14.452 Review session

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Logistics

- Exam next Monday
- Greg will proctor
- Open book & lecture notes
- 3-4 short questions, 1-2 long questions

Determinants of growth

- Definition

$$Y = F(A, K, L, H)$$

  where
  - A = technology
  - K = physical capital
  - L = labor force
  - H = human capital / education

- Only **proximate causes, not fundamental**
  - such as geography, luck, institutions, preferences
  - Acemoglu Naidu Restrepo Robinson (2014): Democracy causes $\approx 1\%$ higher GDP growth

Why write a model of growth?

- For each proximate cause X, want guidance on: (among others)
  - How do **fundamental causes** affect the growth of X?
  - Under what conditions can there be **sustained growth** in X?
  - What kind of **policies** can help **accumulate** more X?
  - What kind of **policies** can increase **welfare**? (if at all?)
  - How can we **measure** contribution of growth in X empirically?
- These Qs require a model with **endogenous accumulation** of X
  - will do this for A, K. H similar to K

Common theme

- In background: $\exists$ "accumulation technology" of X
  - concave $\Rightarrow$ exogenous growth
  - linear $\Rightarrow$ endogenous growth

An aside on TVCs

- TVC: part of **sufficient conditions** for optimum in any **infinite horizon optimal control problem**
  - e.g. a representative household's problem, or a planning problem
- When there is a some lower bound on wealth, it is

$$\lim_{t\to\infty} \underbrace{e^{-(\rho-n)t}u'(c_t)}_{\sim e^{-(r-n)t}} \text{wealth}_t = 0$$

so we can write

$$\lim_{t\to\infty} e^{-rt}\text{TotalWealth}_t = 0$$

where TotalWealth is the whole current generation's wealth

- In pretty much any model, TotalWealth grows at rate g_Y, so along BGP this means

$$r > g_{\text{TotalWealth}} = g_Y$$

Outline

1. Solow model: K
    - Uzawa's theorem
    - Solow models
    - Data
2. NGM and OLG: still K
    - NGM
    - OLG & dynamic inefficiency
3. Neoclassical endogenous growth: still K
4. Endogenous technology: A
5. World technology growth: A
6. DTC: What kind of A?

Section 1

Solow model: K

Subsection 1

Uzawa's theorem

How should technology affect production?

- Could be Hicks, Solow, Harrod neutral
- Uzawa: If $Y = \tilde{F}(K, L, t)$ and
    - capital accumulates as $\dot{K} = Y - C - \delta K$
    - K, Y, C grow exponentially
- Then:
    - $g_K = g_Y = g_C$
    - **can always write it as Harrod neutral**, $Y = F(K, A(t)L)$ for some $A(t)$, $g_A = g_Y - n$
    - if $R = \tilde{F}_K = const \Rightarrow R = F_K = \tilde{F}_K$

Subsection 2

Solow models

Solow model: concave accumulation

- Using Uzawa $\Rightarrow$ focus on $Y = F(K, AL)$
- Constant savings rate s
- Capital accumulation

$$\dot{K} = sF(K, AL) - \delta K$$

- A exogenous, F CRS, with Inada conditions
- Solve? $\rightarrow$ Recitation #2

Results: exogenous growth

- Define $k \equiv K/(AL)$ (more generally $k \equiv e^{-gt}K$)
- **Unique positive steady state k^*, globally stable**

$$\frac{f(k^*)}{k^*} = \frac{\delta + n + g}{s}$$

- **Exogenous growth,** $\dot{Y}/Y = n + g$
- If you can pick s, i.e. $k^* = k^*(s)$, consumption largest if $k^*(s) = k^*_{gold}$ **(golden rule)**

$$f'(k^*_{gold}) = \delta + n + g$$

- $k^* > k^*_{gold}$: have "**dynamic inefficiency"** (but not well defined here)

AK version: sustained growth

- Fix A.
- $F = AK \Rightarrow$

$$\dot{K} = sAK - \delta K$$

$$g_K = sA - \delta$$

- No transitional dynamics

Subsection 3

Data

How much does each proximate cause account for growth?

- Within countries: **Growth accounting**

$$g_Y = s_K g_K + s_L g_L + \underbrace{x}_{\textbf{effect of } A}$$

- OECD countries: 40-50% capital, 30-50% TFP
- LDCs: less TFP, more labor
- mismeasurement issues from capital prices & human capital

How much does each proximate cause account for cross-country GDP differences?

- Across countries: **Development accounting**
- Idea: Make functional form assumption for Y and compare across countries, e.g.

