Recitation 7
September 30, 2010

1. **Problem 2.35, page 130 in the text.** Verify the expected value rule

$$\mathbf{E}[g(X,Y)] = \sum_x \sum_y g(x,y) p_{X,Y}(x,y),$$

   using the expected value rule for a function of a single random variable. Then, use the rule for the special case of a linear function, to verify the formula

$$\mathbf{E}[aX + bY] = a\mathbf{E}[X] + b\mathbf{E}[Y],$$

   where a and b are given scalars.

2. Random variables X and Y can take any value in the set $\{1, 2, 3\}$. We are given the following information about their joint PMF, where the entries indicated by a * are left unspecified:
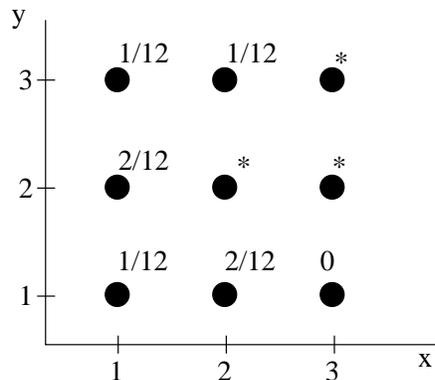
   (a) What is $p_X(1)$?
   (b) Provide a clearly labeled sketch of the conditional PMF of Y given that $X = 1$.
   (c) What is $\mathbf{E}[Y \mid X = 1]$?
   (d) Is there a choice for the unspecified entries that would make X and Y independent?

   Let B be the event that $X \leq 2$ and $Y \leq 2$. We are told that conditioned on B, the random variables X and Y are independent.

   (e) What is $p_{X,Y}(2, 2)$?
       (If there is not enough information to determine the answer, say so.)
   (f) What is $p_{X,Y|B}(2, 2 \mid B)$?
       (If there is not enough information to determine the answer, say so.)

3. **Problem 2.33, page 128 in the text.** A coin that has probability of heads equal to p is tossed successively and independently until a head comes twice in a row or a tail comes twice in a row. Find the expected value of the number of tosses.

6.041SC Probabilistic Systems Analysis and Applied Probability

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