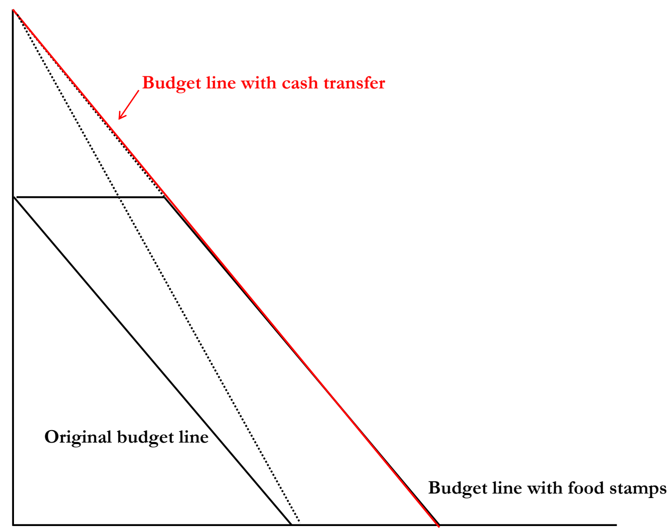
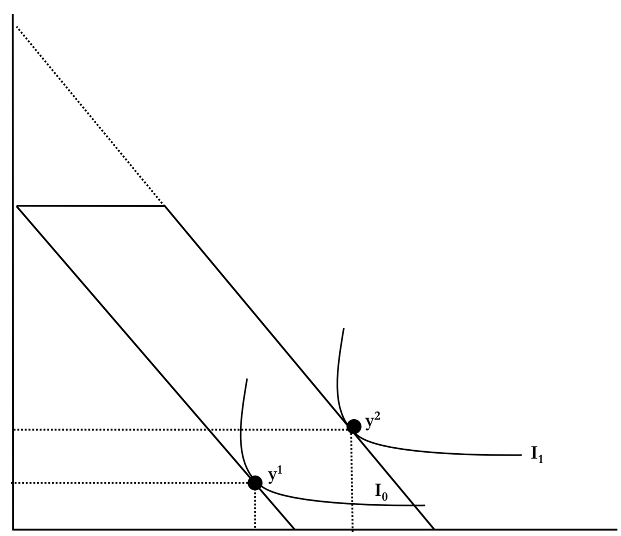
14.01 Fall 2010
Problem Set 3 Solutions

1. (12 points) Assume the government has two policy options, a cash grant of $200 and providing food stamps worth $200.

   (a) (3 points) Draw the budget constraint faced by the consumer in each of these two situations.

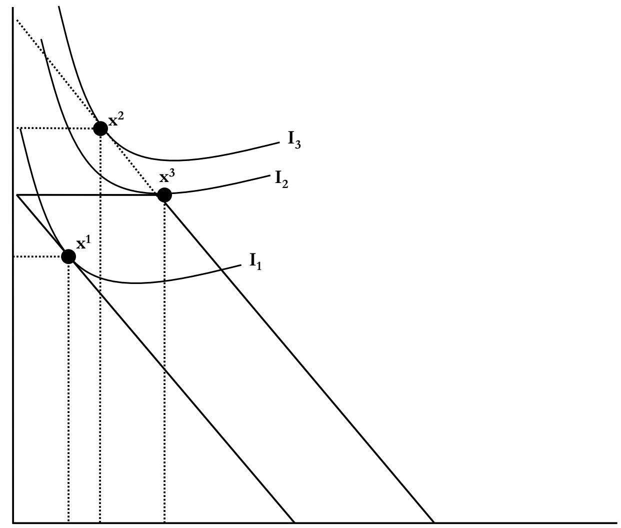


   (b) (3 points) Now, draw a set of indifference curves that corresponds to an individual who would be indifferent between these two policies.



   Indifference curve I_0 and I_1 correspond to an individual who is indifferent between the two policies.

   (c) (3 points) Next, draw a set of indifference curves corresponding to an individual who would prefer to have a cash grant rather than food stamps.



This individual is on indifference curve I_1 prior to the transfer of cash or food stamps. With cash, they could reach indifference curve I_3. However, with food stamps they can only reach indifference curve I_2. Accordingly, this individual would be better off with a cash transfer.

(d) (3 points) Is there a consumer who strictly prefers food stamps to cash grants (i.e., is better off with food stamps rather than a cash grant?) Why does the government use food stamps?

