MIT 6.035
Introduction to Program Analysis and Optimization

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Program Analysis

Compile-time reasoning about run-time behavior of program

- Can discover things that are always true:
  - “x is always 1 in the statement y = x + z”
  - “the pointer p always points into array a”
  - “the statement return 5 can never execute”
- Can infer things that are likely to be true:
  - “the reference r usually refers to an object of class C”
  - “the statement a = b + c appears to execute more frequently than the statement x = y + z”
- Distinction between data and control-flow properties

Transformations

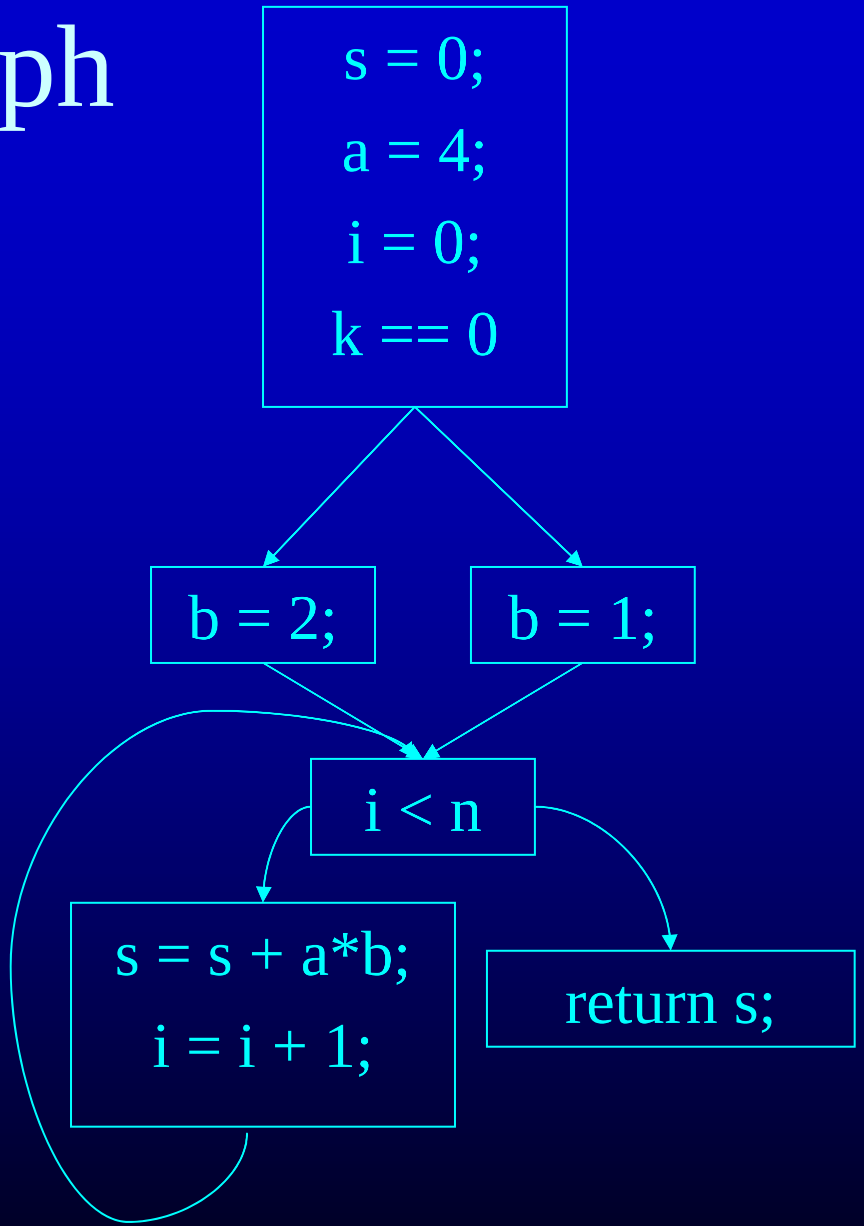
- Use analysis results to transform program
- Overall goal: improve some aspect of program
- Traditional goals:
  - Reduce number of executed instructions
  - Reduce overall code size
- Other goals emerge as space becomes more complex
  - Reduce number of cycles
    - Use vector or DSP instructions
    - Improve instruction or data cache hit rate
  - Reduce power consumption
  - Reduce memory usage

Control Flow Graph

- Nodes Represent Computation
  - Each Node is a Basic Block
  - Basic Block is a Sequence of Instructions with
    - No Branches Out Of Middle of Basic Block
    - No Branches Into Middle of Basic Block
    - Basic Blocks should be maximal
  - Execution of basic block starts with first instruction
  - Includes all instructions in basic block
- Edges Represent Control Flow

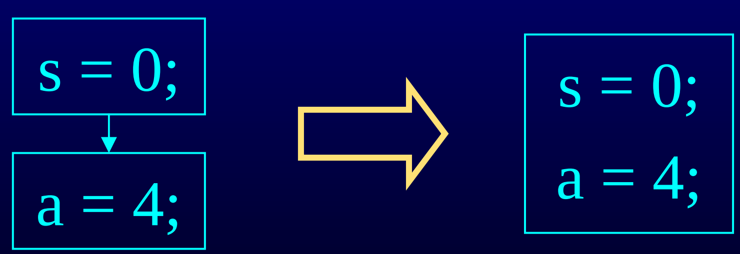
Control Flow Graph

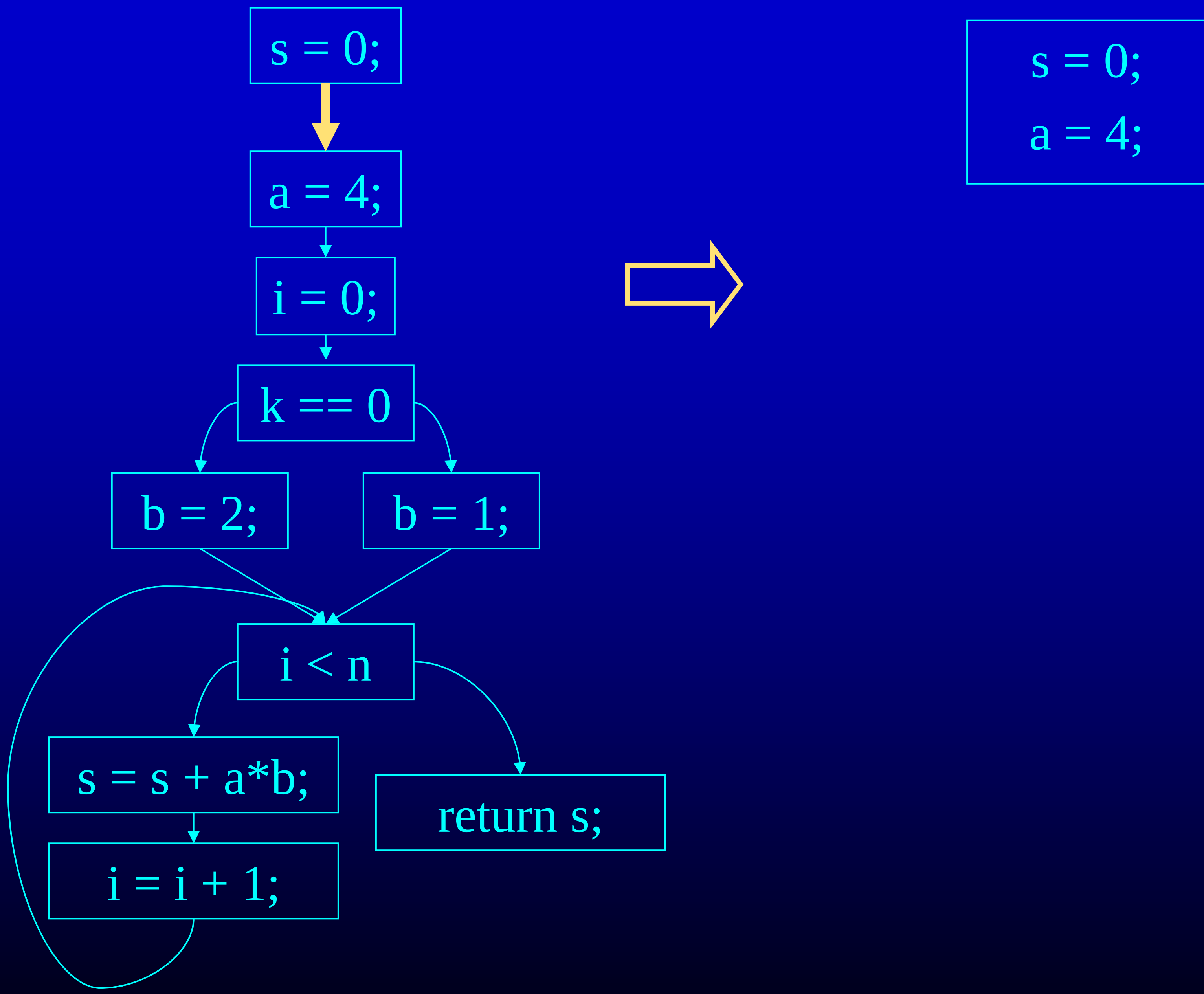
```
into add(n, k) {
    s = 0; a = 4; i = 0;
    if (k == 0) b = 1;
    else b = 2;
    while (i < n) {
        s = s + a*b;
        i = i + 1;
    }
    return s;
}
```



