12.842 / 12.301 Past and Present Climate
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Notes on the Ocean Circulation for Climate Understanding

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1 Introduction

These lectures are intended to provide an overview of how the ocean influences the climate system. This subject is a very large one and no introductory survey can come close to explaining all of the elements and nuances. In particular, since I assume that students have little or no background in fluid dynamics, some things must be taken on faith, or accepted in purely verbal terms when the mathematical argument might be more succinct and convincing. The hope is that students will at least recognize the terminology, and some of the big issues oceanographers face in understanding what the ocean does, and what it may have done in the past and could do in the future.

Modern physical oceanography (the branch of the subject studying the fluid motions and properties) has a history dating back to the middle of the 19th Century (see Deacon, 1971 for a readable review of its historical roots). One of the themes in these lectures is that the ocean has been so difficult to observe and measure that a grossly distorted view of its nature has taken hold in the textbooks of all kinds and levels, and that for anyone interested in really understanding climate, it is necessary to (A) understand how this came about, and (B) to recognize that much of the modern literature claiming to describe the ocean in climate is grossly misleading. Many theoretical books exist, but there exist only one or two that properly describe the modern observations.

A useful, free, oceanographic textbook is Stewart, Robert H., 2006, *Introduction to Physical Oceanography,* http://oceanworld.tamu.edu/resources/ocng_textbook/PDF_files/book_pdf_files.html whose descriptive figures are particularly helpful.

Another free, somewhat dated, but generally accessible discussion of many fundamental aspects of physical oceanography is Warren, B. A. and C. Wunsch, *Evolution of Physical Oceanography,* 1981, http://ocw.mit.edu/ans7870/resources/Wunsch/wunschtext.htm.

Other introductory texts include Open University (2001) and Pond and Pickard (1983). Somewhat more advanced are Gill (1982) and the Chapter by Veronis in Warren and Wunsch (1981). Almost all books are distorted in the sense that they repeat the traditional picture of how the fluid ocean operates, not reflecting the more recent understanding. An encyclopedic, nearly up-to-date summary of much of physical oceanography, can be found in Siedler et al. (2001).

Until very recently (beginning about 10 years ago), the nature and causes of the ocean circulation was a largely academic subject, of interest primarily to a few hundred scientists around the world, who published papers almost solely in their professional journals. As climate change emerged as a source of serious international concern, debates by self-appointed experts about the past, present and future behavior of the ocean, and its impact on climate began to appear in the popular media. Some of the stories being told about the ocean are so fantastical that they stick in the public consciousness as "truth", and begin to influence public policy makers. Thus it is important that anyone studying climate should be able to distinguish science from science fiction. Among the more troublesome distortions now widely accepted one must include the notion that the ocean circulation is a simple "conveyor belt" and that the Gulf Stream is in danger of "turning off." To show that I am not exaggerating, and to give some of the flavor of the discourse, here is an article that appeared in October 2006 in The Guardian, one of the more respectable London newspapers:

> **Sea change: why global warming could leave Britain feeling the cold**
>
> ·No new ice age yet, but Gulf Stream is weakening · Atlantic current came to halt for 10 days in 2004 James Randerson, science correspondent Friday October 27, 2006 *The Guardian, London*
>
> Scientists have uncovered more evidence for a dramatic weakening in the vast ocean current that gives Britain its relatively balmy climate by dragging warm water northwards from the tropics. The slowdown, which climate modellers have predicted will follow global warming, has been confirmed by the most detailed study yet of ocean flow in the Atlantic. Most alarmingly, the data reveal that a part of the current, which is usually 60 times more powerful than the Amazon river, came to a temporary halt during November 2004.
>
> The nightmare scenario of a shutdown in the meridional ocean current which drives the Gulf stream was dramatically portrayed in The Day After Tomorrow. The climate disaster film had Europe and North America plunged into a new ice age practically overnight. Although no scientist thinks the switch-off could happen that quickly, they do agree that even a weakening of the current over a few decades would

have profound consequences.

Warm water brought to Europe's shores raises the temperature by as much as 10C in some places and without it the continent would be much colder and drier.

Researchers are not sure yet what to make of the 10-day hiatus. "We'd never seen anything like that before and we don't understand it. We didn't know it could happen," said Harry Bryden, at the National Oceanography Centre, in Southampton, who presented the findings to a conference in Birmingham on rapid climate change. Is it the first sign that the current is stuttering to a halt? "I want to know more before I say that," Professor Bryden said.

Lloyd Keigwin, a scientist at the Woods Hole Oceanographic Institution, in Massachusetts, in the US, described the temporary shutdown as "the most abrupt change in the whole [climate] record". He added: "It only lasted 10 days. But suppose it lasted 30 or 60 days, when do you ring up the prime minister and say let's start stockpiling fuel? How can we rule out a longer one next year?"

Prof Bryden's group stunned climate researchers last year with data suggesting that the flow rate of the Atlantic circulation had dropped by about 6m tonnes of water a second from 1957 to 1998. If the current remained that weak, he predicted, it would lead to a 1C drop in the UK in the next decade. A complete shutdown would lead to a 4C-6C cooling over 20 years. The study prompted the UK's Natural Environment Research Council to set up an array of 16 submerged stations spread across the Atlantic, from Florida to north Africa, to measure flow rate and other variables at different depths. Data from these stations confirmed the slowdown in 1998 was not a "freak observation"- although the current does seem to have picked up slightly since.

The article (which is not an isolated instance) was followed by a response from a German physical oceanographer, who had been at the same meeting:

Monday October 30, 2006

The Guardian <*http://www.guardian.co.uk*>

You published an article about the Gulf Stream that highlights the most speculative and preliminary finding that was presented at the recent Rapid climate change conference (Sea change: why global warming could leave Britain feeling the cold, October 27). Unfortunately, the information was put in a context that it was never given at the conference and that makes no scientific sense. Some climate models have suggested that as the world will warm, the Atlantic Ocean, overturning circulation (which is only a fraction of the mostly wind-driven Gulf Stream), might dramatically slow down. If that happens it will reduce the atmospheric warming in coastal areas. But in all those scenarios the ocean circulation-induced cooling will not even cancel the (global) warming. So a "new ice age" is not predicted by any model. The current consensus is a 25-30% reduction of the ocean overturning by 2100 and no detectable trend for the next 20 years.

The Atlantic's current changes are no cause for alarm. At the conference, at least six papers were presented that showed that the "apparent" slowdown of the Atlantic Oceans overturning circulation was not consistent with a large number of other observations in the Atlantic. It is also not found in ocean models that have used all the available observations. Thus more than 95% of the scientists at the workshop concluded that we have not seen any significant change of the Atlantic circulation to date, but quite a bit of variability.

As a side point, Harry Bryden showed the short period of apparently no southward flow in the deep. But he also cautioned that there is no explanation for this, meaning that this could also be an artifact of the analysis. In no way has he presented this "oddity" an indication of something alarming.

Harry Bryden's paper last year in Nature claimed that "the comparison suggests that the Atlantic meridional overturning circulation has slowed by about 30 percent between 1957 and 2004". This is a controversial claim. It is strange that such a decease hasn't produce any cooling effects on Europe's climate since a complete shutdown of the system has been estimated by climate models to see a 4°C drop in temperatures.

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Many more instances of nonsense have been published, unfortunately without much response from the science community. So this subject is no longer an academic one (the European community is spending many tens of millions of dollars to monitor the North Atlantic circulation apparently having been convinced that the Gulf Stream is about to disappear).