There is no consumer who strictly prefers food stamps to cash grants. Any allocation that can be achieved with food stamps can also be achieved with cash grants, so there is no reason to strictly prefer a cash grant. However, some allocations that can be achieved with a cash grant cannot be achieved with food stamps, because food stamps require that you spend a certain amount on food. Nonetheless, the government may use food stamps because policymakers prefer to require that consumers spend a certain amount of food rather than on other goods that the policymakers view as undesirable (e.g., alcohol, cigarettes, entertainment.)

Problem 1 solution by MIT OpenCourseWare.

2. (22 points) Suppose there are exactly two consumers (Albie and Bubbie) who demand strawberries. Suppose that Albie's demand for strawberries is given by

$$q_a(p) = p^\alpha f_a(I_a)$$

and Bubbie's demand is given by

$$q_b(p) = p^\beta f_b(I_b)$$

where I_a and I_b are Albie and Bubbie's incomes, and $f_a(\cdot)$ and $f_b(\cdot)$ are two unknown functions.

(a) (5 points) Find Albie and Bubbie's (own-price) elasticities of demand, $\epsilon_{q_a,p}$ and $\epsilon_{q_b,p}$. Use the sign convention that $\epsilon_{y,x} = \frac{\partial y}{\partial x}\frac{x}{y}$.

$$\epsilon_{q_a,p} = \frac{\partial q_a}{\partial p}\frac{p}{q_a(p)} = [\alpha p^{\alpha-1} f_a(I_a s)]\frac{p}{p^\alpha f_a(I_a)} = \alpha$$

and similarly, $\epsilon_{q_b,p} = \beta$.

(b) (5 points) Suppose that $\alpha > 0 > \beta$. Are strawberries a Giffen good for Albie? Are strawberries a Giffen good for Bubbie?

$\epsilon_{q_a,p} = \alpha \geq 0$ which means that strawberries are Giffen for Albie. $\epsilon_{q_b,p} = \beta < 0$, regular for Bubbie.

(c) (12 points) Are strawberries an inferior good for Albie? Are strawberries an inferior good for Bubbie? Assume that these demands arise from utility maximization given linear budget constraints. Hint: This question should not require much/any algebra.

Strawberries must be inferior for Albie, since Giffen goods must be inferior. The substitution effect means a higher price for strawberries encourage Albie to buy less of them. Thus if he buys more, the income effect must make him buy more (i.e., it is inferior), and outweigh the substitution effect.

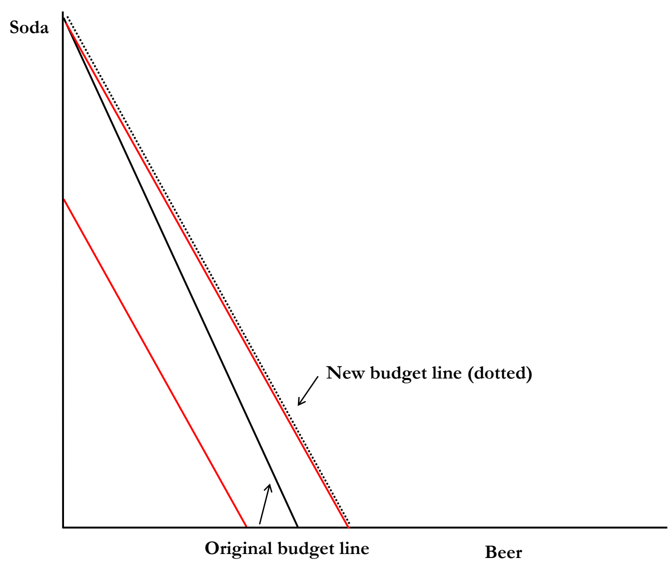
For Bubbie, strawberries could be normal or inferior. If inferior, it's just that the substitution effect outweighs the income effect.

3. (24 points) Joe considers beer and soda to be perfect substitutes at a rate of 1:2, that is, he always receives the same utility if he has one beer or two sodas to drink. He spends \$12 a day on drinks, and beers cost \$3 while sodas cost \$1 each. However, one day the price of beers decreases to \$2; there is no change in his budget.

(a) (6 points) How does consumption change when the price of beers changes? What is Joe's new level of utility?

