for $a \in \mathbb{R}$
Event properties carry
over to RV's directly

Mutally Independent Variables
Alternatively:

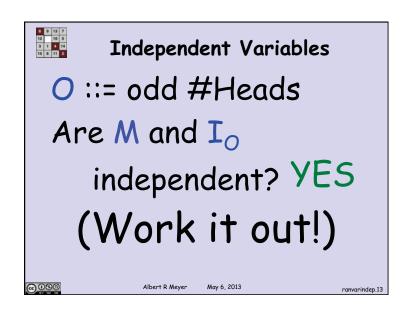
$$Pr[R_1=a_1 \text{ AND } R_2=a_2 \text{ AND}$$

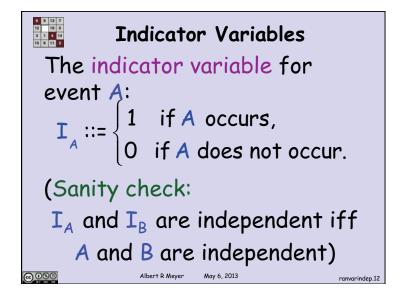
 $\cdots \text{ AND } R_n=a_n]$
 $= Pr[R_1=a_1] \cdot Pr[R_2=a_2] \cdot$
 $\cdots Pr[R_n=a_n]$

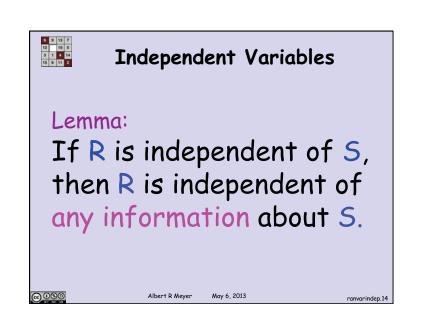
Independent Variables
Are C and M
independent? NO

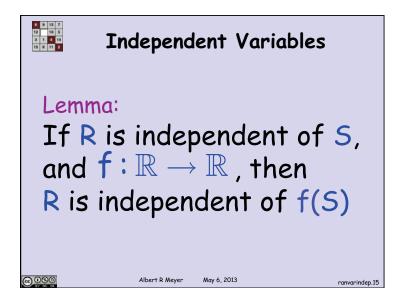
$$Pr[M=1] \cdot Pr[C=1] > 0$$

 $Pr[M=1 \text{ and } C=1] = 0$
NO
 $Pr[M=1 \text{ and } C=1] = 0$









k-way Independent Variables $H_i ::= indicator for Head on flip i \in [1,k]$ $O ::= \bigoplus_{i=1}^{k} H_i$ (mod 2 sum). Any k of them are independent, but not k+1-way independent since any k determine the remaining one.

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@ 080



k-way Independent Variables

k-way Independence:any k of the variables aremutually independent2-way is called pairwise

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@09

ranvar-mutual.1

@090

Pairwise Independent Variables

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ranvar-mutual 17

ranvar-mutual.19

Pairwise Independence sufficient for major applications (in later lecture).

Good to know, since pairwise holds in important cases where mutual does not.

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