14.01 Principles of Microeconomics, Fall 2007
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Lecture 28

Oligopoly

Outline

1. Chap 12, 13: *Stackelberg*

2. Chap 12, 13: *Bertrand*

3. Chap 12, 13: *Prisoner's Dilemma*

In the discussion that follows, all of the games are played only once.
and

1 Stackelberg

Stackelberg model is an oligopoly model in which firms choose quantities sequentially.

Now change the example discussed in last lecture as follows: if firm 1 produces crispy and firm 2 produces sweet, the payoff is $(10, 20)$; if firm 1 produces sweet and firm 2 produces crispy, the payoff is $(20, 10)$ (see Table 1).

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,20
	Sweet	20,10	-5,-5

Table 1: Payoffs of Firm 1 and 2.

$$\begin{pmatrix} -5, -5 & 10, 20 \\ 20, 10 & -5, -5 \end{pmatrix}$$

This is an extensive form game; we use a tree structure to describe it.
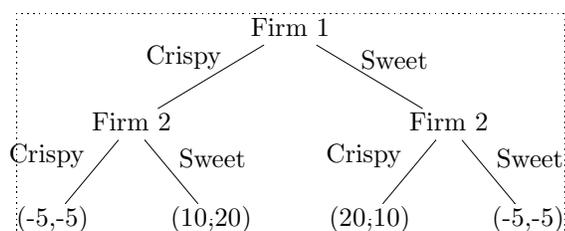
Start from the bottom using backward induction, namely, solve firm 2's decision problem first, and then firm 1's. If firm 1 chooses crispy, firm 2 will choose sweet to get a higher payoff. If firm 2 chooses sweet, firm 2 will choose crispy. Knowing this, firm 1 will choose sweet in the first place. In this case, going first gives firm 1 the advantage. Now consider the case we discussed for the Cournot model, but firm 1 chooses Q_1 first, and firm 2 choose Q_2 later. For firm 2, the first order condition

$$\frac{d}{dQ_2}(30 - Q_1 - Q_2) \times Q_2 = 0$$

gives that

$$Q_2(Q_1) = 15 - \frac{Q_1}{2}.$$

For firm 1,

$$\frac{d}{dQ_1}(30 - Q_1 - Q_2(Q_1) \times Q_1 = 0$$

gives that

$$Q_1 = 15.$$

Thus, the result will be

$$Q_1 = 15,$$
$$\pi_1 = 112.5;$$
$$Q_2 = 7.5,$$
$$\pi_2 = 56.25.$$

In this case, firm 1 also has advantage to go first.

2 Bertrand

The Bertrand model is the oligopoly model in which firms compete in price. First assume that two firms produce homogeneous goods and choose the prices simultaneously. Assume two firms have the same marginal cost

$$MC_1 = MC_2 = 3;$$

consumers buy goods from the firm with lower price. If

$$P_1 = P_2 = 4,$$

the two firms share the market equally, but this is not the equilibrium. The reason is that one firm can get whole demand by lowering the price a little; therefore, the equilibrium will be

$$P_1 = P_2 = 3,$$

when the price is equal to the marginal cost. Now we check if

$$P_1 = 3$$

is the best choice for firm 1 given

$$P_2 = 3.$$

When

$$P_1 = 3,$$
$$\pi_1 = 0;$$

if

$$P_1 > 3,$$

consumers will not buy firm 1's goods, thus

$$\pi_1 = 0;$$

if

$$P_1 < 3,$$

the price is lower than the marginal cost, thus

$$\pi_1 < 0.$$

It follows that

$$P_1 = 3$$

is optimal for firm 1; by analogy, we can get the same conclusion for firm 2. Therefore,

$$P_1 = P_2 = 3 = MC$$

in a Bertrand game with homogeneous goods. This is like the competitive market.

Suppose the goods from the two firms are heterogeneous, but substitutes. Firm 1 and firm 2 face the following demands:

$$Q_1 = 12 - 2P_1 + P_2,$$

and

$$Q_2 = 12 - 2P_2 + P_1.$$

Firm 1's and firm 2's reaction functions are

$$P_1 = 3 + \frac{\overline{P_2}}{4},$$

and

$$P_2 = 3 + \frac{\overline{P_1}}{4}.$$

At equilibrium,
$$P_1 = \overline{P}_1,$$

and
$$P_2 = \overline{P}_2;$$

so
$$P_1 = P_2 = 4,$$
$$Q_1 = Q_2 = 8,$$

and
$$\pi_1 = \pi_2 = 32.$$

Consider the case when the firms choose prices sequentially. Supposing firm 2's first order condition
$$\frac{d}{dQ_2}(12 - P_2 + P_1) \times P_2 = 0$$

and firm 1's first order condition
$$\frac{d}{dQ_1}(12 - 2P_1 + P_2(P_1)) \times P_1 = 0.$$

From the first equation
$$P_2(P_1) = 3 + \frac{P_1}{4},$$

and then substitute it into the second equation, we obtain
$$P_1 = 4\frac{2}{7}.$$

Therefore,
$$\pi_1 = 32\frac{1}{4};$$
$$P_2 = 4\frac{1}{14},$$

and
$$\pi_2 = 33\frac{15}{98}.$$

In this case, we can see that the firm who goes first has disadvantage, when competing in price.

3   Prisoner's Dilemma

Criminals A and B cooperated, and then got caught. However, the police have no evidence; so they have to interrogate A and B separately, trying to make them tell the truth.

		Firm B	
		Betray	Silent
Firm A	Betray	-3,-3	0,6
	Silent	-6,0	-1,-1

Table 2: Payoffs of Firm A and B.

The above matrix shows A and B's payoffs. Given the payoffs, A and B choose to tell the truth (betray) or keep silent. We can see that, if they both keep silence, the result $(-1, -1)$ is best for them; nonetheless, if one of them betrays another, he will be free but his companion will have payoff -6; moreover, if both of them betray, they will face the result $(-3, -3)$.

Consider what A thinks. Whether B keeps silence or betrays him, A will always be better off if he betrays; so will B. Therefore, the result of this problem is $(-3, -3)$, namely, both prisoners betray.