

1 Some Basic Probability Theory and Calculus

Marginals, conditionals

Suppose $(X, Y) \sim f_{XY}(x, y) \implies$ marginals are $f_X(x) = \int f_{XY}(x, y) dy$ and $f_Y(y) = \int f_{XY}(x, y) dx$.
Conditionals are $f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)}$

Inverse function theorem:

$X \sim f_X(x)$ and $Y = g(X)$. Then if $X \in \mathbb{R} \implies f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right| = f_X(g^{-1}(y)) \left| \frac{1}{g'(g^{-1}(y))} \right|$.
If $X \in \mathbb{R}^n \implies f_Y(y) = f_X(g^{-1}(y)) |\det J_{g^{-1}}(y)| = f_X(g^{-1}(y)) |\det J_g(g^{-1}(y))|^{-1}$

Moment generating function: $X \sim f_X(x) \implies M_X(t) = \mathbb{E}(\exp(t'x))$.

Properties:

1. If $Y = AX + b \implies M_Y(t) = \exp(t'b) M_X(A't)$
2. If X, Y independent $\implies M_{X+Y}(t) = M_X(t) M_Y(t)$
3. $M'_X(0) = \mathbb{E}(X)$, $M''_X(0) = \mathbb{E}(X^2)$, ..., $M_X^{(r)}(0) = \mathbb{E}(X^r)$
4. If $\{X_n\}$ is a sequence of r.v and $M_{X_n}(t) \rightarrow M_X(t) \implies X_n \rightarrow^d X$

Order Statistics:

$X_1, \dots, X_n \sim_{i.i.d} f_X(x)$ with cdf $F_X(x)$. And $X_{(k)}$ the k -th order statistic (from smaller to bigger). Then

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!} F_X(x)^{k-1} (1 - F_X(x))^{n-k} f(x)$$

Special cases: $X_{(1)} \implies f_{X_{(1)}}(x) = n(1 - F_X(x))^{n-1} f_X(x)$ and for $X_{(n)} \implies f_{X_{(n)}}(x) = n(F_X(x))^{n-1} f_X(x)$

Taylor Expansion: $f(x) - f(x_0) \simeq \sum_{i=1}^{i=k} f^{(i)}(x_0) \frac{(x-x_0)^i}{i!}$ with $f^{(k)}(x_0) = \frac{\partial^k f(x)}{\partial x^k}$. If f is multivariate and expand up to $k=2 \implies f(x) - f(x_0) \simeq J_f(x_0)(x-x_0) + \frac{1}{2}(x-x_0)^T H_f(x_0)(x-x_0)^T$ with J_f the Jacobian and H_f the hessian

Properties of exponential

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- If $a_n \rightarrow a \implies \lim_{n \rightarrow \infty} \left(1 + \frac{a_n}{n}\right)^n = e^a$