Basic Block Construction

- Start with instruction control-flow graph
- Visit all edges in graph
- Merge adjacent nodes if
  - Only one edge from first node
  - Only one edge into second node

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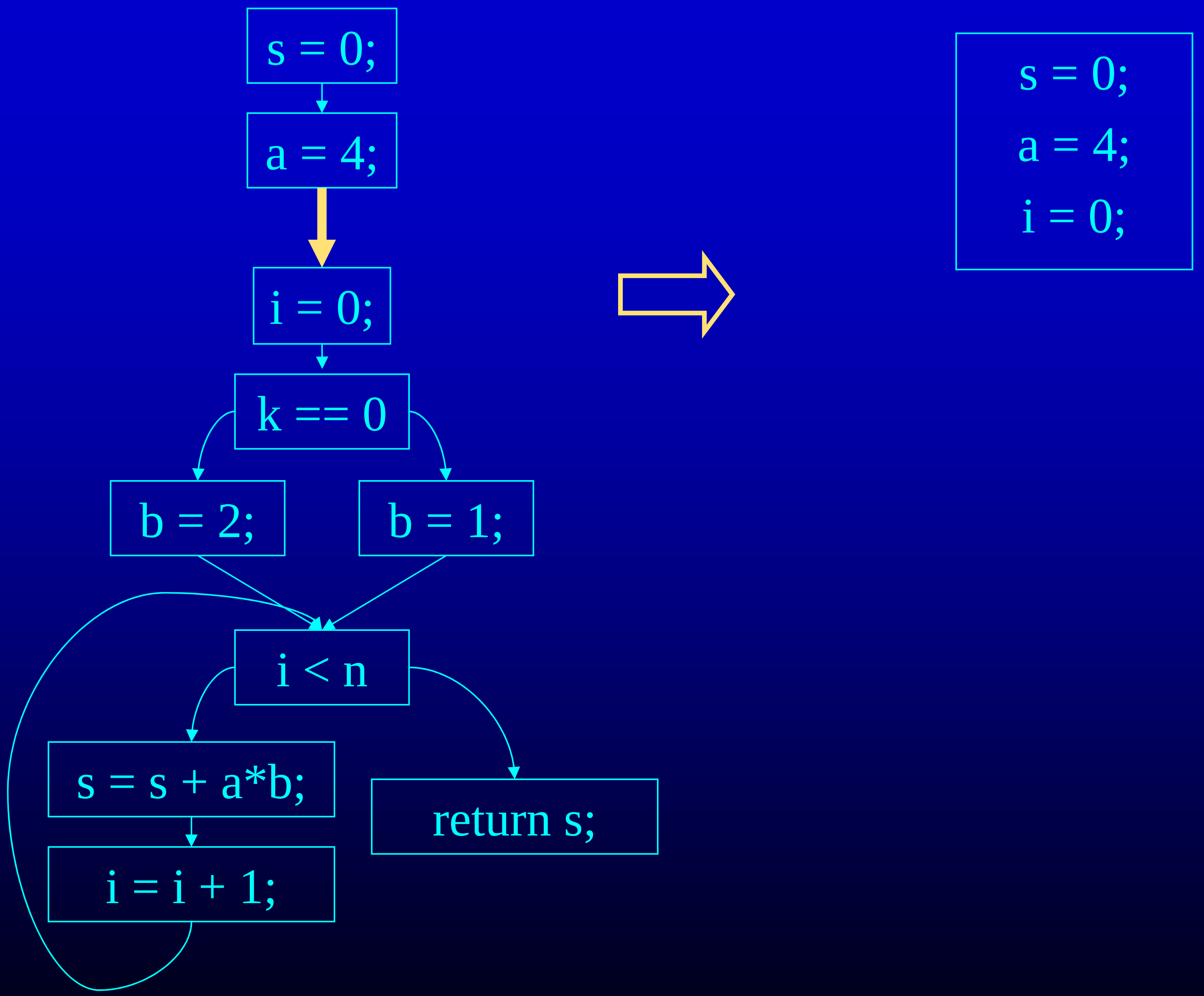

