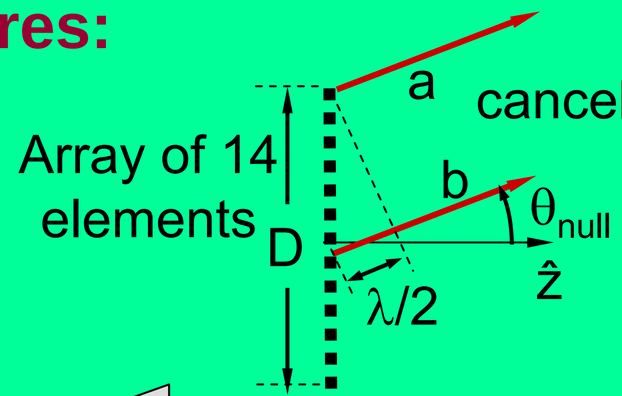
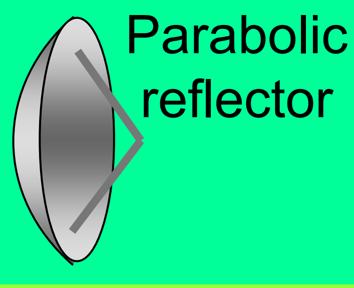
APERTURE ANTENNAS

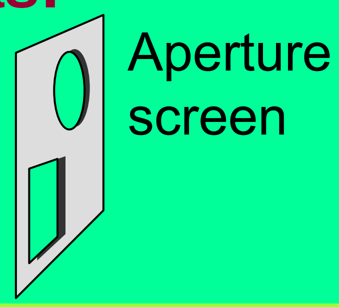
Dense phased arrays ~apertures:

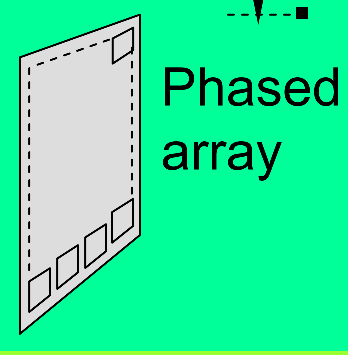
Pairs of radiators cancel (e.g., a,b)
Sum of pairs therefore = 0
First null at θ = ~λ/D



Aperture antennas:







Derivation of far-field aperture radiation:

Find surface current $\overline{J}_s$ that could produce the same aperture fields
Find the far fields radiated by an infinitesimal element of $\overline{J}_s$
Integrate the contributions from the entire aperture
Simplify the expressions by using small-angle approximations
Note the Fourier relationship between aperture and far fields

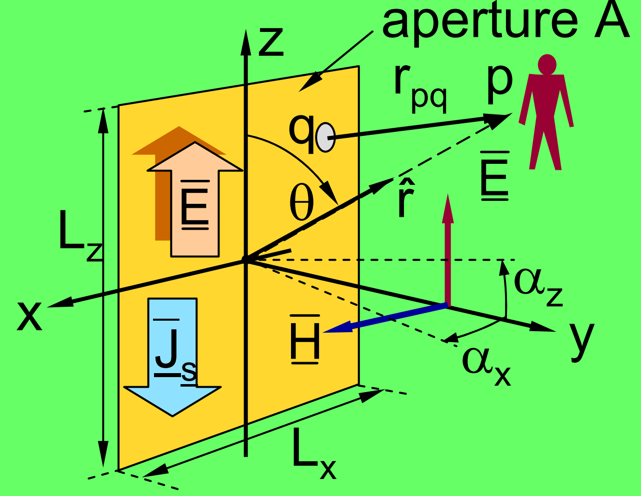
APERTURE ANTENNAS

Assume UPW in aperture:

$\underline{\overline{E}}_+ = \hat{z}\, \underline{E}_o e^{-jky} \Rightarrow \underline{\overline{H}}_+ = \hat{x} \dfrac{\underline{E}_o}{\eta_o} e^{-jky}$