Because the following notes need to take some extended excursions into basic physical principles, it is worth having a couple of examples of where and how the ocean influences climate. Fig. 1 is an estimate (Wunsch, 2005) of the zonally integrated meridional transport of heat by the ocean and atmosphere. The ocean and atmosphere conspire, in ways not well understood, to transport enough heat from the tropics toward the poles so as to radiate back to space an amount

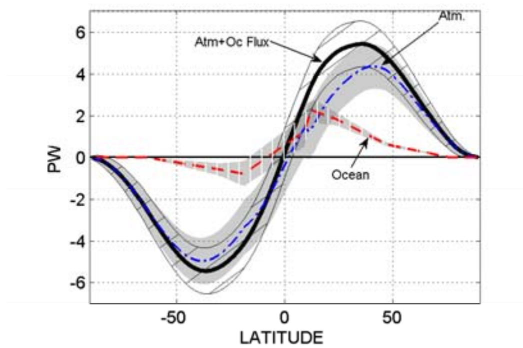

Figure 1: Estimate (Wunsc the ocean and atmosphere.
To maintain overall heat balance has to be exported towards

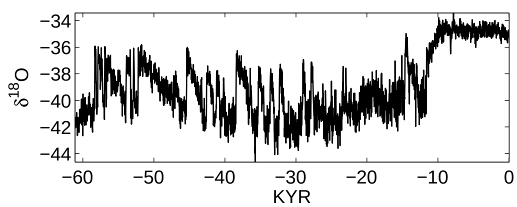

Figure 2: (After M. Bender). δ^{18}O in the Greenland (GISP) ice core. Interpreting this isotope ratio as being an approximate thermometer (temperature increasing upwards), records such as this show rapid warm events (Dansgaard-Oeschger events). Some glaciologists assert that they are caused by abrupt changes in the North Atlantic Ocean circulation. Determining whether that is true, and how it would work dynamically, is a problem in physical oceanography.

almost identical to that absorbed at low latitudes. Fluctuations in the ocean contribution are speculated to change the atmospheric properties and hence the climate.

Fig. 2 is taken from data of M. Bender, and shows the δ^{18}O concentrations in one of the Greenland ice cores through time. The very abrupt increases are usually known as Dansgaard-Oeschger (D-O) events and are interpreted as showing that temperatures increased abruptly. A widely accepted explanation is that the ocean circulation fluctuated at these times, leading to massive changes in the atmosphere. How this would work in practice is, however, obscure.

The annual estimated exchange of CO_2 between the ocean and atmosphere is displayed in Fig. 3

Image removed due to copyright restrictions.
Citation: Takahahsi, T., S. C. Sutherland, C. Seeney, A. Poisson, N. Metzl, B. Tilbrook, N. Bates, R. Wanninhkhof, R. A. Feely, C. Sabine, J. Olafsson, and Y. Nojiri. "Global Sea-air CO2 Flux Based on Climatological Surface pCO2, and Seasonal Biological and Temperature Effects." *Deep-Sea Res II* 49 (2002): 1601-1622.

Figure 3: Annual mean (1995) estimate of carbon dioxide flux to/from ocean (mole CO2 m-2 yr) from Takahashi et al. (2002). The present-day inferred net carbon uptake by the ocean is a small residual sum of exchanges in both directions, and thus presumably subject to fluctuations with the ocean circulation.

2 Some Preliminaries

In contemplating how the ocean affects and is affected by the climate system, one can make an extended list. Among them the effects are that:

(1) The ocean has almost all of the fresh water on the planet

(2) It has an immense heat capacity compared to the atmosphere

Consequently and in addition:

(3) It exchanges energy with the atmosphere (heat, moisture) and transports it in very large amounts

(4) It absorbs, stores, and ejects carbon dioxide in very large amounts

(5) It is the site of a large fraction of the biological activity on Earth.

(6) It is a major component of the biogeochemical cycles (nitrogen, phosphorous, etc.) on Earth

(7) When frozen, it can undergo a very large albedo (reflectivity) change

Understanding the implications of these and other phenomena are major problems in understanding the climate system.

Like the atmosphere, the oceans are a global-scale fluid on a rotating earth. The equations governing the large-scale motions (Newton's Laws) are nearly identical in the two systems. But there are major differences in the way in which the two systems behave (keeping in mind that they are coupled):

The ocean has continental barriers to zonal motions.

The ocean is heated (and cooled) at its upper surface, unlike the atmosphere which is (primarily) heated at the lower surface.

No significant oceanic equivalent to the radiative transfer process in the atmosphere.

No equivalent in the ocean of moist convection (although salinity introduces some analogous issues).

It is much more difficult to observe the ocean (including the facts that we live on the upper edges of the ocean and at the base of the atmosphere).

The last point is an essential one: The depiction of the ocean in textbooks rests primarily on the fundamental observational fact that until very recently obtaining any observation of the ocean at a point involved sending a ship there, and lowering a device to the required depth. Understanding of how the ocean works, and what it does in climate has been largely determined by the observational difficulties, compared to those for the atmosphere. The situation has changed drastically, but the new understanding has not yet been widely disseminated, leaving the public and much of the science community with an incorrect understanding of the nature of the ocean.

Although it sometimes appears to be conveniently forgotten, the ocean is a fluid, and one of it's major characteristics is that it flows! Figure 4 and 5 display two sets of time series from two locations in the North Atlantic at depths of 600 and 640m. The first of these runs for about 1700hours, the second for over a year (about 400 days). One sees by eye how variable the currents are. Figure 6 shows the trajectories of surface floats launched in small regions of the North Atlantic, and the very great complexity of their subsequent motions.

It is not customary to start a description of the ocean circulation with such pictures: almost all textbooks begin their discussion by showing (as we will momentarily) with pictures of the temperature and salinity distributions in the oceans. There are two good reasons for that approach: (1) Velocities appear to be much more variable, in both space and time, and thus difficult to present as simple pictures; (2) It did not become technically possible to obtain records such as these until sometime in the middle 1970s, and thus the variability (whose presence was known to a degree) was regarded simply as a kind of boring "noise" of no particular interest.

We will return later to the problem of flow in the ocean.

3 Observations

The ocean is essentially opaque to electromagnetic radiation, and subsurface measurements, with some rare exceptions, require that one place an instrument at the actual location where a measurement is wanted. Until very recently, that meant one had to have a ship at the actual

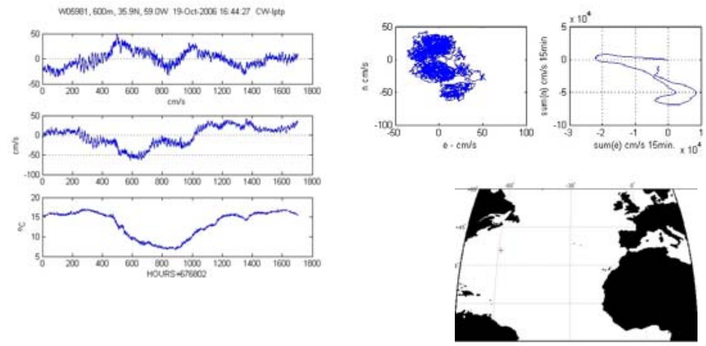

Figure 4: Left panel shows the two components of velocity and temperature at a current meter mooring at the location shown in the lower right chart. The two upper right panels show the so-called hodograph (u, v) and the time summed displacement corresponding to a fluid particle moving at the velocity measured at the current meter (a fictitious particle, as no fluid parcel stays at the current meter).

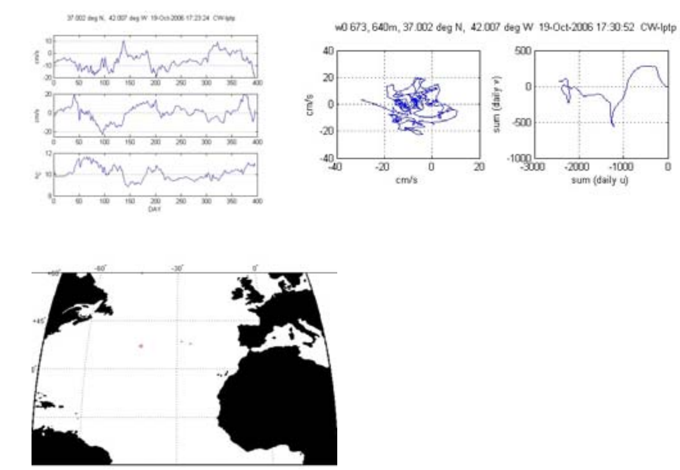

Figure 5: Same as 4 except at a nearby location shown in the chart, and for a much longer period, exceeding a year.