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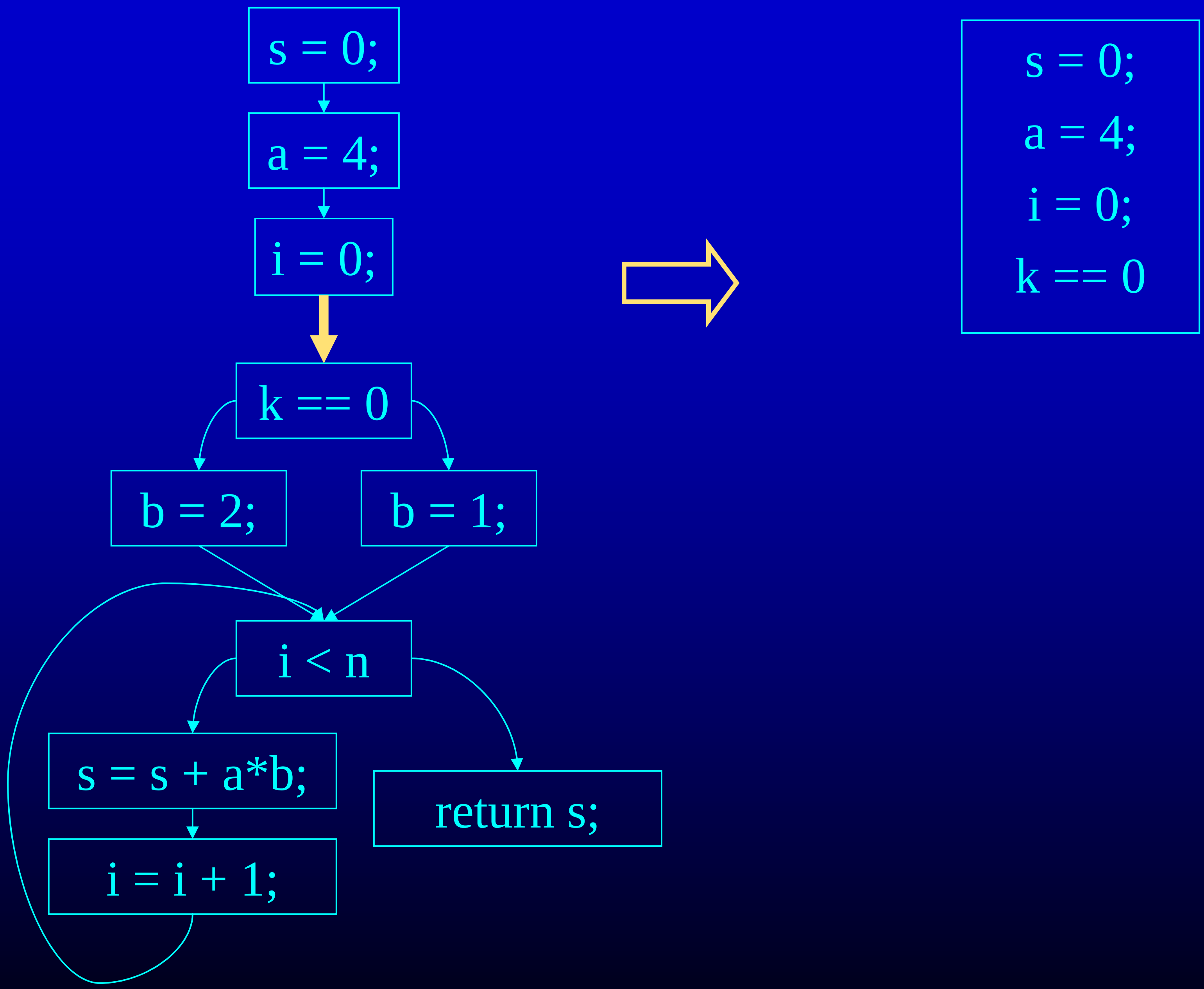
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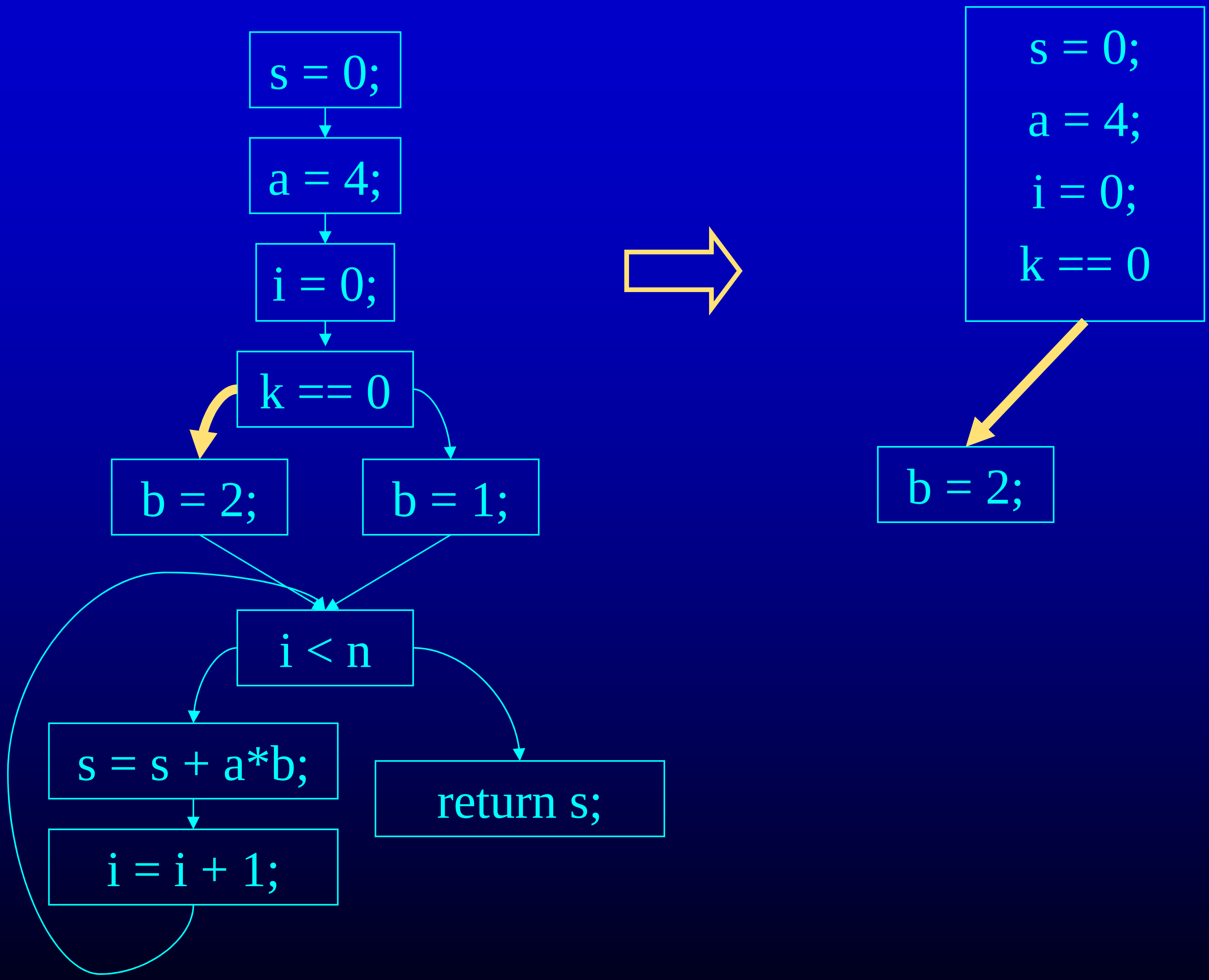
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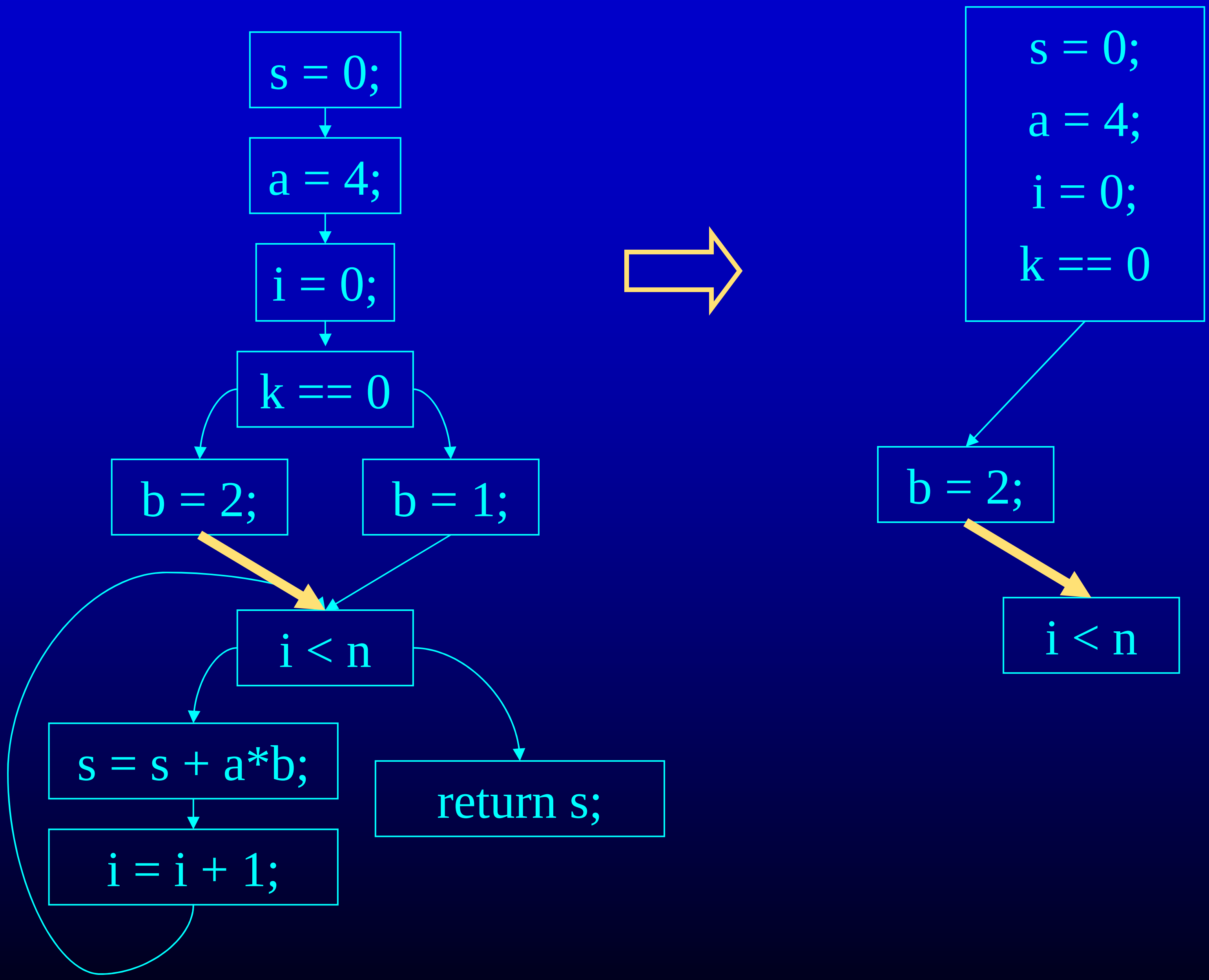
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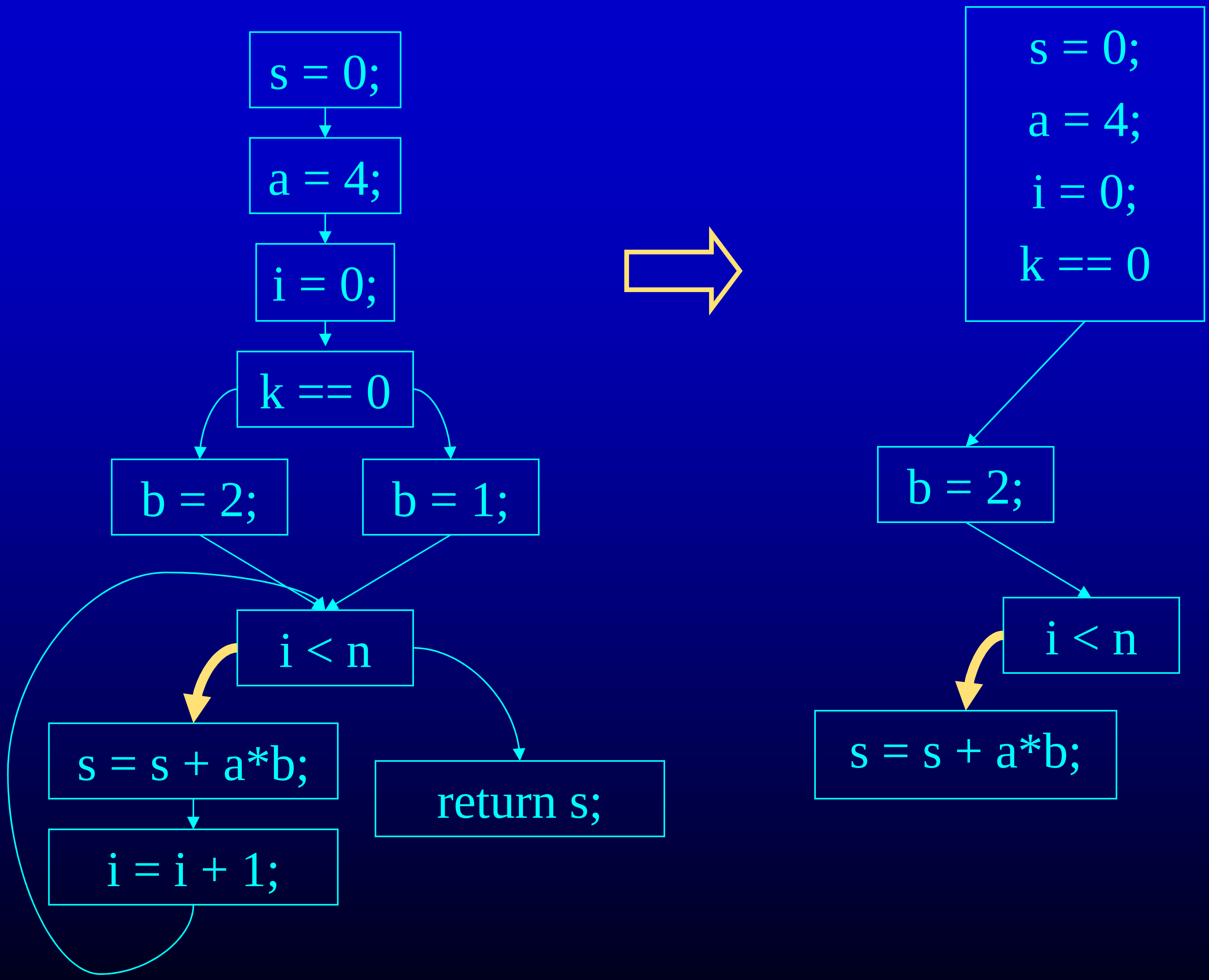