$\underline{\overline{E}}_- = \hat{z}\, \underline{E}_o e^{+jky} \Rightarrow \underline{\overline{H}}_- = -\hat{x} \dfrac{\underline{E}_o}{\eta_o} e^{+jky}$

Satisfies $E_{//}$ continuous at $y = 0$



Equivalent surface current:

$\underline{\overline{J}}_s = \hat{y} \times [\underline{\overline{H}}(y=0_+) - \underline{\overline{H}}(y=0_-)]$

$= \hat{y} \times \hat{x}[\underline{E}_o/\eta_o + \underline{E}_o/\eta_o]$

$\underline{\overline{J}}_s = -\hat{z}\, 2\underline{E}_o/\eta_o \ [\mathrm{A\ m^{-1}}]$

$\hat{z}\underline{I}\, dz = \underline{\overline{J}}_s dx\, dz$ (Hertzian dipole radiator)

Radiated far fields:

$$\underline{\overline{E}}_{ff}(\theta,\phi) = \hat{\theta} \frac{j\eta_o}{2\lambda r} \sin\theta \iint_A \underline{J}_{zs}(x,z)\, e^{-jkr_{pq}} dx\, dz$$

$$= -\hat{\theta} \frac{j}{\lambda r} \sin\theta \iint_A \underline{E}_{oz}(x,z)\, e^{-jkr_{pq}} dx\, dz$$

FAR-FIELD RADIATION

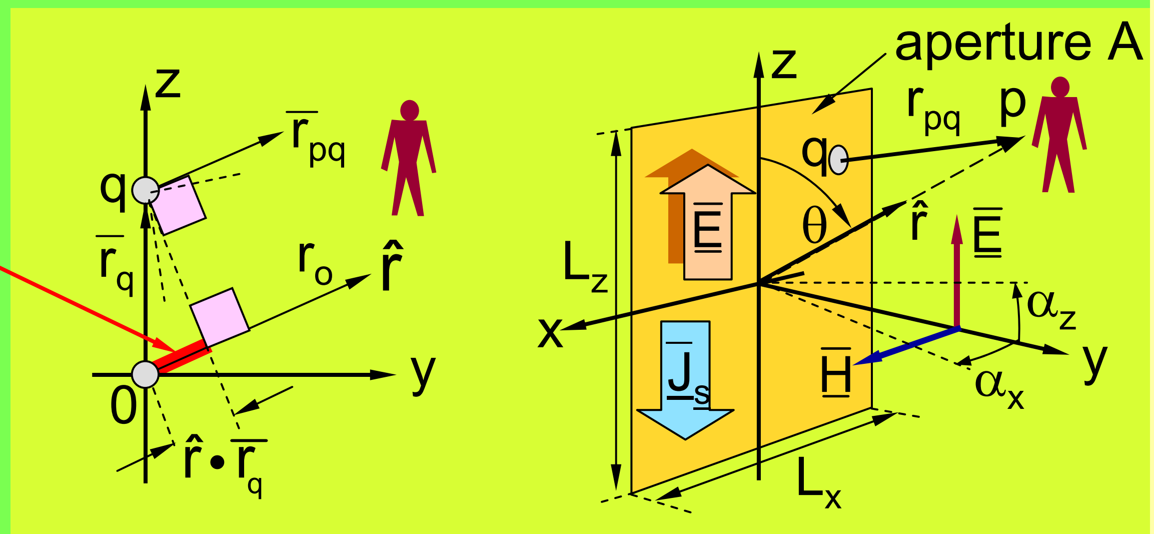
Assume UPW in aperture:

$$\underline{\bar{E}}_{ff}(\theta,\phi) = -\hat{\theta}\,\frac{j}{\lambda r}\,\sin\theta \iint_A \underline{E}_{oz}(x,z)\,e^{-jkr_{pq}}\,dx\,dz$$

$$e^{-jkr} \cong e^{-jkr_o + jk\hat{r}\bullet\bar{r}_q}$$

Fraunhofer approximation

(else, Fresnel)



$$\underline{\bar{E}}_{ff}(\alpha_x,\alpha_z) \cong -\hat{\theta}\,\frac{j}{\lambda r}\,e^{-jkr_o}\iint_A E_{oz}(x,z)\,e^{+j\frac{2\pi}{\lambda}\underbrace{(x\alpha_x + z\alpha_z)}_{\text{Close to the y axis}}}\,dx\,dz$$

