14.271 Fall 2005
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Empirical Problem Set #5
Due October 26, 2005

Question 1   You are asked to conduct a study of the plate glass industry in the United States to assess its market power. In particular, you are instructed to estimate θ from the following generalization of the monopolist's first-order condition:

$$P + \theta Q \frac{\partial P(Q,Y)}{\partial Q} = MC(Q,W) \tag{1}$$

where $P(Q,Y)$ is the inverse demand curve and $MC(Q,W)$ is industry marginal cost. The parameter θ is a measure of conduct, with a value of 0 for perfectly competitive behavior and a value of 1 for perfectly collusive (monopoly) behavior.

You have twenty years of weekly time series data on plate glass prices (P) and quantities (Q). Finally, you have data for a wage variable (W) which affects but does not fully constitute marginal cost. Note that you do not observe marginal cost itself, but you are told that the marginal cost of producing plate glass does not vary with output. There are no fixed costs.

Assume that the demand and marginal cost for plate glass take the following form:

$$\ln Q = a_1 + a_2 \ln P + \nu \tag{2}$$

$$MC(Q,W) = W + \eta \tag{3}$$

(a) Given the data available to you and the assumed functional forms for demand and costs, can you distinguish between competitive and collusive behavior? Derive your estimating equation(s) and explain whether each equation is identified. Can you recover an estimate of θ?

(b) Now suppose that the industry is governed by a cartel. Suppose that, because of difficulties in coordinating the cartel on the monopoly price without attracting the attention of antitrust authorities, the firms use "rule of thumb" pricing. In particular, they always simply add $2 to marginal cost:

$$P = 2 + MC(Q,W) \tag{4}$$

The price of plate glass has averaged $5 over this period. If you are not aware of this rule of thumb pricing and you attempt to infer market power by estimating (1), what will you conclude? Is this a correct inference? Explain.

Question 2   Over the last week, we have discussed several papers where Markov dynamics were central to the analysis. Under Markov dynamics, the probability of moving from one state to another only depends on the current state, and not the path leading to that state. This is a popular method for examining the dynamic behavior of firms, since it allows for a wide range of behavior while remaining tractible. This question asks you to work your way through computing a Markov-perfect Nash equilibrium in a simple environment. You will be required to use Matlab or

1

something similar to program a solution to this problem. You are allowed to work in groups, but I strongly suggest that you try this on your own. If you are planning on doing empirical work in IO, you must be able to program this model by yourself.

The basic setup of the model is that there is a monopoly which sells a single good to a product market in each period, $t = 1, \ldots, \infty$. Its profit function is:

$$\pi^m(s) = 10 \ln(s), \tag{5}$$

where s is a discrete state variable. To guarantee that expected profits are not infinite, there is a positive discount rate equal to $\beta = 0.9$. The dynamics of this problem come through the fact that the firm is able to influence the transition matrix governing s. Typically s is something like quality or capacity. A firm can adjust s from one discrete state to the next by making active, costly investments. For a given investment $x \geq 0$, the probability of moving to the next highest state is:

$$Pr(s' = s + 1 | x) = \frac{x}{1 + x}, \tag{6}$$

Note that the probability of transition increases in x, and is bounded by 0 and 1. The cost of investment is 1 per unit of x. Firms make optimal investments in the sense that they maximize their expected discounted stream of profits:

$$\max_{x_i} \pi^m(s_i) - x_i + \beta EV(s_i'). \tag{7}$$

Note that firms are taking expectations over the value of the future state since the level of s is uncertain for any level of positive investment. Your job is to solve for the optimal policies and value functions for the monopoly for every level of s.

Step One: Setup the Model. In order to make this a little easier, restrict s to go from $s = 10$ to $s = 1,000$ in 10 unit increments. This will generate a state space of 100 states. You want to form the following vectors: the state space, a 100×1 vector containing each element of s, a profit vector containing the payoffs for each of those states, a policy vector which contains the optimal level of investment for each state, and a value vector, which gives the expected net present value of being at that state. Label these vectors S, Π, P, and V, respectively. Fill in S and Π with their appropriate values, give P an initial guess of zeros everywhere, and make V equal Π to start.

Step Two: Solve a simplified version of the model. To solve this model, we are going to use value function iteration. This means that we cycle through the state space, solving for the optimal policy at each state, and then updating the value function. I want you to solve a model without investment to begin. Without any investment, a firm is stuck at whatever state it finds itself initially, and accrues the expected value of being in state forever. Solving this simple model first allows you to check the mechanics of your solving procedure, since can figure out the answer analytically. To do it numerically, start by setting up an outer loop which checks for convergence. Convergence here is defined as two successive value functions being within a small distance of each other. The L^2-norm is a standard measure of distance between two vectors. Stop convergence when this norm is less than a small number, say 1E-5.

What exactly is converging inside this outer loop? You want to cycle through each element of the state space, solving for the optimal investment (which we are going to set equal to zero here), and updating the value function. In this simple example, this just means updating $V(s_i)$:

$$V(s_i) = \pi^m(s_i) + \beta E(V(s_i')), \tag{8}$$

where $E(V(s_i'))$ is the expected continuation value. Since there is no investment here, there is no uncertainty, so we could drop the expectation and write:

$$V(s_i) = \pi^m(s_i) + \beta V(s_i). \tag{9}$$

You can solve for $V(s_i)$ directly from this equation, but set up your loop to solve for it. Given a starting value of Π, you should obtain a solution in 171 iterations. Note that I assume that we do not update V as we go along, but rather change it all at once to reflect the newly computed values after cycling through the entire state space. Report the relationship between the per-period profit and the value function.

Step Three: Solve for the model with investment. Once we have the basic framework for solving for the value function, it is straightforward to add investment. The first step is to derive the optimal investment for each element of the state space. I have chosen functional forms which make an analytic solution easy to obtain.[1] Now when we solve for the value function at each state, we have to update the period payoff function to reflect that we may be making a costly investment:

$$V(s_i) = \pi^m(s_i) - x^* + \beta E(V(s_i')), \tag{10}$$

where x^* is the optimal investment. Now it is not possible to drop the expectation, since there is uncertainty in what state we will be in next period if $x > 0$. Equation 6 gives the probability of being in one of two states given x. Compute the expected continuation value, subtract investment costs, and add per-period payoffs to update the value function. Otherwise our model uses the exact same solution procedure as above.

Report your results in the form of a table which contains the state, the per-period payoff, the optimal policy, and the expected value of being at that state. In this more complex model, I also obtain convergence after 171 iterations. Conjecture why this is the same number as without investment, and if it is true in general.

What happens to investment as the state increases? What is the intuitive explanation for this feature? Even though the monopoly could obtain a maximum $s = 1000$, and profits are increasing in s, in the long-run which level will it end up at?

(Bonus) Suppose that we knew a monopoly operated in accordance with this model, and we observed a long time series of its states, investments, and profits. If investment costs were unknown, could we identify them with this model and data? Explain your intuition.

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[1]Be careful here–investment must be non-negative, and be sure to make sure you won't divide by zero. Make sure you check the corner solution for investment at the maximum state $s = 1000$.