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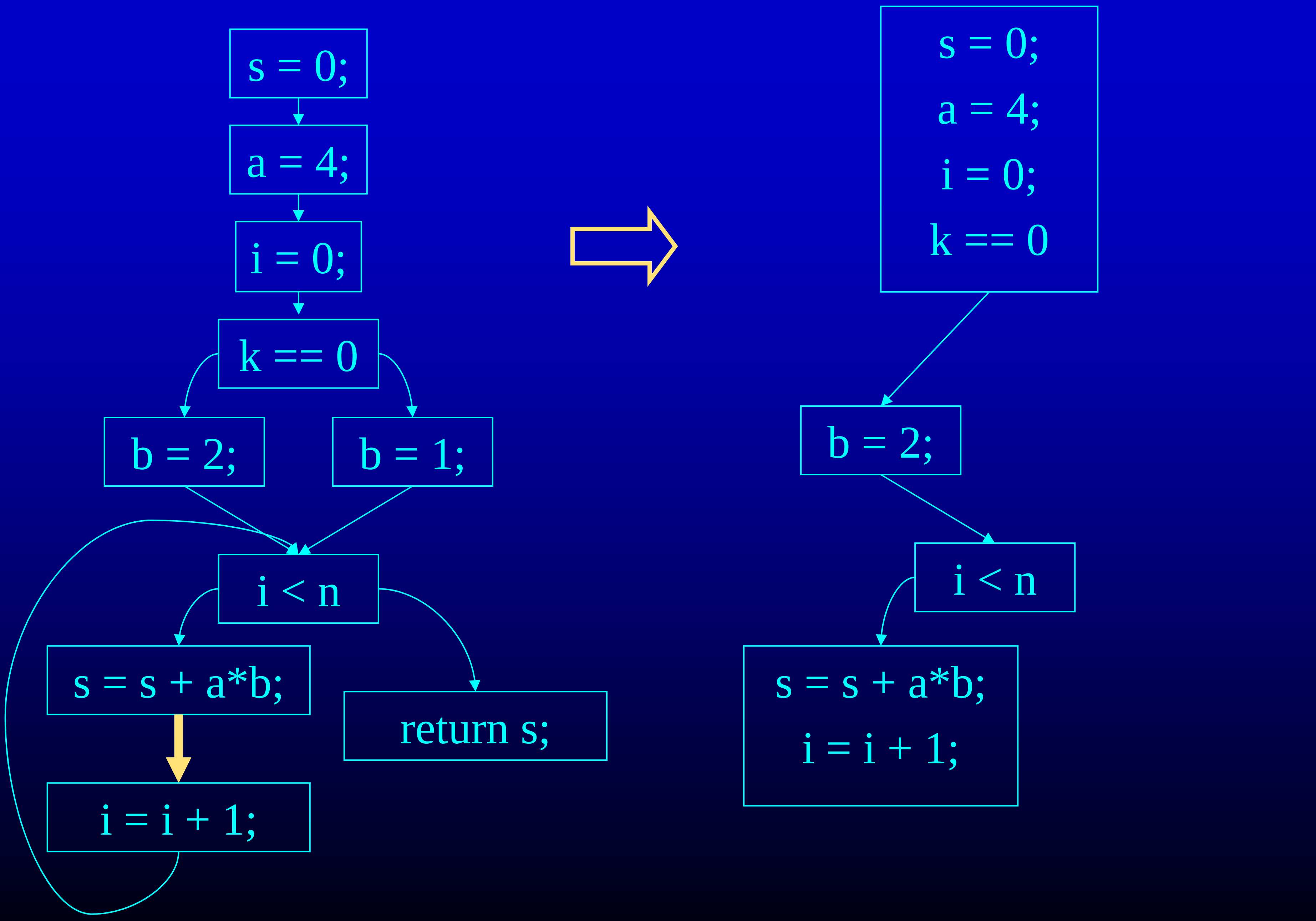