Intuitively, since beer and soda are perfect substitutes, Joe will always spend his entire budget on whichever drink has the lower price per unit of utility. Originally, a unit of utility from soda cost \$2 (for two sodas), while a unit of utility from drinking beer cost \$3, so Joe bought only soda; he would consume 12 sodas, and his utility is 6. Given his budget, he would buy 4 beers. When the price of beer changes, the cost of a unit of utility from soda is now equal to the cost of a unit of utility from beer. Accordingly, Joe is indifferent between soda and beer. He can optimally buy only soda, only beer, or any combination in between. However, his overall utility has not changed.

(b) (6 points) Show with the aid of a graph what happens to the optimal allocation and the level of utility when the price of beer changes.

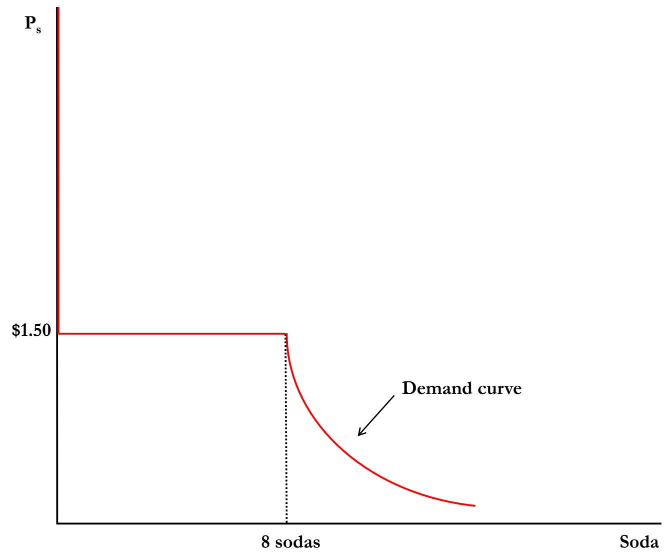


(c) (6 points) How much must Joe's budget decrease to return him to the original utility level?

Joe's utility level has not changed; accordingly, there is no need for a change in the budget to return him to his original level of utility.

(d) (6 points) Now assume the price of beers returns to the original price of \$3. Describe the demand curve for soda holding the price of beer and income constant; a graphic representation is optional.

At the original price of beer, Joe purchased only soda as long as the price of soda was less than \$1.50, and demand is equal to $\frac{I}{P_s}$. Once the price of soda is above \$1.50, demand is zero as Joe will prefer to consume beer.



Problem 3 solution by MIT OpenCourseWare.

4. (42 points) Xiaoyu spends all her income on statistical software (S) and clothes (C). Her preferences can be represented by the utility function: $U(S, C) = 4\ln(S) + 6\ln(C)$.

   (a) (6 points) Compute the marginal rate of substitution of software for clothes. Is the MRS increasing or decreasing in S? How do we interpret this?

   We have that $MRS = \frac{\frac{4}{S}}{\frac{6}{C}} = \frac{2}{3}\frac{C}{S}$. Hence, the MRS is decreasing in S. As Xiaoyu moves down an indifference curve consuming more software and fewer clothes, she is ready to give up fewer clothes for extra software.

   (b) (6 points) Find Xiaoyu's demand functions for software and clothes, $Q_S(p_S, p_C, I)$ and $Q_C(p_S, p_C, I)$, in terms of the price of software (p_S), the price of clothes (p_C), and Xiaoyu's income (I).

   At an interior optimum, $MRS = \frac{p_S}{p_C} \longrightarrow \frac{2}{3}\frac{C}{S} = \frac{p_S}{p_C}$. Hence $C = \frac{3}{2}\frac{p_S}{p_C}S$. Substituting for C into the budget constraint, we get that $p_S S + p_C \frac{3}{2}\frac{p_S}{p_C}S = I$, which means that $Q_S(p_S, p_C, I) = S = \frac{2}{5}\frac{I}{p_S}$. Hence $Q_C(p_S, p_C, I) = \frac{3}{5}\frac{I}{p_C}$.

   (c) (6 points) Draw the Engel curve for software.

   The Engel curve is a linear function with zero intercept and a slope of $\frac{5}{2}p_S$.

   (d) (6 points) Suppose that the price of software is $p_S = 2$, the price of clothes is $p_C = 3$, and Xiaoyu's income is $I = 10$. What bundle of software and clothes (S, C) maximizes Xiaoyu's utility?

   Using the demand functions from part (b), we get that $S = \frac{2}{5}\frac{10}{2} = 2$ and $C = \frac{3}{5}\frac{10}{3} = 2$.