~Fourier (angle ⇔ space)

Fourier relationship:

$$\underline{S}(f) = \int_{-\infty}^{\infty} s(t)\,e^{-j2\pi ft}\,dt \qquad s(t) = \int_{-\infty}^{\infty} \underline{S}(f)\,e^{+j2\pi ft}\,df$$

(frequency ⇔ time)

EXAMPLE: RECTANGULAR APERTURE

Assume UPW in aperture: (Observer close to the y axis)

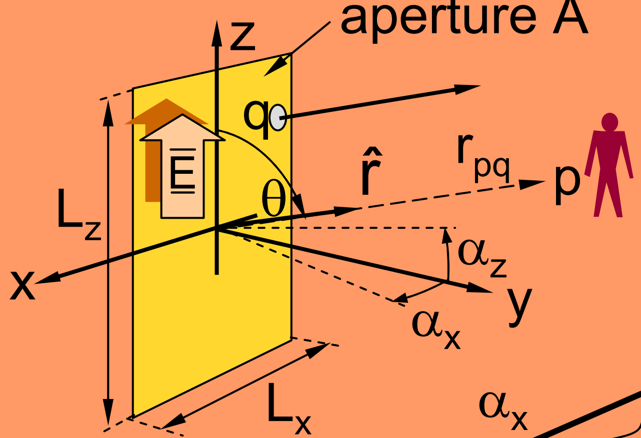
$$\underline{\bar{E}}_{ff}(\alpha_x,\alpha_z) \cong -\hat{\theta}\frac{j}{\lambda r} e^{-jkr_o} \int_{-L_z/2}^{L_z/2} e^{+j2\pi\alpha_z \frac{z}{\lambda}} \int_{-L_x/2}^{Lx/2} \underline{E}_{oz}(x,z)\, e^{+j2\pi\alpha_x\frac{x}{\lambda}}\, dx\, dz$$

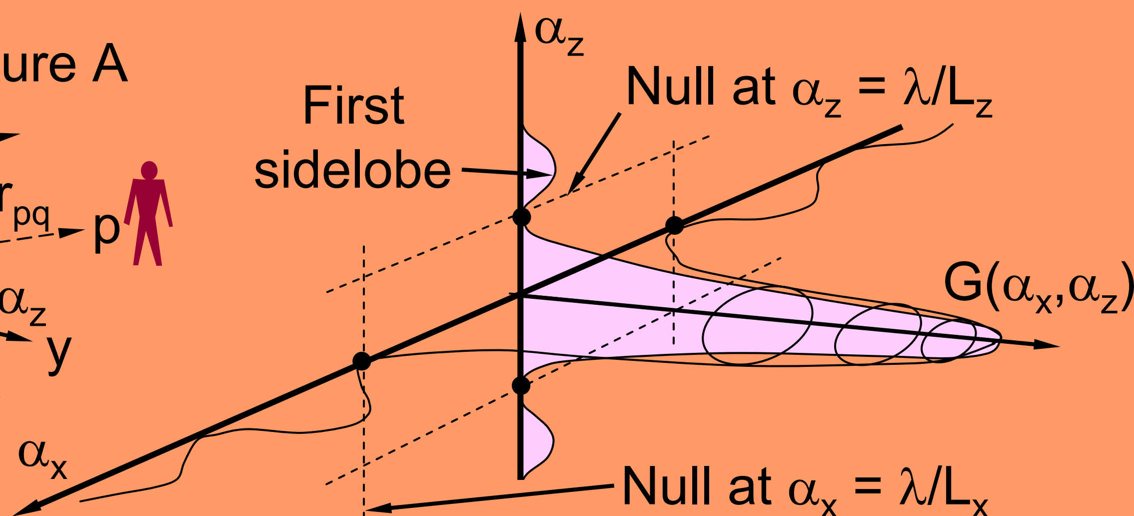
$$\int_{-L_x/2}^{L_x/2}\underline{E}_{oz}(x,z)e^{+j2\pi\alpha_x\frac{x}{\lambda}}\,dx = \underline{E}_{oz}\frac{\lambda}{j2\pi\alpha_x}\left(e^{+j\pi\alpha_x\frac{L_x}{\lambda}} - e^{-j\pi\alpha_x\frac{L_x}{\lambda}}\right) = \underline{E}_{oz}L_x \frac{\sin\frac{\pi\alpha_x L_x}{\lambda}}{\frac{\pi\alpha_x L_x}{\lambda}}$$