2 Most important Distributions

Distribution	Support	pdf	cdf	$\mathbb{E}(X)$	$\mathbb{V}(X)$	$M_X(t)$
Bernoulli (p)	$\{0, 1\}$	$p^x (1-p)^{(1-x)}$	–	p	$p(1-p)$	$pe^t + 1 - p$
$B(n, p)$	$\{0, 1, \dots, n\}$	$\binom{n}{x} p^x (1-p)^{n-x}$	–	np	$np(1-p)$	$(pe^t + 1 - p)^n$
Poisson (λ)	\mathbb{N}	$e^{-\lambda} \frac{\lambda^x}{x!}$	–	λ	λ	$\exp(\lambda(e^t - 1))$
Geometric (p)	\mathbb{N}	$p(1-p)^{(k-1)}$	$1 - (1-p)^k$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$ for $t < -\ln(1-p)$
$\exp(\lambda)$	\mathbb{R}_+	$\frac{1}{\lambda} e^{-\frac{1}{\lambda}x}$ if $x > 0$	$1 - e^{-\frac{1}{\lambda}x}$	λ	λ^2	$(1 - t\lambda)^{-1}$
$\Gamma(\alpha, \beta)$	\mathbb{R}_+	$\frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$ if $x > 0$	–	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
$\chi^2(k)$	\mathbb{R}_+	$\frac{1}{2^{\frac{k}{2}} \Gamma(\frac{k}{2})} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$ if $x > 0$	–	k	$2k$	$(1 - 2t)^{-\frac{k}{2}}$
$N(\mu, \sigma^2)$	\mathbb{R}	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}\right)$	–	μ	σ^2	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$
$N(\mu, \Sigma)$	\mathbb{R}^n	$\frac{1}{(2\pi)^{\frac{n}{2}} \det(\Sigma)^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu)' \Sigma^{-1}(x-\mu)}$	–	μ	Σ	$\exp\left(\mu' t + \frac{1}{2} t' \Sigma t\right)$
$t(p)$	\mathbb{R}	$\frac{\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})} \frac{1}{(p\pi)^{\frac{1}{2}} (1+x^2 \frac{1}{p})^{\frac{p+1}{2}}}$	–	0	$\frac{p}{p-2}$	–
$F(d_1, d_2)$	\mathbb{R}_+	$\frac{\sqrt{\frac{(d_1 x)^{d_1} d_1^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{B(\frac{d_1}{2}, \frac{d_2}{2})}$ with $x > 0$	–	$\frac{d_2}{d_2 - 2}$	$\frac{2d_2^2(d_1 + d_2 - 2)}{d_1(d_2 - 2)^2(d_2 - 4)}$	\nexists
$\mathbf{B}(a, \beta)$	$[0, 1]$	$\frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}$	–	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	–

2.1 Some Special Properties of Distributions

Binomial : If $X \sim B(n, p)$, $Y \sim B(m, p)$ and X indep. of $Y \implies X + Y \sim B(m + n, p)$

Poisson: If $X_i \sim \text{Poisson}(\lambda_i)$ independent, then $\sum_{i=1}^{i=n} X_i \sim \text{Poisson}\left(\sum_{i=1}^{i=n} \lambda_i\right)$

Exponential

- If $X \sim \exp(\lambda) \implies X \sim \Gamma(1, \lambda)$
- If $X_i \sim \exp(\lambda)$ independent $\implies \sum_{i=1}^{i=n} X_i \sim \Gamma(n, \lambda)$
- if $X \sim \exp(\lambda)$ and $\alpha > 0 \implies \alpha X \sim \exp(\alpha\lambda)$

Geometric

- If we conduct an infinite number of iid bernoulli trials (indexed 1,2,3,...) all with success probability p , the number of the trial in which we observe the first success has a geometric distribution with parameter p .

Chi-Squared

- If $X_i \sim N(0, 1)$ independent, then $\sum_{i=1}^{i=n} X_i^2 = \chi^2(n)$
- If $Y_i \sim \chi_{k_i}^2$ independent, then $\sum_{i=1}^{i=n} Y_i = \chi^2\left(\sum_{i=1}^{i=n} k_i\right)$
- If $X \sim \chi_n^2 \implies X \sim \Gamma\left(\frac{n}{2}, 2\right)$

Gamma

- Gamma function: $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$. It satisfies:
 1. $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$
 2. $\Gamma(n) = n!$ if $n \in \mathbb{N}$
 3. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
- If $X_i \sim \Gamma(\alpha_i, \theta)$ indep. $\implies \sum X_i \sim \Gamma(\sum \alpha_i, \theta)$
- If $X \sim \Gamma(\alpha, \theta)$ and $\phi > 0 \implies \phi X \sim \Gamma(\alpha, \phi\theta)$

Normal Distribution

- If $X_i \sim N(\mu_i, \sigma_i^2)$ independent, then $\sum_i X_i \sim N(\sum_i \mu_i, \sum_i \sigma_i^2)$
- If $X \sim N(\mu, \sigma^2)$ and $a, b \in \mathbb{R} \implies aX + b \sim N(a\mu + b, a^2\sigma^2)$
- If $X \sim N(\mu, \Sigma) \in \mathbb{R}^n \implies$ marginals $X_i \sim N(\mu_i, \Sigma_{ii})$
- If $X \sim N(\mu, \Sigma) \implies$ any subvector X_k is multivariate normal
- If $X \sim N(\mu, \Sigma) \implies AX + b \sim N(A\mu + b, A\Sigma A')$

t-student

- Defined as $t = \frac{N(0,1)}{\sqrt{\chi^2(n)/n}} \sim t(n)$
- As $n \rightarrow \infty \implies t(n) \rightarrow N(0, 1)$