Image removed due to copyright restrictions.
Citation: Fratantoni, D. M. "North Atlantic Surface Circulation During the 1990's Observed with Satellite-tracked Drifters." *J Geophys Res* 106 (2001): 22067-22093.

Figure 6: From Fratantoni (2001), showing great dispersion of surface drifters launched in the North Atlantic Ocean. The result does not look much like the simple circulation schemes that elementary theory produces.

horizontal position, and to keep it there for the duration of the measurement. Only beginning in the 1970s did it become possible to leave behind internally recording devices that could produce time series of certain observables without the ship having to remain in position. One still did have to send the ship back, days or as eventually became possible, a year or two later, to retrieve the instrument so as to recover the recorded data. Because vacuum tube electronics worked extremely poorly at sea, the bulk of oceanic measurements made from ships were based upon purely mechanical devices. The problem of measuring temperature, salinity, and depth of an instrument lowered from a ship by purely mechanical means (nothing electrical or electronic being involved) had, remarkably, been essentially completely solved by the middle 19th Century. (The reader might like to think about how she might do that.)

Temperature, salinity and depth were the *only* measurements readily made from ships until about 30 years ago. Ships are slow (even a modern oceanographic ship moves at no more than about 12knots) and very expensive. It thus took decades to conduct a sparse temperature/salinity survey of the global ocean. Fortunately, the large-scale temperature and salinity structure of the ocean appears remarkably stable (see Figs. 7-13) and so one could patch together measurements made over time spans of decades, contouring the results.

These, and thousands of other sections and charts, became the focus of most discussion of the ocean circulation for 100 years. Notice that the reader's eye is called to the large-scale structures, which many years of experience showed are quite stable in time. A slightly more careful scrutiny shows that all the figures display much smaller scale features, which were generally ignored as being "noise." As we will see a bit later, however, ocean dynamics depends not upon temperature and salinity per se, but upon their spatial derivatives, both horizontal and vertical. The tendency of the eye to focus on the large-scale structures led to decades of misinterpretation of what the

Image removed due to copyright restrictions.
Citation: Roemmich, D., and C. Wunsch. "Two Transatlantic Sections: Meridional Circulation and Heat Fux in the Subtropical North Atlantic Ocean." *Deep-sea Res* 32 (1985): 619-664.

Figure 7: Temperature and salinity as a function of depth (vertical access) and longitude (horizontal axis) across the Atlantic at about 25°N (Roemmich and Wunsch, 1985).

Image removed due to copyright restrictions.
Citation: Roemmich, D., and C. Wunsch. "Two Transatlantic Sections: Meridional Circulation and Heat Fux inthe Subtropical North Atlantic Ocean." *Deep-Sea Res* 32 (1985): 619-664.

Figure 8: Density computed from the temperature and salinities shown in Fig. 7.

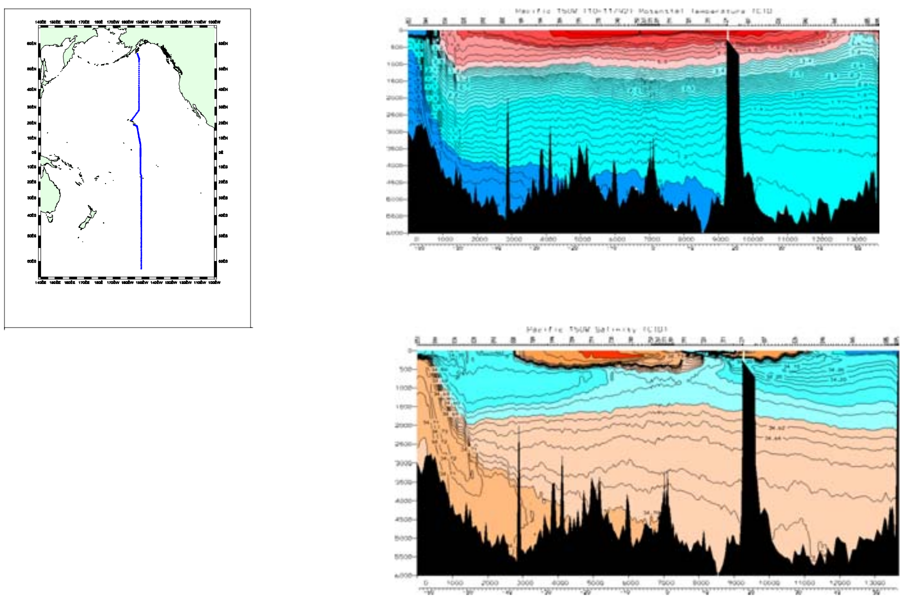

Image courtesy of WOCE.

Figure 9: Temperature and salinity sections down the Pacific Ocean as measured during the World Ocean Circulation Experiment (WOCE). Notice the very cold somewhat saline water near bottom on the southern end.

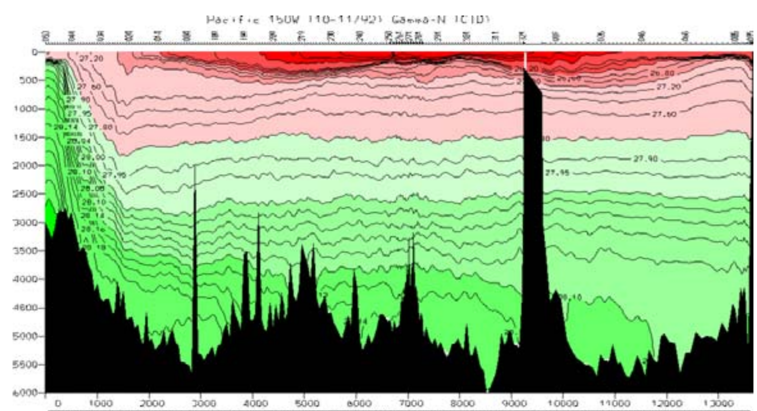

Image courtesy of WOCE.

Figure 10: Density in the section shown in Fig. 9.

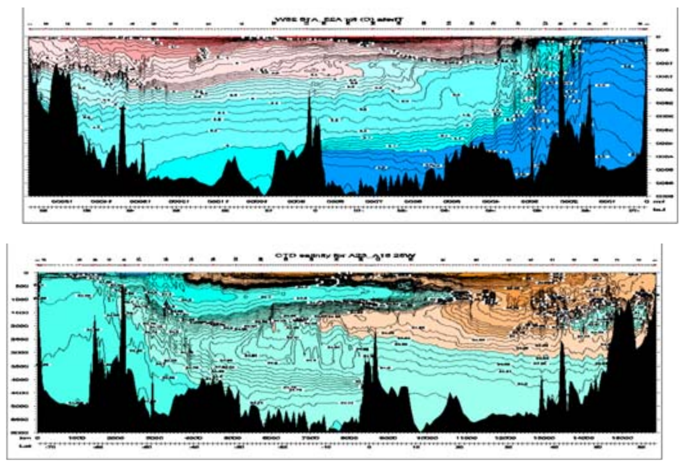

Image courtesy of WOCE.

Figure 11: Temperature and salinity section from a north-south WOCE Atlantic cruise.

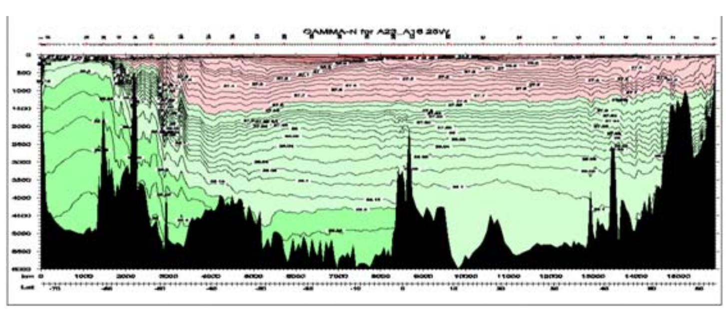

Image courtesy of WOCE.