$$\triangleq \underline{E}_{oz}L_x\,\text{sinc}\frac{\pi\alpha_x L_x}{\lambda}\ ^*$$

$$\underline{\bar{E}}_{ff}(\alpha_x,\alpha_z) \cong -\hat{\theta}\frac{j}{\lambda r}\underline{E}_{oz}e^{-jkr_o} L_x L_z\ \text{sinc}\frac{\pi\alpha_x L_x}{\lambda}\ \text{sinc}\frac{\pi\alpha_z L_z}{\lambda}$$

Antenna Gain: $G(\alpha_x,\alpha_z) \propto |\underline{E}_{ff}|^2 \propto \text{sinc}^2(\pi\alpha_x L_x/\lambda)\text{sinc}^2(\pi\alpha_z L_z/\lambda)$

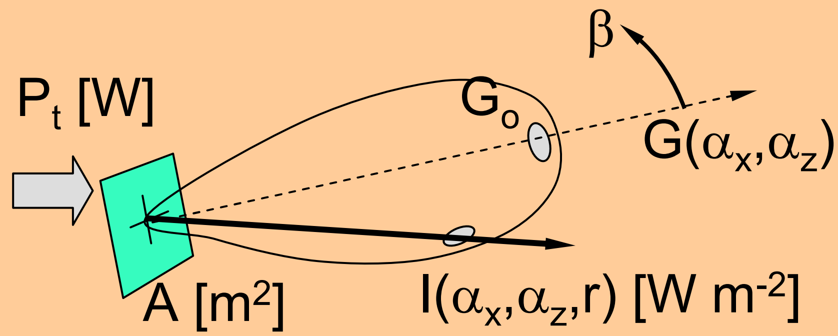




*sinc $\theta \equiv (\sin\theta)/\theta$ [$= 1$ for $\theta = 0$]

RECTANGULAR APERTURE (2)

Antenna Gain $G(\alpha_x,\alpha_z)$:



$$G(\alpha_x,\alpha_z) = \frac{I(\alpha_x,\alpha_z,r)}{(P_t/4\pi r^2)}$$

$$P_t = A\frac{|\underline{E}_{oz}|^2}{2\eta_o} \quad [W] \quad (A=L_xL_z)$$

$$I(\alpha_x,\alpha_z,r) = \frac{|\underline{E}_{ff}|^2}{2\eta_o}[W/m^2] = \frac{1}{2\eta_o}\left(\frac{1}{\lambda r}\right)^2|\underline{E}_{oz}|^2A^2\text{sinc}^2\left(\frac{\pi\alpha_xL_x}{\lambda}\right)\text{sinc}^2\left(\frac{\pi\alpha_zL_z}{\lambda}\right)$$

$$G(\alpha_x,\alpha_z) = A(4\pi/\lambda^2)\text{sinc}^2(\pi\alpha_xL_x/\lambda)\text{sinc}^2(\pi\alpha_zL_z/\lambda)$$

$$G_o = A\frac{4\pi}{\lambda^2} \Rightarrow A = A_e = G_o\frac{\lambda^2}{4\pi}\ ^* \qquad \left[\frac{\sin\theta}{\theta} \to 1 \text{ as } \theta \to 0\right]$$