F-distribution (Snedecor)

- Defined as $F(d_1, d_2) = \frac{\chi^2(d_1)/d_1}{\chi^2(d_2)/d_2}$

Beta ($\mathbf{B}(\alpha, \beta)$)

- **Beta function** = $B(\alpha, \beta) \equiv \int_0^1 u^{\alpha-1} (1-u)^{\beta-1}$
- If $X \sim \Gamma(\alpha_X, \theta)$ and $Y \sim \Gamma(\alpha_Y, \theta) \implies \frac{X}{X+Y} \sim \mathbf{B}(\alpha_X, \alpha_Y)$
- If $X \sim U[0, 1] \implies X^2 \sim \mathbf{B}\left(\frac{1}{2}, 1\right)$

3 Probability Limits

Almost sure convergence: $X_n \xrightarrow{a.s.} X$ almost surely $\iff \Pr(\omega : X_n(\omega) \rightarrow X(\omega)) = 1$

Convergence in Probability: $p \lim_{n \rightarrow \infty} X_n = X \iff$ for all $\varepsilon > 0$, $\lim_{n \rightarrow \infty} \Pr(|X_n - X| < \varepsilon) = 1$

Convergence in Distribution: $X_n \xrightarrow{d} X \iff \lim_{n \rightarrow \infty} F_{X_n}(x) = F_X(x)$ for x continuity point of F_X

Convergence in Quadratic Mean: $X_n \xrightarrow{c.m.} X \iff \mathbb{E}(X_n - X)^2 \rightarrow 0$

Implications: Almost sure convergence \implies Convergence in Probability \implies Convergence in Distribution

Convergence in Quadratic mean \implies Convergence in Probability

Slutsky's Theorem: Let $\{X_n\}, \{Y_n\}$ be seq. of r.v

1. If $X_n \xrightarrow{p} X, Y_n \xrightarrow{p} Y \implies X_n Y_n \xrightarrow{p} XY$
2. If $X_n \xrightarrow{p} X, Y_n \xrightarrow{p} Y \implies X_n + Y_n \xrightarrow{p} X + Y$
3. If $X_n \xrightarrow{p} X$ and $g(x)$ is a continuous function $\implies g(X_n) \xrightarrow{p} g(X)$
4. If $X_n \xrightarrow{a.s.} X$ and $g(x)$ is a continuous function $\implies g(X_n) \xrightarrow{a.s.} g(X)$
5. If $X_n \xrightarrow{d} X$ and $g(x)$ is a continuous function $\implies g(X_n) \xrightarrow{d} g(X)$
6. If $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{d} Y \implies X_n Y_n \xrightarrow{d} XY$

Markov's Inequality: Let X be a random variable and $g(X)$ a nonnegative function. Then for all $r > 0 \implies \Pr(g(X) \geq r) \leq \frac{1}{r} \mathbb{E}(g(X))$

Laws of Large Numbers: Let $\{X_n\}$ be an i.i.d random sequence with $\mathbb{E}(X) = \mu < \infty$ and $\mathbb{V}(X) = \sigma^2 < \infty$. Then

$$\begin{aligned} \bar{X}_n &= \frac{1}{n} \sum_{i=1}^{i=n} X_i \xrightarrow{p} \mu \text{ (weak law of large numbers) and} \\ \bar{X}_n &= \frac{1}{n} \sum_{i=1}^{i=n} X_i \xrightarrow{a.s.} \mu \text{ (strong law of large numbers)} \end{aligned}$$

Central Limit Theorem: Let $\{X_n\}$ be an i.i.d random sequence with $\mathbb{E}(X) = \mu < \infty$ and $\mathbb{V}(X) = \Sigma$. Then

$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \Sigma)$$

Delta method:

• **Univariate:** Suppose $\sqrt{n}(\hat{\theta}_n - \theta) \xrightarrow{d} N(0, \sigma^2)$ and $g(x)$ is a differentiable function. Then

1. If $g'(\theta) \neq 0 \implies \sqrt{n}(g(\hat{\theta}_n) - g(\theta)) \xrightarrow{d} N(0, (g'(\theta))^2 \sigma^2)$
2. If $g'(\theta) = 0$ and $g \in C^2 \implies n(g(\hat{\theta}_n) - g(\theta))^2 \xrightarrow{d} \sigma^2 \frac{g''(\mu)}{2} \chi_1^2$

- **Multivariate:** Let $\hat{\theta}_n \in \mathbb{R}^k$ be a sequence of r.v. and $g : \mathbb{R}^k \rightarrow \mathbb{R}^m$ a differentiable function at $x = \theta$.
Then if $\sqrt{n}(\hat{\theta}_n - \theta) \rightarrow^d N(0, \Sigma)$ and $J_g(\mu) \neq 0 \implies \sqrt{n}(g(\hat{\theta}_n) - g(\theta)) \rightarrow^d N(0, J_g(\mu)\Sigma J_g(\mu)')$
with $J_g(\mu)$ the Jacobian of g at μ

4 Basic Properties of Statistics

Unbiasedness: $\hat{\theta}(\mathbf{X})$ is unbiased $\iff \mathbb{E}_\theta(\hat{\theta}(\mathbf{X})) = \theta$

Consistency: $\hat{\theta}(\mathbf{X})$ is consistent $\iff \hat{\theta}(\mathbf{X}) \rightarrow^p \theta$

Consistency in MSE (mean squared error): $\hat{\theta}(\mathbf{X})$ is consistent in MSE $\iff MSE(\hat{\theta}) \equiv \mathbb{E}(\hat{\theta}(\mathbf{X}) - \theta)^2 \equiv \mathbb{V}(\hat{\theta}(\mathbf{X})) + (\mathbb{E}(\hat{\theta}(\mathbf{X}) - \theta))^2 \rightarrow 0$ as $n \rightarrow \infty$

Properties:

1. $\hat{\theta}(\mathbf{X})$ is consistent in MSE $\iff \mathbb{E}(\hat{\theta}(\mathbf{X})) \rightarrow \theta$ and $\mathbb{V}(\hat{\theta}(\mathbf{X})) \rightarrow 0$.
2. If $\hat{\theta}(\mathbf{X})$ is consistent in MSE \implies is consistent.
3. \bar{X} is consistent in MSE for $\mathbb{E}(X)$

Asymptotic Normality: $\hat{\theta}(\mathbf{X})$ is asympt. normal $\iff \sqrt{n}(\hat{\theta}(\mathbf{X}) - \theta) \rightarrow^d N(0, V_\theta)$.

Cramer-Rao Inequality: Let $W(\mathbf{X})$ be an unbiased estimator for θ that satisfies $\frac{d}{d\theta}\mathbb{E}_\theta(W(\mathbf{X})) = \int_X \frac{\partial}{\partial \theta}(W(\mathbf{x})f(\mathbf{x}|\theta))d\mathbf{x}$. and has finite variance for all θ . Then

$$\mathbb{V}_\theta(W(\mathbf{X})) \geq \frac{1}{n\mathbb{E}_\theta\left(\left(\frac{\partial}{\partial \theta}\ln f(X|\theta)\right)^2\right)} = \frac{1}{J_n(\theta)}$$

With $J_n(\theta)^{-1}$ being the **Cramer-Rao bound**

4.1 Properties of Mean and Variance

- $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$ and $S_X^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$ are unbiased estimators of $\mathbb{E}(X)$ and $\mathbb{V}(X)$ respectively
- Both are asymptotically normal
- If $X_i \sim N(\mu, \sigma^2)$ then
 1. $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
 2. $\frac{(n-1)}{\sigma^2}S_X^2 \sim \chi_{n-1}^2$
 3. \bar{X} and S_X^2 are independent
 4. $\frac{\bar{X} - \mu}{\sqrt{S_X^2/n}} \sim t(n-1)$

4.2 Method of Moments Estimator

Let $X_i \sim_{i.i.d} f(X | \theta)$ with $\theta \in \mathbb{R}^K$

- Calculate $\mathbb{E}(X^r) = f_r(\theta)$ for all $r = 1, 2, \dots, K$
- Replace $\mathbb{E}(X^r)$ with $\overline{X^r}$ (i.e. $\overline{X^r} = f_r(\theta)$)
- Solve for θ the system of R equations in R unknowns.