Figure 12: Density for the WOCE Atlantic section shown in Fig. 11.

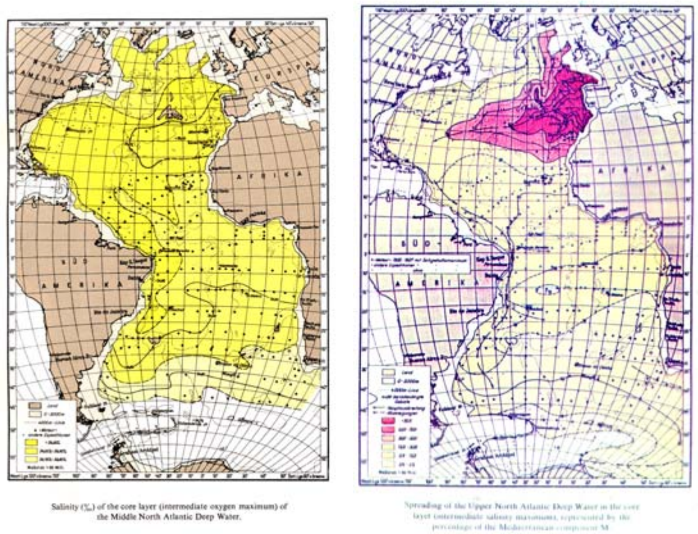

Image courtesy of US Government.

Figure 13: Salinity field at about 2500m depth (left) from Wüst (1935) and at the depth of the Mediterranean outflow. As with the other sections, it is difficult to detect shifts in the large-scale properties even after the elapse of 80+ years.

oceanic flow fields look like.

4 Some Simple Dynamics

In a course like this one, we do not have the time to derive the equations of fluid motion. Unfortunately, it is very difficult to discuss a fluid without some recourse to these equations. Any reader without familiarity with fluids can at best, take what follows as a set of postulates and to seek out a good introductory fluids textbook (e.g., Kundu and Cohen, 2004).

Define a local Cartesian coordinate system (x, y, z), with z measured vertically upward from the resting seasurface, and x by convention is taken in the east-west direction, y in the north-south (Fig. 14). The corresponding components of flow (velocity) are (u, v, w). The ocean has a finite compressibility, but it suffices for our present purposes to treat it as though it is incompressible (see Bohren and Albrecht, 1998 for a stimulating discussion of what true incompressibility implies). It is an easy matter to show that incompressibility leads to the requirement,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

(sometimes called the "continuity" equation). This equation simply asserts that one flows into a small volume of fluid must flow out again because the fluid can neither expand nor contract, and we can't tear holes in it.

To a very good approximation in the ocean (but not always), it suffices to regard the pressure field as "hydrostatic" —that is, given by the weight, at any given depth, z, of the overlying water:

$$p = \int_z^\eta g\rho\left(x,y,z\right)dz \approx g\rho\left(x,y,z=0\right)\eta\left(x,y\right) + \int_z^0 g\rho\left(x,y,z\right)dz. \tag{2}$$

Here $\rho\left(x,y,z\right)$ is the fluid density (kg/m^3), g is the local gravitational acceleration, and $\eta\left(x,y\right)$ is the elevation (or depression, if negative) of the seasurface relative to $z=0$. The density of seawater does not differ greatly from about $\bar{\rho} = 1029$kg/m^3 (about 1.03gm/cm^3 ≈ 1, although it's variations are crucial). Eq. (2) implies that there are two influences on the pressure at depth: (1) the elevation, giving a pressure contribution of $g\bar{\rho}\eta$ and another part from the column integral of density starting at $z=0$ (ignoring the variations in density over the small distance η). Thus if the fluid density were constant, the pressure variations at depth would depend in the horizontal only upon the surface elevation:

$$p_h\left(x,y,z\right) = g\bar{\rho}\eta + \text{constant}\left(z\right)).$$

The last term has no horizontal structure, and so will not contribute to the flow. Note that Eq. (2) can be differentiated in z, giving

$$0 = -\frac{\partial p}{\partial z} - g\rho, \tag{3}$$

and it is this form we will usually start out with.

In classical, ordinary, fluid dynamics, it is known that for sufficiently weak flows, the fluid is approximately governed by the equations,

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \tag{4}$$

in the $x-$direction and ,

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} \tag{5}$$

in the $y-$direction. These are the two, linearized, "horizontal momentum" equations. They make the intuitively plausible assertion that if there is a pressure difference in the fluid, there will be an acceleration of the fluid from high pressure to low. Terms on the left are simply mass $\times$ acceleration per unit volume and terms on the right establish the strength of forcing due to pressure changes.

Equations (1-5), although simplified from the full equations, describe a large variety of fluid phenomena. The difficulty, for someone studying motions in the ocean that exist for more than a few hours, is that the Earth's rotation means that we would need to account for the constant

Image removed due to copyright restrictions.
Citation: Figure 2-8. Cushman-Roisin, B. *Introduction to Geophysical Fluid Dynamics.* Prentice-Hall, Englewood Cliffs, N. J., 1994, 320 pp.

Figure 14: Local Cartesian coordinates superimposed upon a sphere (Cushman-Roisin, 1992). Effective rotation is a function of latitude, vanishing on the equator. (u, v, w) are the three components of velocity in the local x, y, z directions.

reorientation of the flow in space. One can write the equations so that the reflect the way the Earth carries the fluid around 360° every 24 hours as seen by an observer fixed in space. It proves, however, far more convenient to re-write them from the point of view of an observer who is rotating *with* the Earth. It's as though we were doing fluid dynamics on a merry-go-round, but couldn't tell by obvious cues that we were spinning rapidly.

A simple coordinate transformation (carried out in most physics textbooks and any book that considers geophysical fluids, e.g., Kundu and Cohen, 2004) shows that in this new coordinate system, Eqs. (4, 5) become, to a good approximation,

$$\rho\frac{\partial u}{\partial t} - \rho f v = -\frac{\partial p}{\partial x} \tag{6}$$

$$\rho\frac{\partial v}{\partial t} + \rho f u = -\frac{\partial p}{\partial y} \tag{7}$$

where $f = 2\Omega\sin(\phi)\,\Omega$ is the Earth's radian rotation frequency, and ϕ is the latitude where are placing the y origin of our local Cartesian coordinate system on what strictly speaking is a near-spherical geometry. (Fig. 14). The latitudinal dependence of f (almost universally known as the "Coriolis parameter") implies that the *effective* rotation of the hypothetical merry-go-round of the observer goes from essentially zero near the equator to Ω near the pole. That one can think of an effective rotation is the result (not shown here) of treating the ocean as a very thin spherical shell: the mean ocean depth is about 4000m; the radius of the Earth is about 6.3×10^6m so that one cannot even draw a true ocean layer to scale. The new terms $\rho f v, \rho f u$ represent a "fictitious" force, the Coriolis[1] force, and they represent the dominant effect on fluid parcel

[1] It isn't clear why it is named for Coriolis, as the form was known much earlier. Coriolis was apparently best

trajectories of the fact that the observer is rotating rapidly. (Consider an observer on a merry-go-round watching the flight of a ball which in the rest frame is a straight line. It will appear to be curving to the right if the merry-go-round is moving counter-clockwise. It would appear as though a force were acting on it, but we know it's a consequence of the angular acceleration of the observer. Note that since f changes sign with the latitude, that the apparent rotation in the southern hemisphere is in the opposite sense to that in the northern. One must sometimes remember this! The somewhat mysterious Coriolis forces can, with a little effort, be understood as simply a version of centrifugal/centripetal forces with which more people are familiar. (The motion of the Foucault pendulum is intimately connected with the action of Coriolis forces in a thin shell and some readers might find it helpful to study it.)

