### Modal solution of undriven CT LTI state-space models

## 6.011, Spring 2018 Lec 6

### Glucose-insulin system



From Messori et al., IEEE Control Systems Magazine Feb 2018

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### UVA/Padova model (FDA approved!)

$$\begin{cases} \dot{x}_{1}(t) = -k_{gri}x_{1}(t) + d(t), \\ \dot{x}_{2}(t) = k_{gri}x_{1}(t) - k_{empt}(x_{1}(t) + x_{2}(t))x_{2}(t), \\ \dot{x}_{3}(t) = -k_{abs}x_{3}(t) + k_{empt}(x_{1}(t) + x_{2}(t))x_{2}(t), \\ \dot{x}_{4}(t) = EGP(t) + Ra(t) - U_{ii}(t) - E(t) - k_{1}x_{4}(t) + k_{2}x_{5}(t), \\ \dot{x}_{5}(t) = -U_{id}(t) + k_{1}x_{4}(t) - k_{2}x_{5}(t), \\ \dot{x}_{5}(t) = -(m_{2} + m_{4})x_{6}(t) + m_{1}x_{10}(t) + k_{a1}x_{11}(t) + k_{a2}x_{12}(t), \\ \dot{x}_{6}(t) = -(m_{2} + m_{4})x_{6}(t) + m_{1}x_{10}(t) + k_{a1}x_{11}(t) + k_{a2}x_{12}(t), \\ \dot{x}_{7}(t) = -p_{2U}x_{7}(t) + p_{2U}\left(\frac{x_{6}(t)}{V_{I}} - I_{b}\right), \\ \dot{x}_{8}(t) = -k_{i}x_{8}(t) + k_{i}\frac{x_{6}(t)}{V_{I}}, \\ \dot{x}_{9}(t) = -k_{i}x_{9}(t) + k_{i}x_{8}(t), \\ \dot{x}_{10}(t) = -(m_{1} + m_{3}(t))x_{10}(t) + m_{2}x_{6}(t), \\ \dot{x}_{11}(t) = -(k_{d} + k_{a1})x_{11}(t) + i(t), \\ \dot{x}_{12}(t) = k_{d}x_{11}(t) - k_{a2}x_{12}(t), \\ \dot{x}_{13}(t) = -k_{sc}x_{13}(t) + k_{sc}x_{4}(t), \\ \dot{x}_{14}(t) = -n_{G}x_{14}(t) + SR_{H}(t), \\ \dot{x}_{15}(t) = -k_{H}x_{15}(t) + k_{H}\max\{x_{14}(t) - H_{b}, 0\}, \\ \dot{x}_{16}(t) = \dot{S}R_{H}^{8}(t), \end{cases}$$

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# Linearization at an equilibrium yields an LTI model

CT case: 
$$\mathbf{q}(t) = \bar{\mathbf{q}} + \widetilde{\mathbf{q}}(t)$$
,  $x(t) = \bar{x} + \widetilde{x}(t)$ ,

$$\dot{\mathbf{q}}(t) = \mathbf{f}(\mathbf{q}(t), x(t))$$
 $\downarrow$ 

$$\dot{\widetilde{\mathbf{q}}}(t) \approx \left[\frac{\partial \mathbf{f}}{\partial \mathbf{q}} \right]_{\bar{\mathbf{q}},\bar{x}} \widetilde{\mathbf{q}}(t) + \left[\frac{\partial \mathbf{f}}{\partial x} \right]_{\bar{\mathbf{q}},\bar{x}} \widetilde{\mathbf{q}}(t)$$

for small perturbations  $\widetilde{\mathbf{q}}(t)$  and  $\widetilde{x}(t)$  from equilibrium

### Phase plane trajectories



### Complex eigenvalue pairs (CT case)

If  $\lambda_i$  is a (complex) eigenvalue with eigenvector  $\mathbf{v}_i$ , then its complex conjugate  $\lambda_i^*$  is also an eigenvalue, with associated eigenvector  $\mathbf{v}_i^*$ .

Write  $_{i} = _{i} + j\omega_{i}$ ,  $\mathbf{v}_{i} = \mathbf{u}_{i} + j\mathbf{v}_{i}$ . Then the contribution of the complex pair to the modal solution is

$$\alpha_i \mathbf{v}_i e^{-it} + \alpha_i^* \mathbf{v}_i^* e^{-it} =$$

$$K_i e^{-it} \left[ \mathbf{u}_i \cos(\omega_i t + \theta_i) - \mathbf{w}_i \sin(\omega_i t + \theta_i) \right]$$

### Acoustics and Vibration Animations

#### Have fun exploring the animations created by

Prof. Dan Russell, Penn State

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