If $f^{-1} = (f_1(\theta), f_2(\theta), \dots, f_K(\theta))^{-1}$ exists and is continuous, then $\widehat{\theta}_{MM}$ is consistent. If f^{-1} is differentiable, then because of delta method it is asymptotically normal

5 Properties of Maximum Likelihood Estimation

Invariance: If $\widehat{\theta}_{MLE}$ is the MLE of θ , then $\tau(\widehat{\theta}_{MLE})$ is the MLE of $\tau(\theta)$ for any function τ

Regularity Conditions

1. X_1, X_2, \dots, X_n i.i.d with $X_i \sim f(x | \theta)$
2. **Identifiably:** If $\theta \neq \theta' \implies f(x | \theta) \neq f(x | \theta')$
3. $f(x | \theta)$ has support that does not depend on θ
4. True parameter θ_0 is interior to Θ
5. $\frac{\partial^3 f(x|\theta)}{\partial \theta^3}(\theta)$ exists, is continuous, and satisfies that for all θ_0
6. There exist a function $M_{\theta_0}(x)$ such that $\mathbb{E}_{\theta}(M_{\theta_0}(X)) < \infty$ and c_{θ_0} such that for all x and for all $\theta \in (\theta_0 - c_{\theta_0}, \theta_0 + c_{\theta_0})$ we have that $\left| \frac{\partial^3 \ln f(x|\theta)}{\partial \theta^3}(\theta) \right| \leq M_{\theta_0}(x)$

Information equality: Under regularity conditions, $\mathbb{E}_{\theta} \left(\left(\frac{\partial}{\partial \theta} \ln f(X | \theta) \right)^2 \right) = -\mathbb{E} \left(\frac{\partial^2 \ln f(X|\theta)}{\partial \theta^2} \right) \equiv I_1(\theta)$, so $J_n(\theta) = nI_1(\theta)$

Asymptotic Properties of MLE

1. $\widehat{\theta}_{MLE}$ is consistent for θ
2. **Asymptotic Normality:** $\sqrt{n}(\widehat{\theta}_{MLE} - \theta) \rightarrow^d N(0, I_1(\theta)^{-1})$
3. **Asymptotic Efficiency:** Asymptotically attains cramer-rao bound

6 Testing

Type 1 Error: reject H_0 | H_0 true

Type 2 Error: not reject H_0 | H_0 false

Power of a test: $\Pr(\text{reject } H_0 | H_0 \text{ false})$.

Power function: $P(\theta) = \Pr(\text{reject } H_0 | \theta)$

Neyman-Pearson Lemma: Consider test $H_0 : \theta = \theta_0$ vs. $H_1 : \theta = \theta_1$. Then there exist a UMP

test, with rejection region

$$\text{reject} \iff \frac{f(\mathbf{X} | \theta_1)}{f(\mathbf{X} | \theta_0)} > k$$

Mean Test:

1. $H_0 : \mu = \mu_0$ vs. $\mu > \mu_0$. Use test statistic

$$t = \sqrt{n} \left(\frac{\bar{X} - \mu_0}{S_X} \right)$$

and reject $\iff t > K$. If $X \sim N(\mu, \sigma^2) \implies$ reject if $t > t_{1-\alpha}(n-1)$ being the $1-\alpha$ quantile of t -student. If X is not normal, then reject if $t > z_{1-\alpha}$, with z quantile of $N(0, 1)$

2. $H_0 : \mu = \mu_0$ vs. $\mu \neq \mu_0$ use same statistic, and If $X \sim N(\mu, \sigma^2)$ then reject if $t \notin (-t_{1-\frac{\alpha}{2}}(n-1), t_{1-\frac{\alpha}{2}}(n-1))$. If not normal, reject if $t \notin (-z_{1-\frac{\alpha}{2}}, z_{1-\frac{\alpha}{2}})$