The same coordinate rotation leaves the continuity and hydrostatic equations unchanged. A reader not acquainted with fluid dynamics might simply accept that the collection of equations (1, 3, 6, 7), are found empirically to describe many important oceanographic phenomena, at least qualitatively, and that the approximate forms we have written down can in fact be mathematically justified for the motions we will consider. That they are *approximate* equations must be borne in mind if one seeks to extend the simple analyses we use here.

In the rest frame, the only steady-state in our equations would be $\partial p/\partial x = \partial p/\partial y = 0$, implying $u = v = w = 0$—that is no flow. But in the rotating frame, if we set the time derivatives to zero, we have the possibility of a steady flow with,

$$-fv = -\frac{\partial p}{\partial x}, \tag{8}$$

$$fu = -\frac{\partial p}{\partial y}, \tag{9}$$

known as "geostrophic balance". It is both an empirical fact, and something readily justified mathematically, that observed motions in the ocean changing more slowly than about once/day, on spatial scales larger than about 50km, are in geostrophic balance to a very high order. That the balance is not actually perfect is very important, but numerically, the deviations from balance are small compared to any of the terms in Eqs. (8, 9).

Write the horizontal flow as a vector,

$$[u, v] = \frac{1}{f}\left[-\frac{\partial p}{\partial y}, \frac{\partial p}{\partial x}\right]$$

If we take the dot product of this vector with the horizontal pressure gradient, $[\partial p/\partial x, \partial p/\partial y]$, the result vanishes. Thus in a geostrophic flow, the water velocity instead of being from high pressure to low pressure, is exactly *along* the lines of constant pressure—it cannot flow from high

known in his own time (19th Century) for an extended treatment of the physics of billiards.

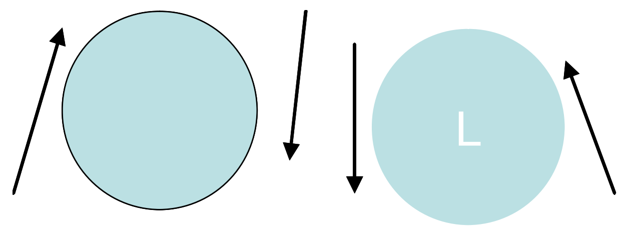

Figure 15: In geostrophic flow in the northern hemisphere, flow follows the lines of constant pressure with the highest pressure on the right. (Left region is high.)

to low. In particular, if the pressure lines are closed (Fig. 15) in the northern hemisphere, the flow is clockwise around a high (so-called anti-cyclonic flow), and counterclockwise around a low (reversed in the southern hemisphere). The same physics applies to large scale weather systems, and the rule has been known for hundreds of years as Buys-Ballot's law: that if you stand with your back to the wind, then the high pressure is to the right (in the northern hemisphere).

The Dynamic Method and the "Thermal Wind"

As we have already noted, oceanographers concluded that the large-scale flows in the ocean were essentially steady, so we might try using these equations to understand what is observed. This brings us back to the point that for one hundred years, shipboard observations were confined essentially to temperature, salinity and pressure (as well as horizontal position). Fortunately, that proves adequate to say a great deal, because from laboratory measurements, it is possible to construct an "equation of state", $\rho(T, S, p)$, where T, S are salinity, and pressure and thus ρ can be regarded as known from shipboard measurements.

The pressure field can be eliminated by cross-differentiation between the hydrostatic equation and either of the momentum equations (6, 7) to produce,

$$f \frac{\partial (\rho v)}{\partial z} = -g \frac{\partial \rho}{\partial x}, \tag{10}$$

$$f \frac{\partial (\rho u)}{\partial z} = g \frac{\partial \rho}{\partial y}, \tag{11}$$

and which can be integrated in z :

$$\rho v(x, y, z) = -g \int_{z_0}^{z} \frac{\partial \rho}{\partial x} dz + b(x), \tag{12}$$

$$\rho u(x, y, z) = g \int_{z_0}^{z} \frac{\partial \rho}{\partial y} dz + c(x). \tag{13}$$

Here $b(x), c(x)$ are integration constants that depend upon the arbitrarily chosen integration depth z_0. The beauty of these equations is that given only measurements of density as a function of position, precisely what one could obtain on a ship, one can compute the geostrophic flow. These equations (in either integrated or unintegrated form) are called the "thermal wind" equations in analogy to their use in the atmosphere. Note that it is the *derivative* of ρ that is critical; the value of ρ where it multiplies u or v, can be treated as constant (permitting slightly sloppy reference to the velocity rather than the mass transport. And as noted, in CGS units, $\rho \approx 1\mathrm{gm}/\mathrm{cm}^3$)

Eqs. (12, 13) became the mainstay of physical oceanographic determination of flow beginning before 1900, and remain in wide use today. There is only one (apparently) small difficulty—the integration constants need to be determined. It suffices to consider one of the equations, e.g., (12). Evidently $b(x)$ is the velocity at the "reference depth," z_0, which could itself be a function of x . One way of determining b would be to measure the flow at a single depth, making $b(x)$ the "reference level" velocity. It would then be a "level-of-known-motion." The trouble is that early practitioners of observational physical oceanography realized that the flow field behaved as it does in Figs. 4, 5—that is was unstable, and its connection to the component of flow in Eq. (12) which was supposed to be a steady component, was unclear.

It was concluded that the variability seen in those figures was "noise", and that it did not represent the geostrophic flow. Instead, and in the absence of any other ideas, it was generally concluded that because the flow appears (as we will see) to be largely wind-driven, it could be assumed to weaken with depth, and actually vanish if one went deep enough. That is, it was assumed that at some depth z_0, $b = 0$ (and equivalently for u , although probably at some other depth, in (13)), which became known as the "level-of-no-motion".

A level-of-no-motion permitted calculation of the absolute flow field, but it also seemed to imply that one could forego measurements below that depth—resulting in enormous savings in time, money, and equipment wear and tear. Implicitly, it was assumed that the ocean circulation was dominantly a consequence of the wind, whose effects should be small at great depths, nearly vanishing below the level-of-no-motion. The textbooks should be consulted for discussion of how choices of level-of-no-motion were made, but most commonly they corresponded to isotherms, isopycnals[2], or fixed depths (e.g., 1000m), or by assuming them to lie between water masses believed to be moving in different directions (e.g., above the North Atlantic Deep Water and below the Circumpolar Intermediate Water).

Another useful characteristic of the physics leading to Eq. (10) is that the total amount of water moving normal to the station pair above z_0 was independent of the distance between the pair (because of the x derivative), if bottom topography did not intervene above $z = z_0$. Thus widely separated stations could be used to find the total mass or volume transport between them. Close pairs (separated, e.g., by Δx) gave temporally much less stable horizontal derivatives e.g.,

[2] Isotherms are lines or surfaces of constant temperature, and isopycnals are lines or surfaces of constant density.

$\partial\rho/\partial x \approx \Delta\rho/\Delta x$, than did widely separated ones, and this experience also implied that only widely separated station positions are required.

Along with the contoured property fields (see e.g., Fig. 13) a reasonably self-consistent picture of the ocean as a large-scale, essentially steady flow emerged, one in which the abyssal components were thought to be so weak as to be hardly worth discussing, and in which the upper ocean flows, believed known, varied significantly only on time scales of hundreds or thousands of years. That the results were only qualitative can be inferred from the variety of contradictory upper ocean circulation patterns that were published over the years; Reid (1981) conveniently displays some of them.

It's worth asking how this works. Why not, for example, use the pressure in Eqs. (10, 11) where the vertical derivatives of the velocity field do not appear? The difficulty with that arises from the contribution of the pressure field from the elevation term in Eq. (2): if one uses numerical values, it is found that surface elevation changes of tens of centimeters give pressure changes comparable to the contributions from the density variations. Thus an elevation change of a few centimeters can overwhelm the pressure derivatives produced from the density field part. Because one cannot measure elevation changes of centimeters on a ship, there was no way to determine the absolute pressure.