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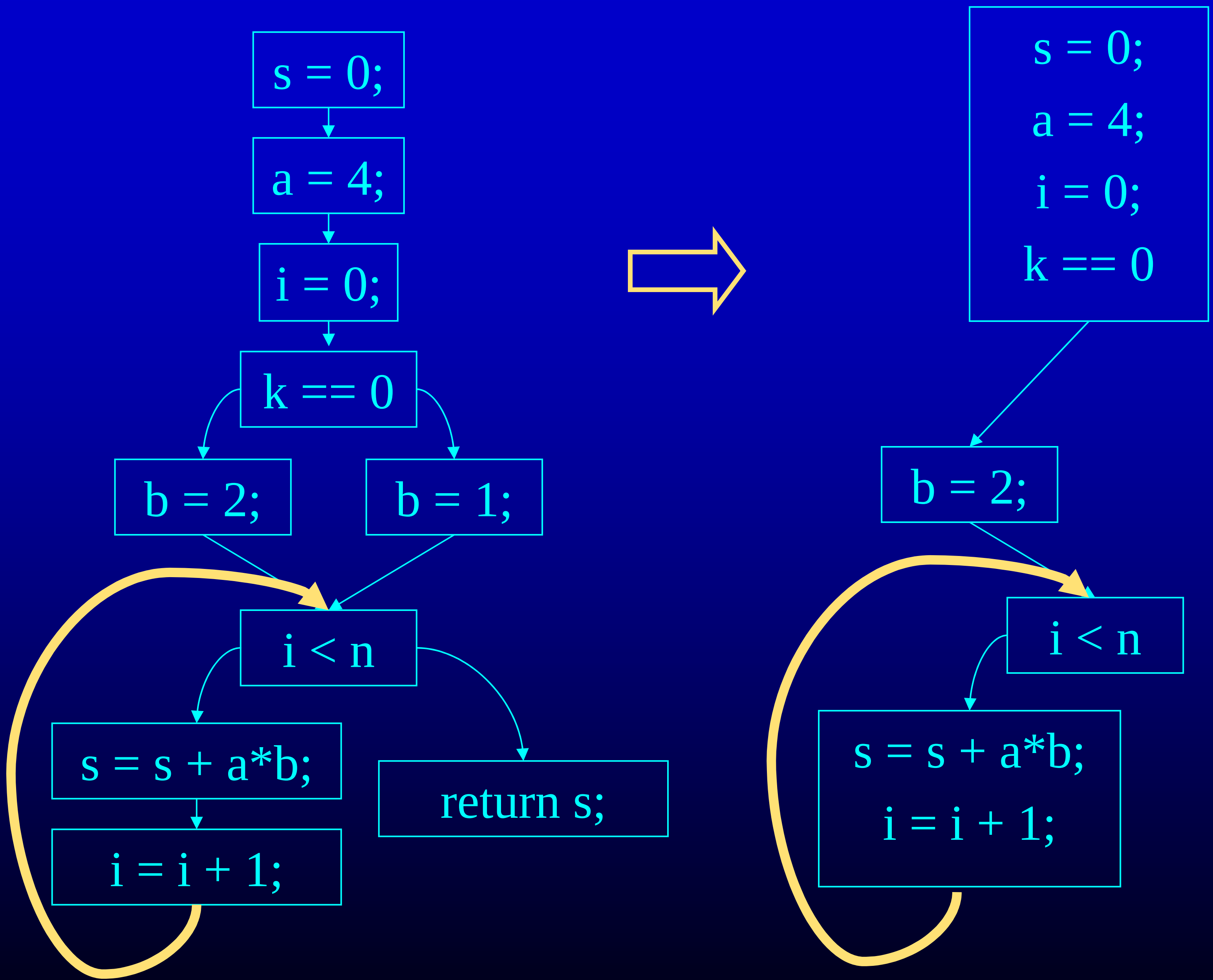
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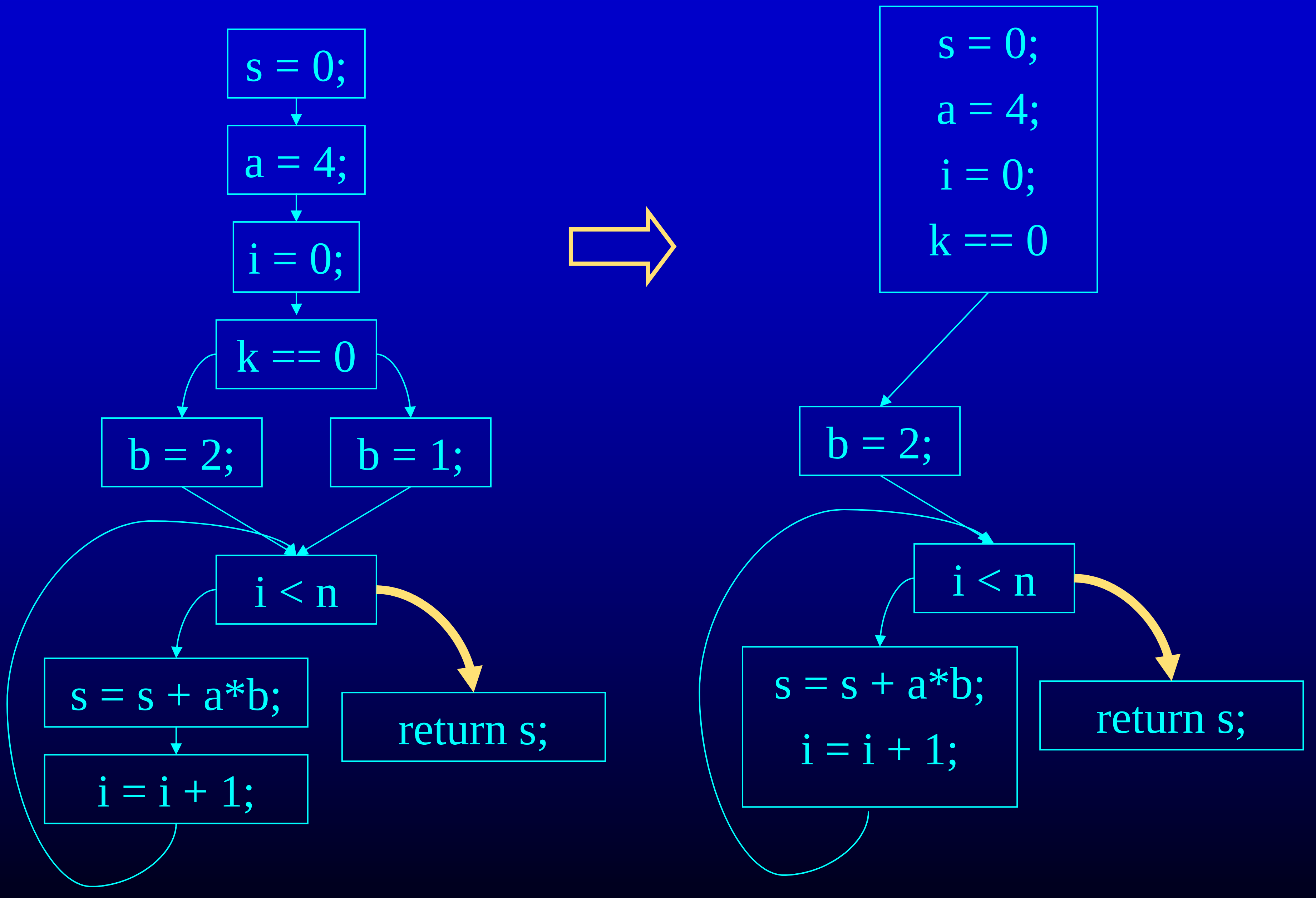

