In-Class Problems Week 6, Wed.

Problem 1.

Find the remainder of $26^{1818181}$ divided by 297. *Hint:* $1818181 = (180 \cdot 10101) + 1$; use Euler's theorem.

Problem 2. (a) Prove that 2012^{1200} has a multiplicative inverse modulo 77.

(b) What is the value of $\phi(77)$, where ϕ is Euler's function?

(c) What is the remainder of 2012^{1200} divided by 77?

Problem 3.

Prove that for any prime, p, and integer, $k \ge 1$,

$$\phi(p^k) = p^k - p^{k-1},$$

where ϕ is Euler's function. *Hint*: Which numbers between 0 and $p^k - 1$ are divisible by p? How many are there?

Note: This is proved in the text. Don't look up that proof.

Problem 4.

At one time, the Guinness Book of World Records reported that the "greatest human calculator" was a guy who could compute 13th roots of 100-digit numbers that were 13th powers. What a curious choice of tasks....

In this problem, we prove

$$n^{13} \equiv n \pmod{10} \tag{1}$$

for all *n*.

(a) Explain why (1) does not follow immediately from Euler's Theorem.

(b) Prove that

$$d^{13} \equiv d \pmod{10} \tag{2}$$

for $0 \le d < 10$.

(c) Now prove the congruence (1).

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