η_A = "aperture efficiency" $\equiv A_e/A \approx 0.65$ typically

(non-uniform aperture amplitude and/or phase $\Rightarrow A_e < A$)

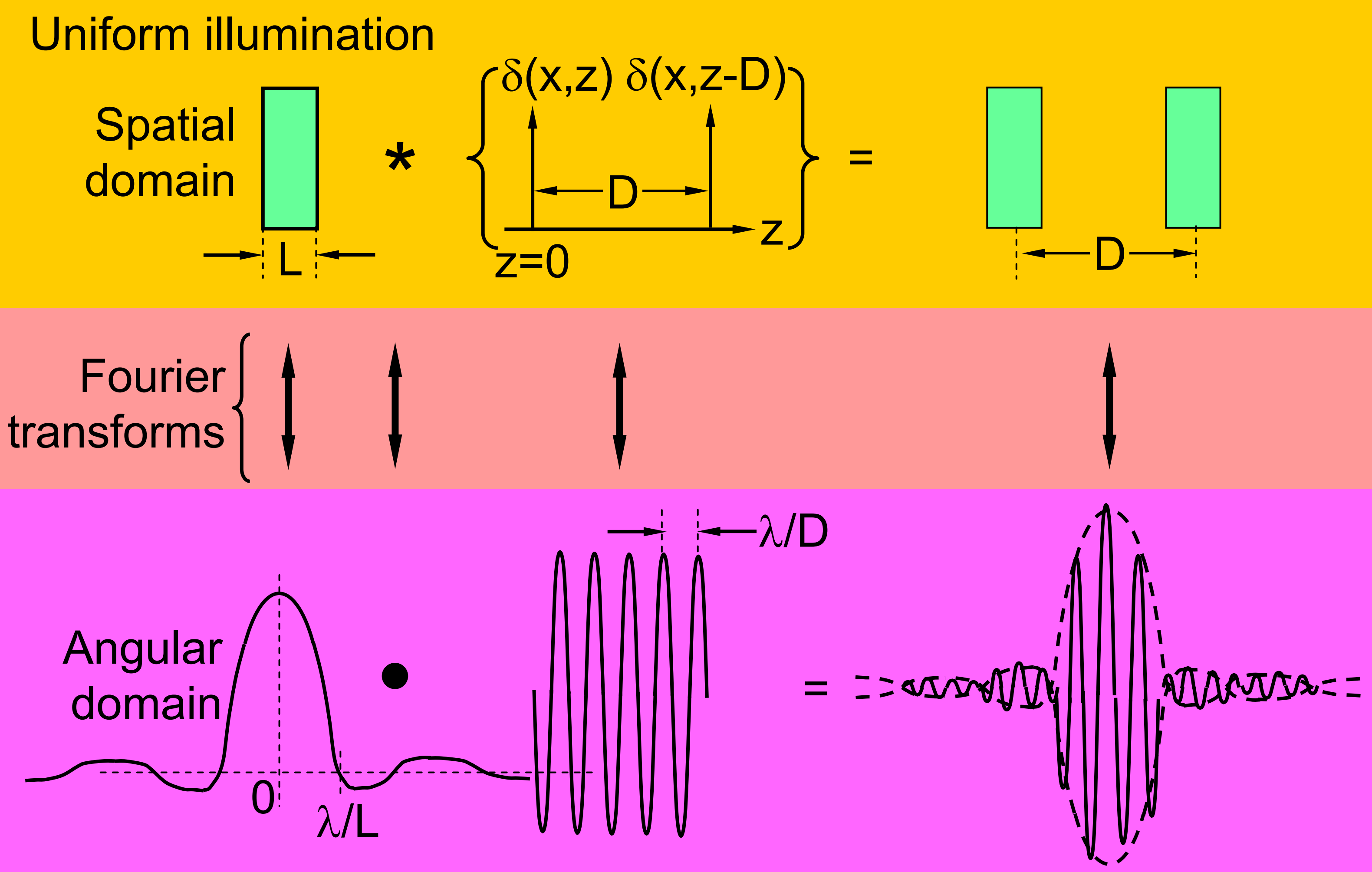
Huygen's approximation:

$$\underline{\underline{E}}_{ff} \cong \hat{\theta}\,(j/2\lambda r)\,(1+\cos\beta)\oiint_A \underline{E}_{oz}(x,z)e^{-jkr_{pq}}\,dx\,dz$$