It is useful to understand then, what a level-of-no-motion implies. Consider Fig. 16. It is assumed that the isotherms slope as depicted, but that the horizontal pressure gradient vanishes at the dashed line. Because the water on the left of the figure is warmer than that on the right, its weight at the dashed line, z_0, if the sea surface were flat, would be less on the left than on the right, and there would be a pressure gradient there. On the other hand, if the sea surface is heaped up over the warm water, then the lower weight at z_0 could be compensated by the extra water overhead. Notice that immediately under the surface elevation, the pressure is higher than it is to the right until one reaches z_0, the depth where the elevation is compensated by the lower density. Once a level-of-no-motion is chosen, one can then calculate the surface elevation required to just compensate the density changes (we have ignored salinity here, but the effects are similar). Roughly speaking, one expects the parts of the ocean that are warmer (lighter) to stand higher than the parts that are cold (denser)—assuming the velocity diminishes with depth, which is often, but not always true.

Fig. 17 shows an example of two nearby hydrographic stations producing the temperature and salinity profiles in the North Atlantic. The thermal wind calculation leads to the velocity field displayed, with the zero-crossing chosen arbitrarily. Moving large distances in the vertical has only a small effect on the near-surface flow, but qualitatively changes the inferred volume transports in the deep water.

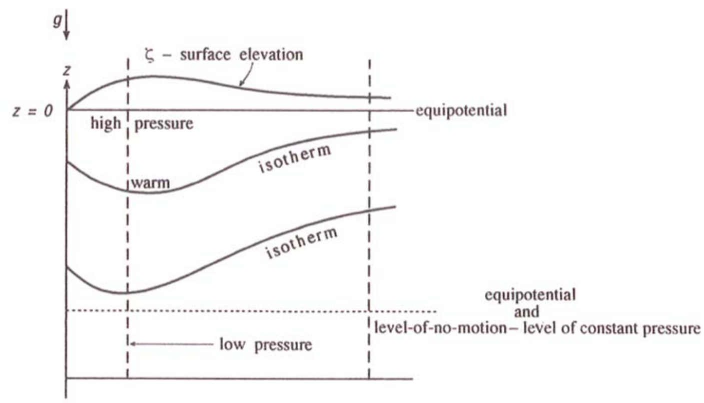

Figure 16: Warm water is lighter than cold, so a column of warm water of the same thickness as that of cold would exert a smaller pressure at any given depth. But the pressure difference can be overcome by having a taller column where it is warm, in such a way that the two contributons just "compensate" at a level of no motion where the pressure is presumed constant.

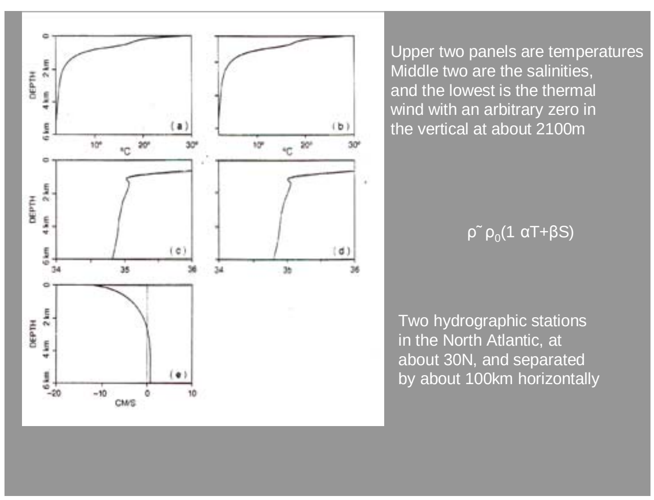

Figure 17: Temperature and salinity at two nearby (separated zonally) stations in the Sargasso Sea. Visually, it is difficult to distinguish them, but the implied density difference is sufficient to produce the vertical shear shown in the Figure in the lowest panel. The derived thermal wind is set arbitrarily to zero as shown. Adjusting this depth over a wide interval makes little change in the near-surface velocity but a large difference in the abyssal values.

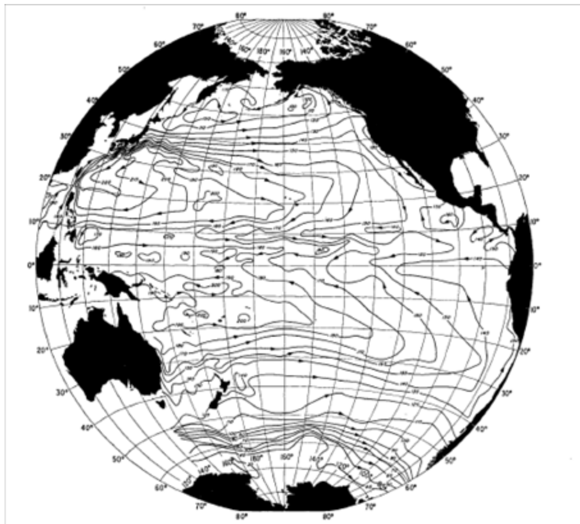

Fig. 1. Mean annual dynamic topography (dyn-cm) of the sea surface relative to 1000 db: 36,356 observations.

Image courtesy of AMS.

Figure 18: Wyrtki (1975), calculated the absolute shape of the Pacific Ocean sea surface by assuming that the 1000m depth was a level of no motion. Compare to Fig. 19. Arrows denote the direction of the geostrophic flow at the surface, computed from the pressure force exerted by the estimated elevations.

Charts such as Fig. 18 were constructed showing the elevation inferred from various assumptions about where the level-of-no-motion lay. Beginning in 1992, however, with the launch of the TOPEX/POSEIDON altimeter satellite, one could finally measure the absolute height of the seasurface (Fig. 19) and it was found that the general inferences made by oceanographers over the preceding decades were at least qualitatively correct.

Later, we will revisit this level-of-no-motion problem.

By exploiting the density measurements that gradually accumulated, combined with various assumptions about the level-of-no-motion, oceanographers gradually built up a picture of the ocean circulation, similar to the North Atlantic one depicted in Fig. 20. One striking feature that emerged was the tendency for the circulation in all oceans to be strongest on the western side. It had been long known that the Gulf Stream in the North Atlantic and the Kuroshio in the North Pacific were powerful currents with no counterparts on the east.

Descriptions such as Fig. 20 ultimately called for an explanation.

5 A Bit of Wind-Driven Dynamics

Why is there an ocean circulation at all? All real fluids are subject to friction, and some forces must sustain the movement of water. A list of possibilities is not long:

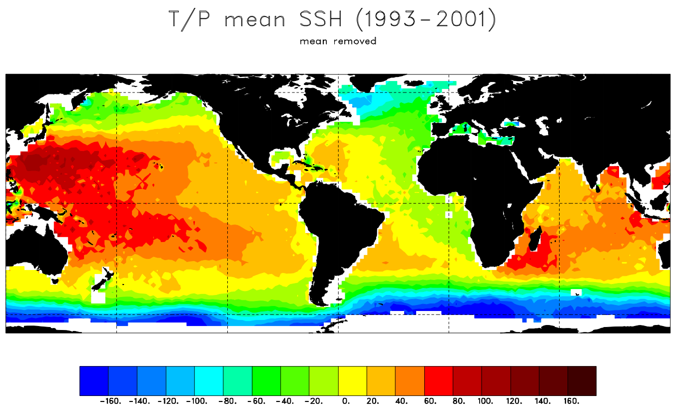

Figure 19: Absolute topography of the sea surface relative to an estimated gravitational equipotential as measured from satellite altimetry. Compare to the inference in Fig. 18 for the Pacific Ocean.

