Homework 1 (problems marked with * will be graded)
14.124. Spring 2017

1. *Show that the monotone likelihood ratio condition ($f(x)/g(x)$ increasing in x, where x is a real number) implies first-order stochastic dominance ($F(x) \leq G(x)$ for all x).

2. Let $\theta = \theta_1$ or θ_2 denote the state of nature in a two state world. Let $p_0 = \Pr(\theta = \theta_1)$. Let f(p) be an arbitrary distribution on the unit interval such that $\int pf(p)dp = p_0$. Show that f(p) can be construed as a distribution over posterior probabilities of p, stemming from an experiment with outcomes y, a prior probability $p = p_0$ and two likelihood functions $p(y|\theta_1)$ and $p(y|\theta_2)$. [It may be easier to solve the problem if you assume that f(p) consists of two mass points so that the experiment will have only two relevant outcomes, say, y_L and y_H.]

3. An agent has to decide between two actions a_1 and a_2, uncertain of the prevailing state of nature s, which can be either s_1 or s_2. The agent's payoff as a function of the action and the state of nature is as follows:

$$u(a_1,s_1) = 10,\ u(a_1,s_2) = 7$$

$$u(a_2,s_1) = 5,\ u(a_2,s_2) = 11$$

   The prior probability of state s_1 is $p = \Pr(s_1)$.

   a. Give a graphical representation of the agent's decision problem as a function of the probability of state s_1. If $p = (.4)$, what is the agent's optimal decision?

   b. Let the agent have access to a signal y before taking an action. Assume y has two possible outcomes with the following likelihoods

$$\Pr(y_1|s_1) = \lambda_1$$

$$\Pr(y_1|s_2) = \lambda_2$$

   How valuable is this information system if $\lambda_1 = \lambda_2 = ½$? How valuable is it if $\lambda_1 = ½$ and $\lambda_2 = 0$?

   c. Show that, irrespectively of the prior, the information system $\lambda_1 = ½$ and $\lambda_2 = 0$ is preferred to the information system $\lambda_1 = ½\alpha + ½\beta$ and $\lambda_2 = \beta$, where α and β are any numbers between 0 and 1. (Hint: show that latter system is garbling of former).

4. *An agent can work hard ($e = e_H$) or be lazy ($e = e_L$), where $e_H > e_L > 0$. There are two profit levels, x_1 and x_2; $x_1 < x_2$. Hard work makes the high profit level more likely. Specifically, Prob($x = x_2$) is f_H if the agent works hard and $f_L < f_H$, if the agent is lazy. The principal can only observe realized profits x. The principal is risk neutral and the agent is risk averse with preferences u(w) - e. The utility function u is strictly concave and increasing.

   a. Assume that the Principal's reservation utility is 0; that is, the Principal has to be assured a non-negative profit. Show how to solve for the Pareto Optimal action and incentive scheme
   $s(x_i) = s_i$, i = 1,2, where s_i represents the payment to the agent if the outcome is x_i.

   b. Suppose that the agent is given the opportunity to choose a third action e_M with the features that $Prob(x_2|e_M) = ½f_H + ½f_L$ and $e_M > ½e_H + ½e_L$. Can the Principal implement e_M?

   c. Suppose that the second-best solution in Part a is such that it is optimal to implement e_H. Let the agent have access to the action e_M described above, but assume now that $e_M < ½e_H + ½e_L$. Will the solution to Part a stand once the agent is given access to e_M?

5. Consider the risk-sharing problem we saw in class. Show that the Pareto frontier is concave (in expected utility space), or, equivalently, that the feasible set of utility pairs is convex.

14.124 Microeconomic Theory IV
Spring 2017