*True for any uniformly illuminated aperture

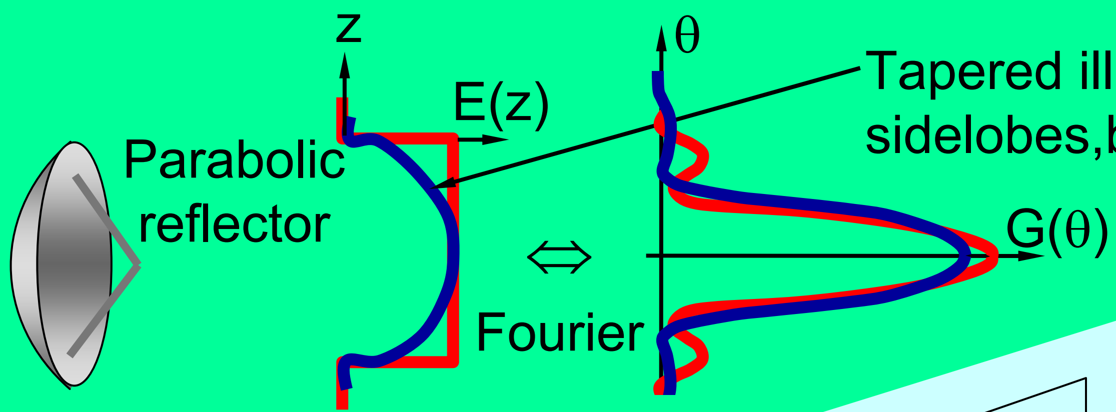
EXAMPLE: SLIT DIFFRACTION

Pair of rectangular slots:



DIFFRACTION

Aperture tapering: $\underline{E}_{ff} \cong \hat{\theta}\,(j/2\lambda r)\,(1+\cos\beta) \oiint_A \underline{E}_{oz}(x,z)e^{-jkr_{pq}}\,dx\,dz$

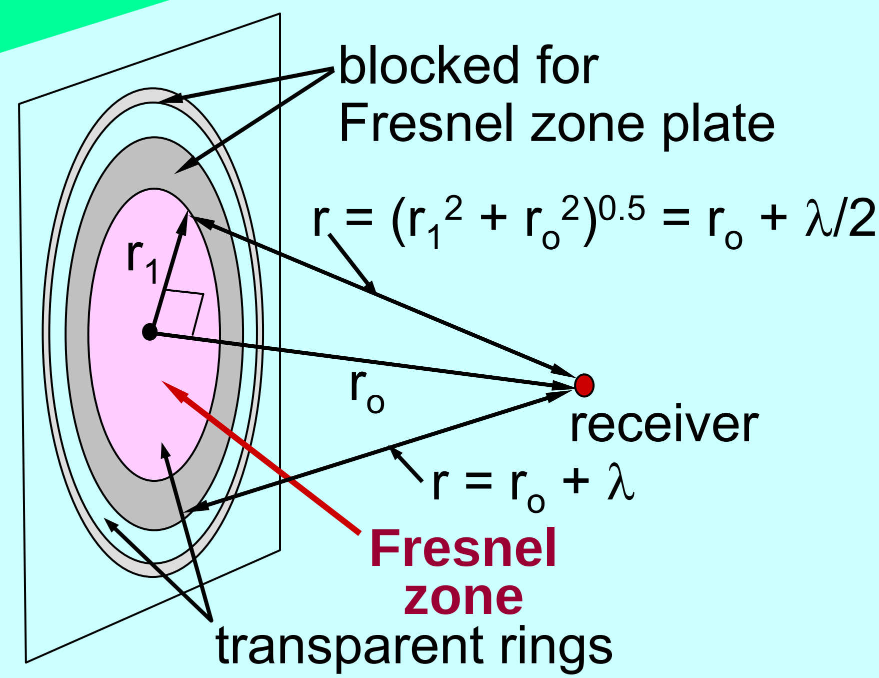


Fresnel zone plate:

Blocks rings of negative phase, maximizing Huygen's integral

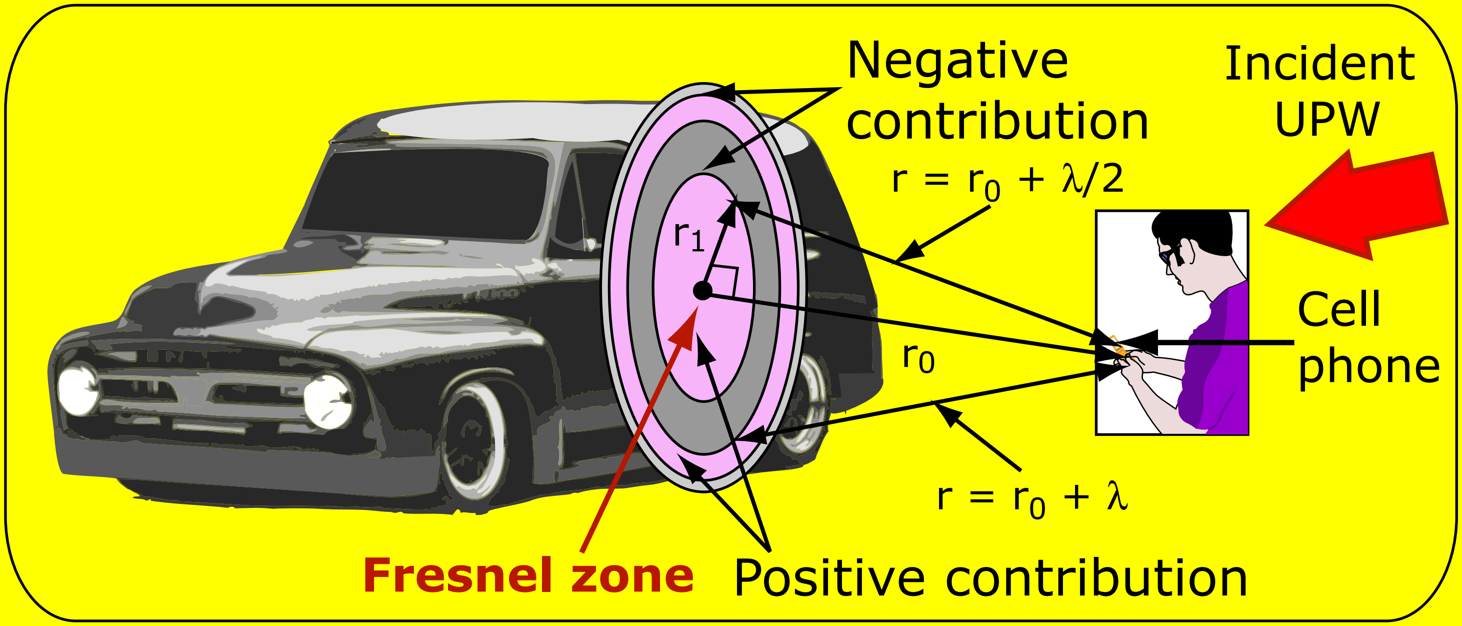
Aperture radius = r_1 maximizes received signal. Additional transparent rings increase it further, yielding lense behavior

Adjacent rings cancel if $\beta \approx 1$



FRESNEL ZONE SIZE, REFLECTIONS

Fresnel zone radius $r_1 = \sqrt{(r_o + \frac{\lambda}{2})2 - r_o^2} \cong \sqrt{r_o\lambda}$ for $r_o \gg \lambda$



When the truck is much larger than the Fresnel zone, it acts like a large mirror. When the truck is smaller, the power reflected declines with illuminated zone area on the truck. Thus the reflecting power of a mirror depends on its size and distance relative to a wavelength.