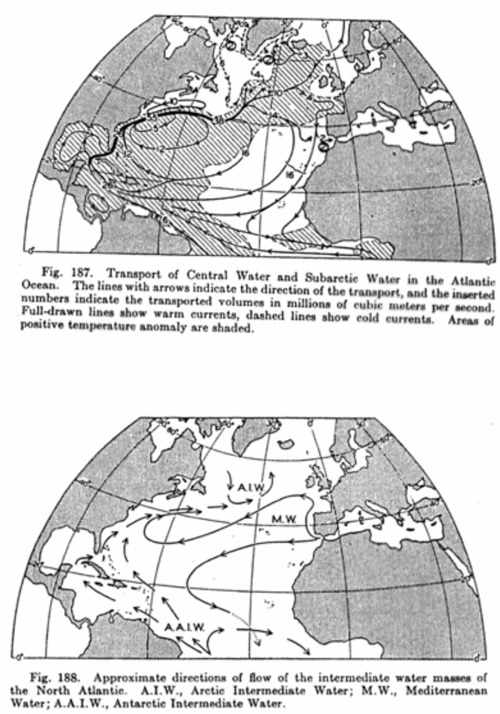

Figure 20: The near-surface and intermediate depth geostrophic flow as calculated by Sverdrup et al. (1942) using the thermal wind and a deep reference level. Note the emergence of the Gulf Stream circulation.

(1) the wind

(2) heating and cooling at the surface

(3) atmospheric pressure loading/unloading

(4) precipitation and evaporation

(5) heating through the seafloor

(6) tides

All of these things do make the fluid move, but some analysis suggests that only (1), (2) and (6) are really significant, and as we will see, it is (1) that is most important even for motions in which (2) and (4) appear to dominate.

Wind Driving

How does the wind set the fluid ocean into motion? To answer that question, it is necessary to understand how friction comes into the fluid equations of motion. In the full form of the fluid equations, usually known as the Navier-Stokes equations, friction modifies the non-rotating equations (4, 5) to

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u \tag{14}$$
$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \nabla^2 v \tag{15}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. (It is far from obvious that this is the proper form. See the description in Darrigol, 2005 of the many decades of struggle that went into developing the equations.) Before proceeding, it is useful to pause and ask what happens when a fluid encounters a solid surface (the boundary conditions). For *any* fluid, it is intuitively reasonable that there should be no flow into a solid surface; that is, the normal component should vanish (a definition of a solid surface). In a viscous fluid as described by Eqs. (14, 15) it is found, empirically, that the tangential component of flow also vanishes (called "no slip"). Thus for a flat sea floor at $z = -h$, one expects $w\,(z = -h) = 0$ and the tangential components u, v also must vanish there if $\mu \neq 0$. If $\mu = 0$, then there is no restriction on u, v at $z = -h$ (called "free slip").

At the sea surface there is no wall. If there is no wind blowing, one requirement is that there be "no stress", or

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = \left.\frac{\partial v}{\partial z}\right|_{z=0} = 0.$$

If there is a wind blowing, the stress exerted by the wind on the ocean is a vector $\tau = [\tau_x, \tau_y]$, and it is found that

$$\mu \left.\frac{\partial u}{\partial z}\right|_{z=0} = \tau_x, \ \mu \left.\frac{\partial v}{\partial z}\right|_{z=0} = \tau_y$$

(Again, these relations are not very obvious, and readers are advised to seek the help of a fluid dynamics textbook.)[3] The units of stress, τ, are of a force/unit area.

Suppose then that the ocean were not rotating, and a completely uniform wind is blowing in the $x-$direction. Then absent any x or y dependence in the forcing, one might seek solutions similarly independent of the horizontal coordinates. Suppose further that everything is steady, so that the time derivatives can be set to zero. Then Eqs. (14, 15) become

$$0 = \mu\frac{\partial^2 u}{\partial z^2}$$
$$0 = \mu\frac{\partial^2 v}{\partial z^2}$$

both of which can be integrated easily so that

$$u(z) = a + bz$$
$$v(z) = c + dz$$

where a, b, c, d are constants. At the bottom, the flow is no-slip:

$$0 = a - bh$$
$$0 = c - dh$$

At the top, we have

$$\mu \left.\frac{\partial u}{\partial z}\right|_{z=0} = \mu b = \tau_x$$
$$\left.\frac{\partial v}{\partial z}\right| = d = 0.$$

Thus,

$$u(z) = \frac{1}{\mu}\tau_x h + \frac{1}{\mu}\tau_x z = \frac{\tau_x}{\mu}(h+z)$$
$$v = 0$$

a parallel shear flow (Fig. 21). It is common to denote $\mu/\rho = \bar{\nu}$, the "kinematic viscosity." (The bar is added here because in this LaTeX typeface, the letter "vee," v, looks too much like "nu", ν.) In water, $\bar{\nu} \approx 1.4 \times 10^{-6}\mathrm{m}^2/\mathrm{s}= 1.4 \times 10^{-2}\mathrm{cm}^2/\mathrm{s}$ (e.g., Batchelor, 1967). A somewhat typical wind strength gives rise to a stress of $\tau_x \approx 0.1\mathrm{N/m}^2$ (about 1 dyne/cm^2 in the old CGS

[3] Note that meteorologists describe a wind coming from the west as "westerlies", while an oceanographer descrbing a current coming from the west as an *eastward* current. Similar confusion exists for "easterly" winds and "westward" currents.

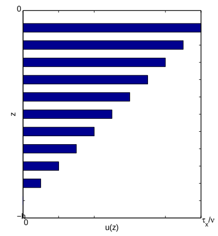

Figure 21: Linear shear flow driven in a non-rotating fluid by a constant surface stress. The flow is zero at the sea floor.

units). Thus at the sea surface, one predicts $u \approx \left(1\text{dyne}/10^{-2}\text{cm}^2/\text{s}\right) \times 4 \times 10^5\text{cm} = 10^7\text{cm/s}$, an alarmingly fast flow.

It is not hard to solve the problem of having the fluid at rest, and then turning the wind on abruptly. One finds that the time required to establish the flow exceeds $h^2/\bar{\nu}$ (which one can confirm has the dimensions of a time), and thus exceeds $4000^2/\left(1.4 \times 10^{-6}\right) \approx 10^{13}$s or about 300,000 years! This solution seems so unrealistic for the ocean that something fundamental must be wrong.

We know the system is actually rotating, and so let us reconsider the identical problem as seen by the observer on the spinning Earth. Equations (14, 15) become,

$$\rho\frac{\partial u}{\partial t} - \rho f v = -\frac{\partial p}{\partial x} + \mu\nabla^2 u \tag{16}$$

$$\rho\frac{\partial v}{\partial t} + \rho f u = -\frac{\partial p}{\partial y} + \mu\nabla^2 v \tag{17}$$

Because we are treating ρ as a constant, it saves some writing to divide through by ρ, to get,

$$\frac{\partial u}{\partial t} - f v = -\frac{\partial p'}{\partial x} + \bar{\nu}\nabla^2 u \tag{18}$$

$$\frac{\partial v}{\partial t} + f u = -\frac{\partial p'}{\partial y} + \bar{\nu}\nabla^2 v, \tag{19}$$

where $p' = p/\rho$, although we will normally omit the prime. When ρ is not visible in the equations, it is an indication that the pressure has been so redefined (or else $\rho = 1$, which is a good approximation in cm-gm-s (CGS) units). In the steady-state, with a wind that is uniform in the rotating x, y, z domain, all x, y derivatives can once again be dropped, and we are faced with

$$-f v = \bar{\nu}\frac{\partial^2 u}{\partial z^2} \tag{20}$$

$$f u = \bar{\nu}\frac{\partial^2 v}{\partial z^2} \tag{21}$$

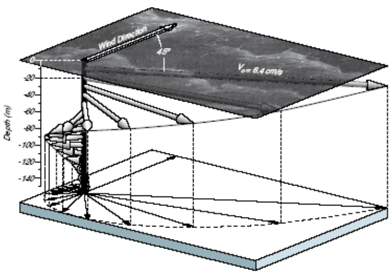

Figure 9.3. Ekman current generated by a 10 m/s wind at 35° N.