   (e) (6 points) Suppose the price of software increases to $p_S = 4$. What bundle of software and clothes does Xiaoyu demand now?

   Using the demand functions from part (b), we get that $S' = \frac{2}{5}\frac{10}{4} = 1$ while $C' = 2$ as before.

   (f) (6 points) Given the price increase, how much income does Xiaoyu need to remain as happy (have the same utility) as she was before the price change? What bundle of software and clothes would Xiaoyu consume if she had that additional income, given the new prices?

   Prior to the price change, Xiaoyu's utility is:

$$U(2, 2) = 4\ln(2) + 6\ln(2) = 10\ln(2)$$

   We want to find the income level I^* which would give her the same utility $U = 10\ln(2)$ after the price increase. Plugging Xiaoyu's demand functions from part (b) into her utility function, we get that:

$$\begin{aligned} U(S, C) &= 4\ln\left(\frac{2}{5}\frac{I}{p_S}\right) + 6\ln\left(\frac{3}{5}\frac{I}{p_C}\right) \\ &= 4\ln(2) + 6\ln(3) + 10\ln(I) - 10\ln(5) - 4\ln(p_S) - 6\ln(p_C) \end{aligned}$$

Now, I^* solves:

$$\begin{aligned} U(2,2) = 10\ln(2) &= 4\ln(2) + 6\ln(3) + 10\ln(I^*) - 10\ln(5) - 4\ln(4) - 6\ln(3) \\ 6\ln(2) &= 10\ln(I^*) - 10\ln(5) - 4\ln(4) \\ \ln(I^*) &= \frac{3\ln(2) + 5\ln(5) + 2\ln(4)}{5} = \ln(5 \cdot 2^{\frac{7}{2}}) \end{aligned}$$

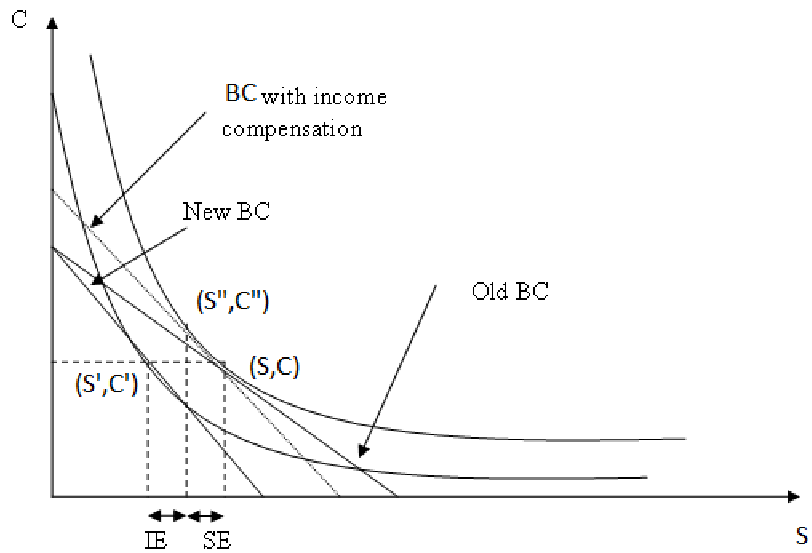
Hence $I^* = 2^{1.4}5 = 13.1951$. If Xiaoyu had I^* she would consume $S'' = 1.3195$ and $C'' = 2.639$.

(g) (6 points) Going back to the situation in part (e) ($p_S = 4$ and $I = 10$), decompose the total change of software and clothes demanded into substitution and income effects. In a clearly-labeled diagram with software on the horizontal axis, show the income and substitution effects of the increase in the price of software.

The total effect is the difference between the bundles consumed after and before the price change, i.e. $(S', C') - (S, C) = (-1, 0)$, i.e. the total effect is a decrease in the consumption of software by 1 and no decrease in the consumption of clothes.

The substitution effect (SE) is the difference between the bundles consumed and before the price change staying on the same indifference curve, i.e. $(S'', C'') - (S, C) = (-0.6805, .639)$. Hence, the pure substitution effect is a decrease in the consumption of books by .6805 and an increase in the consumption of clothes by .639.

The income effect (IE) is the difference between (S', C') and (S'', C''), i.e. a decrease of .3195 in software and a decrease of .639 in clothes.



Problem 4 solution courtesy of William Wheaton. Used with permission.