Variance test: Suppose $X \sim N(\mu, \sigma^2)$ and want to test $H_0 : \sigma^2 = \sigma_0^2$ vs $H_1 : \sigma^2 < \sigma_0^2$. Use test statistic

$$\chi^2 = \frac{(n-1)}{\sigma_0^2} S_X^2 \sim \chi_{n-1}^2 \text{ under null.}$$

and reject if $\chi^2 < \chi_{n-1}^2(1-\alpha)$. If test $H_0 : \sigma^2 = \sigma_0^2$ vs $H_1 : \sigma^2 \neq \sigma_0^2$ use same statistic and reject if $\chi^2 \notin [\chi_{n-1}^2(\frac{\alpha}{2}), \chi_{n-1}^2(1-\frac{\alpha}{2})]$

Some critical values:

$$\begin{array}{lll} z_{0.9} & = & 1.2816 & z_{0.975} & = & 1.9600 & z_{0.995} & = & 2.5758 \\ z_{0.95} & = & 1.6449 & z_{0.99} & = & 2.3263 & & & \end{array}$$

and for χ_1^2 :

$$\begin{array}{lll} \chi_1^2(0.95) & = & 3.8415 & \chi_1^2(0.99) & = & 6.6349 & \chi_1^2(0.01) & = & 0.00015 \\ \chi_1^2(0.975) & = & 5.0239 & \chi_1^2(0.05) & = & 0.0039 & & & \end{array}$$

6.1 MLE Tests

Want to test $H_0 : \theta = \theta_0$ vs. $\theta \neq \theta_0$

Wald test:

- **Univariate:** $\sqrt{n} \frac{\hat{\theta}_{MLE} - \theta_0}{\sqrt{I_1^{-1}(\theta_0)}} \sim N(0, 1)$. Do a test like mean test. Or equivalently $n \left(\hat{\theta}_{MLE} - \theta_0 \right)^2 I_1 \left(\hat{\theta}_{MLE} \right) \simeq \chi_1^2$
- **Multivariate:** $n \left(\hat{\theta}_{MLE} - \theta_0 \right)' I_1 \left(\hat{\theta} \right) \left(\hat{\theta}_{MLE} - \theta_0 \right) \sim \chi_p^2$ with $p = \#$ of parameters

LR Test

- Let $\lambda(\mathbf{X}) = \frac{\mathcal{L}(\theta_0)}{\mathcal{L}(\hat{\theta}_{MLE})} = \frac{f(\mathbf{X}|\theta_0)}{\max_{\theta \in \Theta} f(\mathbf{X}|\theta)}$. Then $-2 \ln(\lambda(\mathbf{X})) \simeq \chi_1^2$ and rejection region is $-2 \ln(\lambda(\mathbf{X})) > \chi_1^2(1 - \alpha)$.
- In general, if we test $H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \notin \Theta_0$ use LR $\lambda(\mathbf{X}) = \frac{\max_{\theta \in \Theta_0} f(\mathbf{X}|\theta)}{\max_{\theta \in \Theta} f(\mathbf{X}|\theta)} = \frac{\mathcal{L}(\hat{\theta}_{RESTRICTED})}{\mathcal{L}(\hat{\theta}_{MLE})}$ and then $-2 \ln(\lambda(\mathbf{X})) \simeq \chi_p^2$ with $p = \#$ restrictions.

LM test (score test)

- **Univariate:** $\frac{1}{\sqrt{n}} \frac{s(\theta_0)}{\sqrt{I_1(\theta_0)}} \simeq N(0, 1)$ and do mean test. Equivalently $\frac{1}{n} s(\theta_0)^2 \frac{1}{I_1(\theta_0)} \simeq \chi_1^2$
- **Multivariate:** $\frac{1}{n} s(\theta_0)^T I_1(\theta_0)^{-1} s(\theta_0) \sim \chi_p^2$ with $p = \#$ of parameters

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