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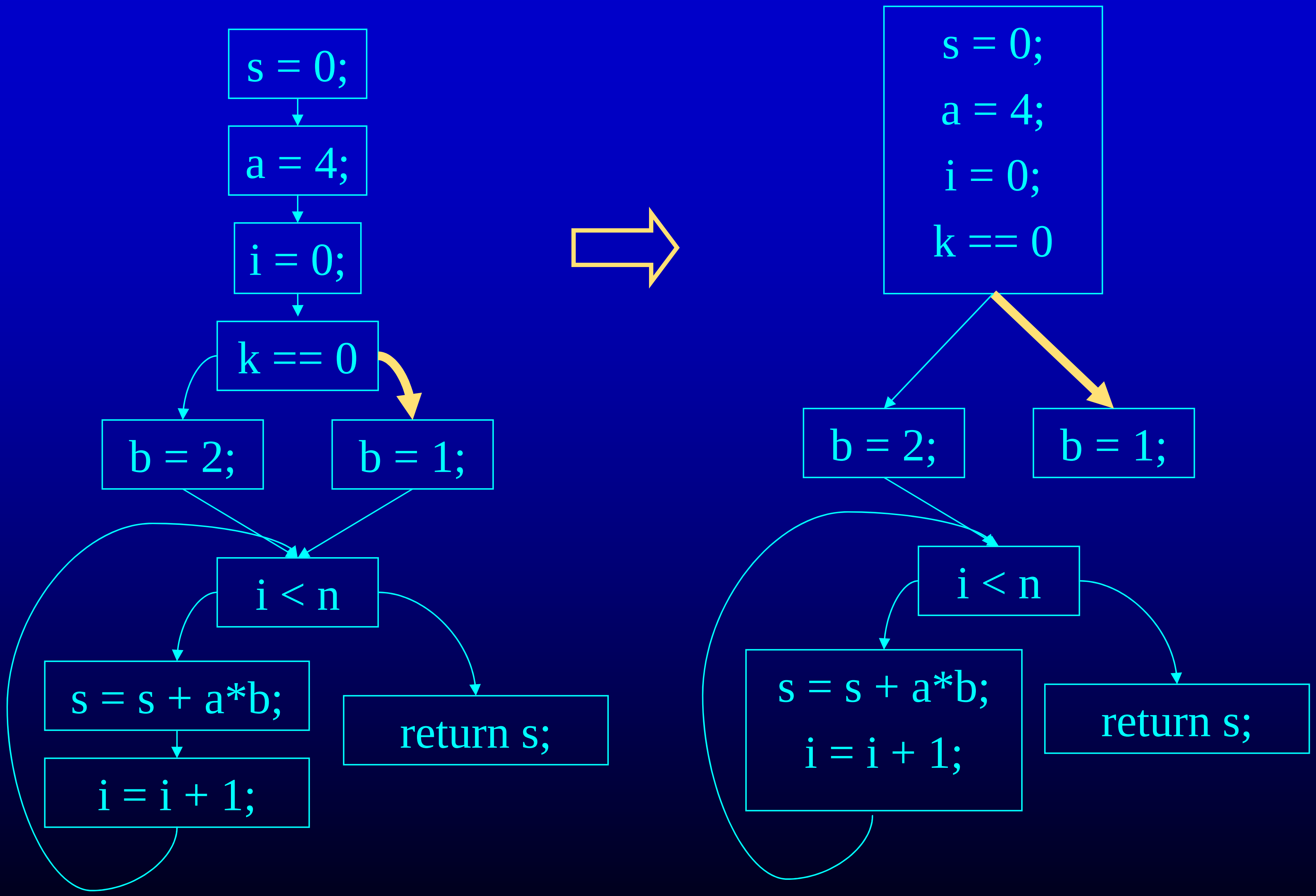

