

Problem Set 8

Due: November 21

Reading: Counting Notes, II.§4–5; Notes on Generating Functions

Problem 1. Find the coefficients of

(a) x^{10} in $(x + (1/x))^{100}$

(b) x^k in $(x^2 - (1/x))^n$.

Problem 2. Suppose a generalized World Series between the Sox and the Cardinals involved $2n + 1$ games. As usual, the generalized Series will stop as soon as one team has won more than half the possible games.

(a) Suppose that when the Sox finally win the GSeries, the Cards have managed to win *exactly* r games (so $r \leq n$). How many possible win-loss patterns are possible for the Sox to win the GSeries in this way? Express your answer as a binomial coefficient.

(b) How many possible win-loss patterns are possible for the Sox to win the GSeries when the Cards win *at most* r games? Express your answer as a binomial coefficient.

(c) Give a combinatorial proof that

$$\sum_{i=0}^r \binom{n+i}{i} = \binom{n+r+1}{r}. \quad (1)$$

(d) Verify equation (1) by induction using algebra.

Problem 3. (a) ¹ Let a_n be the number of length n ternary strings (strings of the digits 0, 1, and 2) that contain two consecutive digits that are the same. For example, $a_2 = 3$ since the only ternary strings of length 2 with matching consecutive digits are 11, 22, and 33. Also, $a_0 = a_1 = 0$, since in order to have consecutive matching digits, a string must be of length at least two.

Find a recurrence formula for a_n .

(b) Show that

$$\frac{-x}{1-2x} + \frac{x}{(1-3x)(1-2x)}$$

is a closed form for the generating function of the sequence a_0, a_1, \dots

(c) Find real numbers r and s such that

$$\frac{1}{(1-2x)(1-3x)} = \frac{r}{1-2x} + \frac{s}{1-3x}.$$

(d) Use the previous results to write a closed form for the n th term in the sequence.

Problem 4. Suppose there are four kinds of doughnuts: plain, chocolate, glazed, and butterscotch. Write generating functions for the number of ways to select the flavors of n doughnuts, subject to the following different constraints.

(a) Each flavor occurs an odd number of times.

(b) Each flavor occurs a multiple of 3 times.

(c) There are no chocolate doughnuts and at most one glazed doughnut.

(d) There are 1, 3, or 11 chocolate doughnuts, and 2, 4, or 5 glazed.

(e) Each flavor occurs at least 10 times.

¹From Rosen, 5th ed., §6.1, Exercise 34.

Student's Solutions to Problem Set 8

Your name:

Due date: November 21

Submission date:

Circle your TA: David Jelani Sayan Hanson

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:
got help from:¹
and referred to:²

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	
2	
3	
4	
Total	

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¹People other than course staff.

²Give citations to texts and material other than the Fall '02 course materials.