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{H}{L}\right)^{\beta}$$

- Two approaches:
  1. assume A_j exogenous $\Rightarrow$ figure out α, β & R^2
  2. pick value for $\alpha, \beta \Rightarrow$ Recover A_j's

1) Mankiw Romer Weil

- Assume Solow-type accumulation of K and H $\rightarrow$ **evaluate at steady state**

$$\log y_j^* = gt + \frac{\alpha}{1-\alpha-\beta} \log \frac{s_{k,j}}{n_j + g + \delta_k} + \frac{\beta}{1-\alpha-\beta} \log \frac{s_{h,j}}{n_j + g + \delta_h} + \log A_j$$

- Large R^2 around 70%, $\alpha, \beta \approx 0.30$
- **But:**
- Strong assumption that $\log A_j$ is uncorrelated with $s_{k,j}, s_{h,j}$
  - biases α, β, R^2 upwards
- Huge value of β relative to Mincerian estimates

2) Hall Jones 1999

- Construct H from Mincerian regression
- Recover

$$\frac{A_j}{A_{US}} = \left(\frac{Y_j}{Y_{US}}\right)^{3/2} \left(\frac{K_{US}}{K_j}\right)^{1/2} \left(\frac{H_{US}}{H_J}\right)$$

- Find larger role for technology
- Assumptions
  - no human capital externalities + other assumptions to construct K, H
  - Cobb-Douglas Y with same α! ($\rightarrow$ can be somewhat more flexible)

Section 2

NGM and OLG: still K

Subsection 1

NGM

Baseline NGM

- Endogenize savings rate: Representative household solving

$$\max_{c,k} \int_0^\infty e^{-(\rho-n)t} u(c_t) dt$$

- Assume $\rho > n$, $u(c) = \frac{c^{1-\theta}}{1-\theta}$. For now: $A = 1$.
- Equilibrium efficient (single agent) $\Rightarrow$ Planner

$$\max_{c,k} \int_0^\infty e^{-(\rho-n)t} u(c_t) dt$$

$$c_t + \dot{k}_t = f(k) - (\delta + n)k$$

$$k_0 \text{ given}$$

NGM FOCs

- Euler (always holds for **per capita** c)

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(f'(k) - \delta - \rho\right)$$

- TVC

$$\lim_{t\to\infty} e^{-(\rho-n)t} u'(c_t) k_t = 0$$

- Illustrate dynamics in **phase diagram.** TVC pins down a single stable arm!
- Can do comparative dynamics ...
- With growth: Use c/A and k/A

Subsection 2

OLG & dynamic inefficiency

The problem with infinite households

- With ∞ households, planner is allowed to redistribute along an infinite chain of households
- Can violate FWT if value of endowments is infinite $\rightarrow$ **dynamic inefficiency**
- Here: only canonical OLG model with
  - $L = const$
  - Cobb-Douglas technology $f(k) = k^{\alpha}$
  - log utility
  - $\delta = 1$

Canonical OLG model

- Generation t solves

$$\max \log c_1(t) + \beta \log c_2(t)$$

$$c_1(t) + k(t) \leq w(t)$$

$$c_2(t) \leq R(t+1)k(t)$$

giving

$$k(t) = \frac{\beta}{1+\beta} w(t) = \frac{\beta}{1+\beta}(1-\alpha)k(t)^{\alpha}$$

- Unique positive steady state k^*, globally stable

Dynamic inefficiency

- **But:** possibly $k^* > k^*_{gold}$, i.e. $R^* < 1$: **dynamic inefficiency**
- Can be cured by
  - redistribution from young to old (unfunded social security)
  - less saving
  - government debt
  - money

Section 3

Neoclassical endogenous growth: still K

Neoclassical AK model

- Except for the Solow AK economy: No **endogenous** growth model so far! Here: NGM version of AK...
- Assume $f(k) = Ak \Rightarrow$

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(A - \delta - \rho)$$

$$\dot{k}_t = Ak - (\delta + n)k - c$$

- Hence $g_c = \frac{1}{\theta}(A - \delta - \rho)$, $r = A - \delta$
- Need:

$$r > g_Y = g_C = g_c + n$$

- Here: Tax changes affect growth rates!

Rebelo AK

- Same AK structure now produces capital, using capital as input
- Final output is consumed $C = BK_C^{\alpha} L_C^{1-\alpha}$, relative price of capital goes to zero
- Easiest way to analyze: Planning problem!

Romer 1986: Growth with externalities

- Assume $Y = F(K, AL)$ with $A = BK$ uninternalized "learning by doing"
- Then:
$$R = F_K(K, AL) = F_K(1, BL) = const$$
so from Euler we get $g_C = \frac{1}{\theta}(R - \delta - \rho)$
- TVC requires
$$r > g_Y = g_C$$
- Not Pareto optimal due to externalities!

