14.04 - Problem Set 1
Due Sept 22nd in recitation

1) Start with an arbitrary utility function $u(x_1, x_2)$ that is differentiable. Let $v(u)$ be a monotonic transformation of u.

a) Solve:

$$\max u(x_1, x_2)$$

$$ST : p_1x_1 + p_2x_2 = m$$

b) Solve:

$$\max v(u(x_1, x_2))$$

$$ST : p_1x_1 + p_2x_2 = m$$

c) Discuss the relationship between these problems. What characteristics of the utility function is generating this result?

2) Consider the following problem:

$$\max_{x,y} x^\alpha y$$

Subject to:

$$x + py \leq 10, x \geq 0, y \geq 0$$

a) Show formally that the utility function $x^\alpha y$ is at least weakly monotonic and strongly convex for $\alpha > 0$. You may use ideas from problem 1 to simplify the problem.

b) Find $V(\alpha, p), x(\alpha, p), y(\alpha, p)$

3) Solve the following:

$$\max_{x,y} \ln x + y$$

$$ST : 2x + y \leq 10, x \geq 0, y \geq 0$$

4) One way to rule out the potential that the non negativity constraints aren't binding is to look at the marginal rate of substitution (MRS) when one of the factors gets arbitrarily close to zero. Suppose that we have a function $f(x_1, x_2)$. The $MRS_{12}(x_1, x_2)$ is the amount of x_1 required to keep the function f the same when x_2 changes by a small amount. $MRS_{12}(x_1, x_2)$ is read "the marginal rate of substitution of good 1 for good 2 at (x_1, x_2)" Formally:

$$MRS_{12}(x_1, x_2) = \left.\frac{dx_1}{dx_2}\right|_{(x_1,x_2)} = \frac{\frac{\partial f(x_1,x_2)}{\partial x_2}}{\frac{\partial f(x_1,x_2)}{\partial x_1}}$$

a) Consider the function $f(x, y) = xy$. Starting from a point where $x, y > 0$, what happens to the MRS$_{xy}$ as y grows smaller and approaches zero (ie $lim_{y \to 0} MRS_{xy}(x, y)$) ? What happens to $lim_{x \to 0} MRS_{yx}$?

b) Consider the function $x + y$. What is $lim_{y \to 0} MRS_{xy}$? What is $lim_{x \to 0} MRS_{yx}$?

c) Consider $\ln x + y$. What is $lim_{y \to 0} MRS_{xy}$? What is $lim_{x \to 0} MRS_{yx}$?