14.04 Midterm Exam 1 Prof. Sergei Izmalkov

Wed, Oct 1

1. Let $(\mathbf{p}^t, \mathbf{y}^t)$ for $t = 1, \dots, N$ be a set of observed choices that satisfy WAPM, let YI and YO be the inner and outer bounds to the true production set Y. Let $\pi^+(\mathbf{p})$, $\pi(\mathbf{p})$, and $\pi^-(\mathbf{p})$ be profit functions associated with YO, Y, and YI correspondingly.

   (a) Show that for all $\mathbf{p}$, $\pi^+(\mathbf{p}) \geq \pi(\mathbf{p}) \geq \pi^-(\mathbf{p})$.

   (b) If for all $\mathbf{p}$, $\pi^+(\mathbf{p}) = \pi(\mathbf{p}) = \pi^-(\mathbf{p})$, what you can say about YO, Y, and YI? Provide formal arguments.

   (c) For $(\mathbf{p}^1, \mathbf{y}^1) = ([1, 1], [-3, 4])$, and $(\mathbf{p}^2, \mathbf{y}^2) = ([2, 1], [-1, 2])$ construct YI and YO (graphically). What can you say about returns to scale in the technology these observations are coming from? Hint: think $\mathbf{y} = (-x, y)$.

2. Given the production function $f(x_1, x_2, x_3) = x_1^a \min\{x_2, x_3\}^a$,

   (a) Calculate profit maximizing supply and demand functions, and the profit function. What restriction you have to impose on a?

   (b) Fix y. Calculate conditional demands and the cost function $c(w_1, w_2, y)$.

   (c) Solve the problem $py - c(w_1, w_2, y) \to \max_y$, do you obtain the same solution as in 2a? Explain your findings.

3. Given the production function $f(x_1, x_2) = x_1 + x_2^b$, where $b > 0$, calculate the cost function $c(1, 1, y)$. How would costs respond to the changes in w_1, w_2, and y? How would factor demands respond?

4. Consider a firm with conditional factor demand functions of the form (output has been set equal to 1 for convenience):

$$\begin{aligned} x_1 &= 1 + w_1^{-\frac{1}{3}} w_2^a, \\ x_2 &= 1 + d w_1^b w_2^c. \end{aligned}$$

   What are the values of the parameters a, b, c, and d and why?

5. The cost function is $c(w_1, w_2, y) = w_1^a w_2^b y^d$.

   (a) What do we know about a and b?

   (b) What are the conditional factor demands? What is the production function?

   (c) What can you tell about returns to scale?

6. Let $c(w_1, w_2, \bar{y}) = \bar{c}(\bar{y})$ be the isocost and $y = f(x_1, x_2) = \bar{y}$ be the isoquant corresponding to a fixed output level $\bar{y} = 1$.

   (a) What are the slopes of these lines?

   (b) Draw the isocost and isoquant for Cobb-Douglas technology $y = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$ corresponding to $\bar{y} = 1$.

   (c) Suppose $c(w_1, w_2, 1) = aw_1 + bw_2$. Draw the isocost and the corresponding isoquant (use the slopes to obtain the shape of the isoquant).

   (d) Repeat for $c(w_1, w_2, 1) = \min\{aw_1, bw_2\}$.

   (e) Draw conditional demand x_1 as a function of $\frac{w_1}{w_2}$ for 7c and 7d.
   Hint: If you have trouble in $7c - 7e$, think what technology these costs came from.