# State-Space Models, Equilibrium, Linearization 

### 6.011, Spring 2018

Lec 5

# State variables are (relevant) "memory" variables 

In physical systems, the natural state variables are typically related to energy storage mechanisms:
capacitor voltages or charges, inductor currents or fluxes, positions and velocities of masses,

## Defining properties of CT state-space models

$$
\begin{aligned}
\dot{\mathbf{q}}(t) & =\mathbf{f}(\mathbf{q}(t), x(t), t) \\
y(t) & =g(\mathbf{q}(t), x(t), t)
\end{aligned}
$$

- State evolution property
- Instantaneous output property

Numerical solution of CT state-space model


## Integrator-adder-gain system



## Mechanistic model for capnography



## ... and the governing equations

$$
\begin{gathered}
L \ddot{V}(t)+R \dot{V}(t)+\frac{V(t)}{C}=\Delta P \\
\dot{p}_{D}(t)=\frac{-p_{D}(t)+p_{A}}{V_{D}} \dot{V}(t), \quad \dot{V}(t)>0 \\
\dot{p}_{D}(t)=\frac{p_{D}(t)}{V_{D}} \dot{V}(t), \quad \dot{V}(t)<0
\end{gathered}
$$

## Equilibrium

For a time-invariant nonlinear system with a constant input, an initial state that the system remains at:

$$
\begin{aligned}
& \text { DT : } \overline{\mathbf{q}}=\mathbf{f}(\overline{\mathbf{q}}, \bar{x}) \\
& \mathrm{CT}: \\
& \mathbf{0}=\mathbf{f}(\overline{\mathbf{q}}, \bar{x})
\end{aligned}
$$

## Linearization at an equilibrium yields an LTI model

DT case: $\quad \mathbf{q}[n]=\overline{\mathbf{q}}+\widetilde{\mathbf{q}}[n], \quad x[n]=\bar{x}+\widetilde{x}[n]$,

$$
\begin{array}{r}
\mathbf{q}[n+1]=\mathbf{f}(\mathbf{q}[n], x[n]) \\
\downarrow \\
\widetilde{\mathbf{q}}[n+1] \approx\left[\left.\frac{\partial \mathbf{f}}{\partial \mathbf{q}}\right|_{\overline{\mathbf{q}}, \bar{x}}\right] \widetilde{\mathbf{q}}[n]+\left[\left.\frac{\partial \mathbf{f}}{\partial x}\right|_{\overline{\mathbf{q}}, \bar{x}}\right] \widetilde{x}[n]
\end{array}
$$

for small perturbations $\widetilde{\mathbf{q}}[n]$ and $\widetilde{x}[n]$ from equilibrium

## Linearization at an equilibrium yields an LTI model

$$
\text { CT case: } \quad \mathbf{q}(t)=\overline{\mathbf{q}}+\widetilde{\mathbf{q}}(t), \quad x(t)=\bar{x}+\widetilde{x}(t),
$$

$$
\dot{\mathbf{q}}(t)=\mathbf{f}(\mathbf{q}(t), x(t))
$$

$$
\dot{\tilde{\mathbf{q}}}(t) \approx\left[\left.\frac{\partial \mathbf{f}}{\partial \mathbf{q}}\right|_{\overline{\mathbf{q}}, \bar{x}}\right] \widetilde{\mathbf{q}}(t)+\left[\left.\frac{\partial \mathbf{f}}{\partial x}\right|_{\overline{\mathbf{q}}, \bar{x}}\right] \widetilde{x}(t)
$$

for small perturbations $\widetilde{\mathbf{q}}(t)$ and $\widetilde{x}(t)$ from equilibrium

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