Section 4

Endogenous technology: A

Endogenous technology models

- Discussed the mechanics in Recitation #4 at length. Here: **Overview**
- 3 models of endogenous A:
  - Lab Equipment, Knowledge Spillovers: expanding varieties N
  - Schumpeterian: quality Q
- Key: **Technology is excludable**, even if non-rival
  - hence inventors can earn monopoly rents
- Abstract from K

Lab Equipment (Romer 1990)

- Innovation possibilities frontier: $\dot{N} = \eta Z$
- Find BGP with $r = \eta \beta L$ and $g_C = \frac{1}{\theta} (\eta \beta L - \rho)$
- Two types of externalities
  - "new good" externalities
  - monopoly distortion / aggregate demand externalities
- $\Rightarrow$ **social planner values varieties more & prefers higher growth!**
- Implement using two instruments:
  - subsidies to research
  - subsidies to intermediate good inputs
- More competition lowers growth! (but raises current output)

Knowledge spillovers

- Innovation possibilities frontier: $\dot{N} = \eta N L_R$
- Find BGP with $r = (1-\beta)(\eta L - g)$
- New externality: Spillovers $\rightarrow$ even stronger reason for planner to boost growth!

Scale effects

- These models have **scale effects**
- Higher $L \Rightarrow$ higher growth rate
- **Problematic** because
  - L grows in practice
  - higher $L \not\Rightarrow$ higher growth
- Variant: $\dot{N} = \eta N^{\phi} L_R$, $\phi < 1$ but population growth
- akin to "concave" technology, hence exogenous growth $g_Y = \frac{n}{1-\phi} + n$

Schumpeterian model

- Quality improvements, rather than more gadgets
- Creative destruction
- Find $r = \eta\lambda\beta L - \frac{g}{\lambda - 1}$
- New **business stealing** externality
- Planner does not necessarily want to boost growth!

Section 5

World technology growth: A

Model with technology spillovers

- Lab Equipment model in each country, "anchored" to world technology $N_t = e^{gt} N_0$

$$\dot{N}_j = \eta_j \left(\frac{N}{N_j} \right)^{\phi} Z_j$$

where $\phi > 0$. At BGP:

$$g_{N_j} = g$$

$$\frac{N_j}{N} = \left(\frac{\eta_j \beta L_j}{\zeta_j r^*} \right)^{1/\phi}$$

- If $N = \frac{1}{J} \sum N_j \Rightarrow$

$$g = \frac{1}{\theta} \left(\frac{1}{J} \sum \left(\frac{\eta_j \beta L_j}{\zeta_j} \right)^{1/\phi} \right)^{\phi}$$

Remarks

- g taken as given by each country, but endogenously determined by the countries
- Instead of modelling technology spillovers, **terms of trade effects** can also synchronize growth rates along the world
    - opposite also interesting: trade causing asymmetric growth rates (e.g. "infant industries")

Section 6

DTC: What kind of A?

Why DTC?

- Technology often **directed at certain factors** (e.g. skill biased techn change)
- E.g.

$$Y = F(A_L L, A_H H)$$

- What determines profitability of that? e.g.

$$\frac{\partial Y}{\partial A_H} = \underbrace{F_{A_H H}}_{\text{price per efficiency unit}} \times \underbrace{H}_{\text{market size}}$$

- Let s_H be share of income going to $A_H H$
- Then:

$$\frac{\partial Y}{\partial A_H} = \frac{Y}{A_H} s_H$$

Relative profitability

- This gives a measure for relative profitability:

$$\frac{\frac{\partial Y}{\partial A_H}}{\frac{\partial Y}{\partial A_L}} = \left(\frac{A_H}{A_L}\right)^{-1} \frac{s_H}{s_L}$$

- with CES with ES ϵ: s_H / s_L depends on $A_H H / A_L L$
  - increasing if $\epsilon > 1$
  - decreasing if $\epsilon < 1$

Equilibrium bias

- **Weak equilibrium bias:** Increase in $H/L \Rightarrow$
  - A_H/A_L increases if $\epsilon > 1$
  - A_H/A_L decreases if $\epsilon < 1$
- Both times: technology response biased towards H/L!
- **Strong equilibrium bias:** Increase in $H/L \Rightarrow$ relative wage w_H/w_L *increases*
- Upward sloping demand curve

Endogenous DTC model

- Benefit of innovating in sector H

$$V_H = \frac{\beta p_H^{1/\beta} H}{r^*}$$

$$\frac{V_H}{V_L} = const \times \left(\frac{N_H}{N_L}\right)^{-1} \underbrace{\left(\frac{N_H H}{N_L L}\right)^{(\sigma-1)/\sigma}}_{\sim s_H / s_L}$$

- BGP: $V_H / V_L = \eta_L / \eta_H \Rightarrow$

$$\frac{N_H}{N_L} = const \times \left(\frac{H}{L}\right)^{\sigma-1}$$