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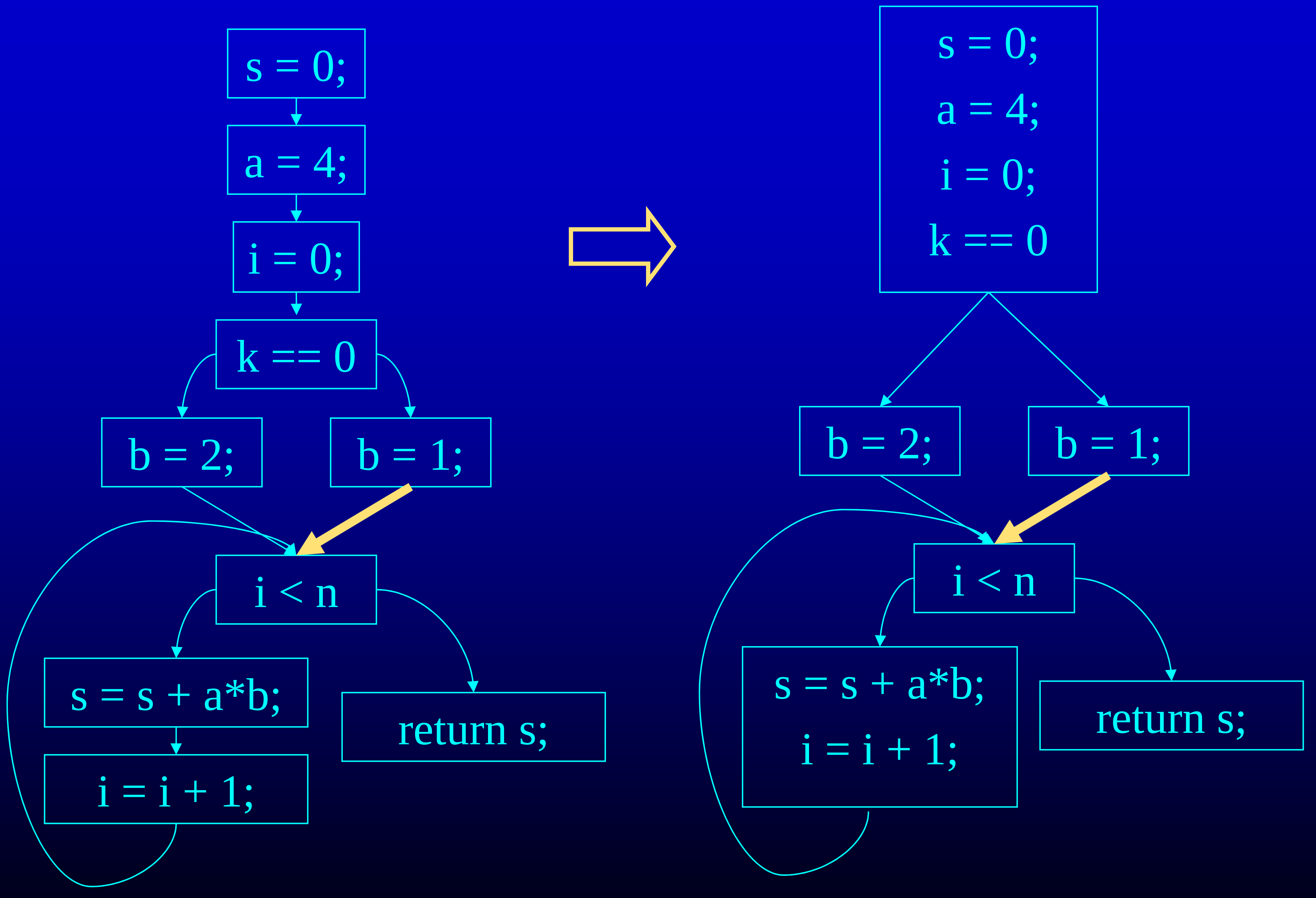
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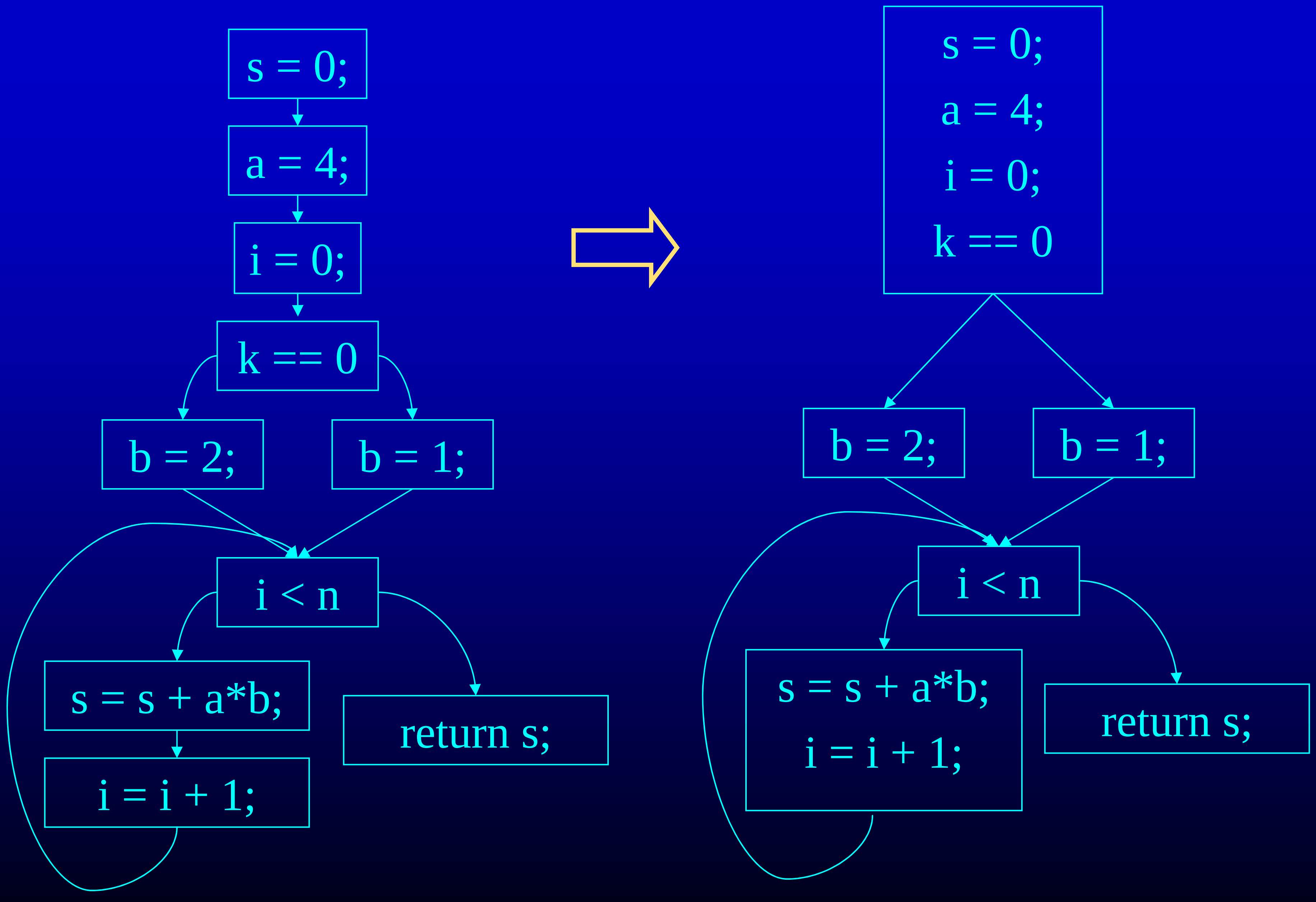
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i = i + 1;
return s;

Program Points, Split and Join Points

- One program point before and after each statement in program
- Split point has multiple successors – conditional branch statements only split points
- Merge point has multiple predecessors
- Each basic block
  - Either starts with a merge point or its predecessor ends with a split point
  - Either ends with a split point or its successor starts with a merge point

Two Kinds of Variables

- Temporaries Introduced By Compiler
  - Transfer values only within basic block
  - Introduced as part of instruction flattening
  - Introduced by optimizations/transformations
  - Typically assigned to only once
- Program Variables
  - Declared in original program
  - May be assigned to multiple times
  - May transfer values between basic blocks

Basic Block Optimizations

- Common Sub-Expression Elimination
  - a = (x+y)+z; b = x+y;
  - t = x+y; a = t+z; b = t;
- Constant Propagation
  - x = 5; b = x+y;
  - b = 5+y;
- Algebraic Identities
  - a = x * 1;
  - a = x;
- Copy Propagation
  - a = x+y; b = a; c = b+z;
  - a = x+y; b = a; c = a+z;
- Dead Code Elimination
  - a = x+y; b = a; c = a+z;
  - a = x+y; c = a+z
- Strength Reduction
  - t = i * 4;
  - t = i << 2;

Basic Block Analysis Approach

- Assume normalized basic block - all statements are of the form
  - var = var op var (where op is a binary operator)
  - var = op var (where op is a unary operator)
  - var = var
- Simulate a symbolic execution of basic block
  - Reason about values of variables (or other aspects of computation)
  - Derive property of interest

Value Numbering

- Reason about values of variables and expressions in the program
  - Simulate execution of basic block
  - Assign virtual value to each variable and expression
- Discovered property: which variables and expressions have the same value
- Standard use:
  - Common subexpression elimination
  - Typically combined with transformation that
    - Saves computed values in temporaries
    - Replaces expressions with temporaries when value of expression previously computed

Original Basic Block

a = x+y
b = a+z
b = b+y
c = a+z

New Basic Block

a = x+y
t1 = a
b = a+z
t2 = b
b = b+y
t3 = b
c = t2

Var to Val

x → v1
y → v2
a → v3
z → v4
b → v6
c → v5

Exp to Val

v1+v2 → v3
v3+v4 → v5
v5+v2 → v6

Exp to Tmp

v1+v2 → t1
v3+v4 → t2
v5+v2 → t3

Value Numbering Summary

- Forward symbolic execution of basic block
- Each new value assigned to temporary
  - a = x+y; becomes a = x+y; t = a;
  - Temporary preserves value for use later in program even if original variable rewritten
    - a = x+y; a = a+z; b = x+y becomes
    - a = x+y; t = a; a = a+z; b = t;
- Maps
  - Var to Val – specifies symbolic value for each variable
  - Exp to Val – specifies value of each evaluated expression
  - Exp to Tmp – specifies tmp that holds value of each evaluated expression

Map Usage

- Var to Val
  - Used to compute symbolic value of y and z when processing statement of form x = y + z
- Exp to Tmp
  - Used to determine which tmp to use if value(y) + value(z) previously computed when processing statement of form x = y + z
- Exp to Val
  - Used to update Var to Val when
    - processing statement of the form x = y + z, and
    - value(y) + value(z) previously computed

Interesting Properties

- Finds common subexpressions even if they use different variables in expressions
  - y = a+b; x = b; z = a+x becomes
  - y = a+b; t = y; x = b; z = t
  - Why? Because computes with symbolic values
- Finds common subexpressions even if variable that originally held the value was overwritten
  - y = a+b; x = b; y = 1; z = a+x becomes
  - y = a+b; t = y; x = b; y = 1; z = t
  - Why? Because saves values away in temporaries

One More Interesting Property

- Flattening and CSE combine to capture partial and arbitrarily complex common subexpressions
  - w = (a+b)+c; x = b; y = (a+x)+c; z = a+b;
  - After flattening:
  - t1 = a+b; w = t1+c; x = b; t2 = a+x; y = t2 + c; z = a+b;
  - CSE algorithm notices that
    - t1+c and t2+c compute same value
    - In the statement z = a+b, a+b has already been computed so generated code can reuse the result
  - t1=a+b; w = t1+c; t3 = w; x = b; t2=a+x; y = t3; z = t1;

Problems

- Algorithm has a temporary for each new value
  - a = x+y; t1 = a;
- Introduces
  - lots of temporaries
  - lots of copy statements to temporaries
- In many cases, temporaries and copy statements are unnecessary
- So we eliminate them with copy propagation and dead code elimination

Copy Propagation

- Once again, simulate execution of program
- If can, use original variable instead of temporary
  - a = x+y; b = x+y;
  - After CSE becomes a = x+y; t = a; b = t;
  - After CP becomes a = x+y; b = a;
- Key idea:
  - determine when original variable is NOT overwritten between its assignment statement and the use of the computed value
  - If not overwritten, use original variable

Copy Propagation Maps

- Maintain two maps
  - tmp to var: tells which variable to use instead of a given temporary variable
  - var to set: inverse of tmp to var. tells which temps are mapped to a given variable by tmp to var

Copy Propagation Example

- Original

  a = x+y

  b = a+z

  c = x+y

  a = b

- After CSE

  a = x+y

  t1 = a

  b = a+z

  t2 = b

  c = t1

  a = b

- After CSE and Copy Propagation

  a = x+y

  t1 = a

  b = a+z

  t2 = b

  c = a

  a = b

Copy Propagation Example

Basic Block
After CSE

a = x+y
t1 = a

Basic Block After
CSE and Copy Prop

a = x+y
t1 = a

tmp to var

t1 $\rightarrow$ a

var to set

a $\rightarrow$ {t1}

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b

tmp to var

$t1 \rightarrow a$
$t2 \rightarrow b$

var to set

$a \rightarrow \{t1\}$
$b \rightarrow \{t2\}$

Copy Propagation Example

Basic Block
After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1

Basic Block After
CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b

tmp to var

t1 $\rightarrow$ a
t2 $\rightarrow$ b

var to set

a $\rightarrow$ {t1}
b $\rightarrow$ {t2}

Copy Propagation Example

Basic Block
After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1

Basic Block After
CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a

tmp to var

t1 $\rightarrow$ a
t2 $\rightarrow$ b

var to set

a $\rightarrow$ {t1}
b $\rightarrow$ {t2}

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1
a = b

tmp to var

$t1 \rightarrow a$
$t2 \rightarrow b$

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b

var to set

$a \rightarrow \{t1\}$
$b \rightarrow \{t2\}$

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1
a = b

tmp to var

t1 → t1
t2 → b

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b

var to set

a → {}
b → {t2}

Dead Code Elimination

- Copy propagation keeps all temps around
- May be temps that are never read
- Dead Code Elimination removes them

Basic Block After CSE and Copy Prop	Basic Block After CSE and Copy Prop
a = x+y	a = x+y
t1 = a	b = a+z
b = a+z	c = a
t2 = b	a = b
c = a	
a = b	

Dead Code Elimination

- Basic Idea
  - Process Code In Reverse Execution Order
  - Maintain a set of variables that are needed later in computation
  - If encounter an assignment to a temporary that is not needed, remove assignment

Basic Block After CSE and Copy Prop

$a = x+y$

$t1 = a$

$b = a+z$

$t2 = b$

$c = a$

$\Rightarrow$ $a = b$

Needed Set

$\{b\}$

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
⇨ c = a
a = b

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

a = x+y

t1 = a

b = a+z

⇒ t2 = b

c = a

a = b

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

a = x+y

t1 = a

b = a+z

⇨

c = a

a = b

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z

c = a
a = b

Needed Set

{a, b, z}

Basic Block After CSE and Copy Prop

a = x+y

t1 = a

b = a+z

c = a

a = b

Needed Set

{a, b, z}

Basic Block After CSE and Copy Prop

a = x+y

⇨

b = a+z

c = a

a = b

Needed Set

{a, b, z}

Basic Block After , CSE Copy Propagation, and Dead Code Elimination

$\Rightarrow$ a = x+y

b = a+z

c = a

a = b

Needed Set

{a, b, z}

Basic Block After , CSE Copy Propagation, and Dead Code Elimination

a = x+y

b = a+z

c = a
a = b

Needed Set
{a, b, z}

Interesting Properties

- Analysis and Transformation Algorithms Symbolically Simulate Execution of Program
  - CSE and Copy Propagation go forward
  - Dead Code Elimination goes backwards
- Transformations stacked
  - Group of basic transformations work together
  - Often, one transformation creates inefficient code that is cleaned up by following transformations
  - Transformations can be useful even if original code may not benefit from transformation

Other Basic Block Transformations

- Constant Propagation
- Strength Reduction
  - a << 2 = a * 4; a + a + a = 3 * a;
- Algebraic Simplification
  - a = a * 1; b = b + 0;
- Do these in unified transformation framework, not in earlier or later phases

Summary

- Basic block analyses and transformations
- Symbolically simulate execution of program
  - Forward (CSE, copy prop, constant prop)
  - Backward (Dead code elimination)
- Stacked groups of analyses and transformations that work together
  - CSE introduces excess temporaries and copy statements
  - Copy propagation often eliminates need to keep temporary variables around
  - Dead code elimination removes useless code
- Similar in spirit to many analyses and transformations that operate across basic blocks