Figure 22: From Stewart (2006). The structure of the Ekman spiral. At the sea surface the flow is at 45° to the right of the wind (in the northern hemisphere), but the vertically integrated flow is 90° to the right of the wind.

subject to the same boundary conditions. These equations are discussed and solved in essentially every oceanographic book written since 1903, and so I will just write down the solution for a wind blowing only in the $x-$direction:

$$
\begin{aligned}
u_E &= \tau_x \cos\left(\frac{\pi}{4} + \frac{\pi z}{D_E}\right) \exp\left(\frac{\pi z}{D_E}\right) \\
v_E &= -\tau_x sin\left(\frac{\pi}{4} + \frac{\pi z}{D_E}\right) \exp\left(\frac{\pi z}{D_E}\right), \\
D_E &= \pi\left(\frac{2\bar{\nu}}{f}\right)^{1/2}
\end{aligned} \tag{22}
$$

This solution is a rotating spiral, decaying exponentially downward from the sea surface. At 30°N, the thickness of the moving layer is about $D_E = \pi(2\times 1.4\times 10^{-6})/(2.908\,9\times 10^{-4})$, which is about 3cm. Rotation has taken the solution which before produced enormous velocities deep into the water column, and confined the fluid flow to a minute upper layer. The form of the solution (22) is usually called the "Ekman spiral" or "Ekman layer" after the scientist who first worked it out about 1903. The subscript E is used to denote the variables within this layer. If one here calculates the motion starting from a state of rest, it is found that it is fully established in one or two rotation periods. Rotation has made a huge difference to the nature of the flow.

But an ocean driven by the wind whose response extended only to about 1cm does not describe any ocean anyone has ever seen in nature. What is wrong? The generally accepted hypothesis of what is wrong is that the ocean is in fact turbulent. In practice what that means is that superimposed upon the larger scale motions we are trying to determine, are much smaller scale motions—call them "eddies" (Fig. 23) whose main role is to dissipate the larger scale flows in a way analogous to the working of molecular viscosity, but with enormously greater

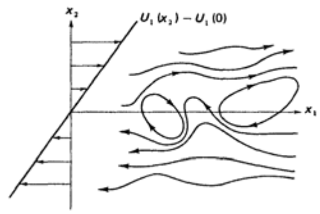

Figure 2.4. Turbulent pure shear flow. The mean velocity is steady: $U_2 = U_3 = 0$ and $U_1 = U_1(x_2)$. The instantaneous streamline pattern sketched refers to a coordinate system that moves with a velocity $U_1(0)$.

Figure 23: Schematic of eddies superimposed upon a larger scale flow. The effect of the eddies is supposed to act like a frictional dissipation of the larger scale. From Tennekes and Lumley (1972).

numerical value. In this "Reynolds analogy," one hypothesizes that equations such as (20, 21) that we should replace $\bar{\nu} \rightarrow A$, where A is the "eddy coefficient" whose numerical value we have to determine.

The solution will be identical to Eqs. (22) but with $\bar{\nu}$ everywhere replaced by A. It is commonly believed that the direct effects of the wind extend to a depth of order 100m in the ocean. Accepting that might be correct, we can then ask what value of A would extend the Ekman depth to 100m. One obtains $A \approx 100^2 f/2 \approx 0.4\text{m}^2/\text{s}$ at mid-latitudes which then becomes an estimate of the turbulent intensity in the ocean.

If one asks where the water *transport* is directed ("transport" is used to imply an integrated total flow, here the net flow from top to bottom), it is found easily that

$$\int_{-h}^{0} (u_E, v_E)\, dz = (0, -\tau_x/f) \tag{23}$$

That is the *total* transport is (1) directed at 90° to the right of the wind (in the northern hemisphere), and (2) is independent of A. The latter result is particularly reassuring, as the determination of A is so poorly justified. The result is general, not confined to a purely zonal wind (we can always reorient the x, y axes to lie along the direction of the mean wind). Note that in he integral in (23) although carried to the sea floor, contributions are exponentially small until very near the surface.

The Ekman solution is extremely elegant, and discussion of it exists in essentially every oceanographic textbook or paper on ocean dynamics written in the last 100 years. In practice, it has proven extremely difficult to demonstrate its existence (among other issues, the wind rarely holds still long enough for a true steady-state to be established, and turbulence does not really work as is implied by replacing $\bar{\nu}$ by A in Eq. (20, 21).) Nonetheless, the concept is extremely useful, and Eq. (23) shows that the details of the flow field are immaterial for some purposes.[4]

The Sverdrup Relation

The Ekman theory shows that the direct effect of the wind, and of the inferred turbulence, really only matters within about 100m of the sea surface. It seemingly does nothing to explain the inference that the deeper parts of the ocean are set in motion by the wind. Indeed, it would seem to contradict it. To get any further, we need to look a bit further at the solution (23). Suppose the wind is purely zonal from west to east, and spatially completely uniform; then the Ekman layer is driving a flow to the right of magnitude τ_x/f. We previously noted that f is in fact a function of latitude, and this implies then that the amount of fluid moving to the right must be changing with latitude. To be specific, if the wind really is latitudinally uniform, more fluid is moving to the right as one goes southward. Where does this excess fluid come from?

There are only two possibilities: (1) The sea surface is being drawn downward in the north, to supply the extra fluid required in the south. (2) It is coming from below. The trouble with (1), which is physically possible, is that it conflicts with the assumption of a steady-state. So we conclude that fluid is being supplied from below the Ekman layer. How much is that? To proceed, we need to represent the latitudinal dependence of the Ekman layer in our local Cartesian coordinate system. To that end, write $f = f_0 + \beta y$, where f_0 is the value of the Coriolis parameter at some reference latitude. $\beta = df/dy$. We know u_E, v_E and we have not yet used Eq. (1), and so can calculate

$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u_E}{\partial x} + \frac{\partial v_E}{\partial y}\right) = -\frac{\partial v_E}{\partial y}$$

because f depends only upon y. This last equation can be integrated once, producing a constant of integration which can be used to force the vertical velocity to go to zero at the sea surface (an exercise left for the reader). We instead proceed with a mainly physical argument.

The latitudinal dependence of the Ekman transport implies that there has to be a supply of water from below. But the flow in the ocean below the Ekman layer is observed to be essentially geostrophic in nature, and that is consistent with the Ekman solution which shows the turbulent viscosity terms becoming exponentially unimportant at depth. Thus the flow below the Ekman layer must satisfy (8, 9). However, we know that geostrophic flow cannot cross the lines of constant pressure. So let us ask how much fluid is moving north-south ($y-$direction) between pressure lines found at x_1, x_2. From Eq. (8), we have

$$V_{gtot} = -\frac{p(x_2) - p(x_1)}{f(y)} h \tag{24}$$

[4] For an example of actual measurement, see Price et al. (1987).

which does *not* depend upon the distance $x_2 - x_1$. (We have also integrated in the vertical all the way to the sea floor, noting that the contribution to the integral in the Ekman layer will be too slight to notice as it is so thin compared to the full depth.) If this geostrophic transport is to provide fluid to the overlying Ekman layer, it has to somehow reduce its own transport correspondingly. But the flow cannot cross the pressure lines. Thus there is only one way to decrease V_{gtot} : move the fluid toward a region where f increases—that is move northward (poleward more generally).

The conclusion here is very simple, but not intuitive: if the Ekman layer is moving more fluid toward the south as one goes toward the equator, the geostrophic fluid underneath must be moving in the opposite direction so it can provide the additional mass.

There is another way to change the amount of fluid being moved meridionally in the Ekman layer: change τ_x. If τ_x/f diminishes with decreasing latitude, one must find a place to absorb the water, and this can only be in the fluid below. Thus one would infer that in this case, the interior (geostrophic) flow would have to move to the south so as to accommodate an increased meridional transport.

By using the Ekman solution, and the geostrophic equations below the Ekman layer, one can work out all of the details of the flow field. It is possible to eliminate some of the relative complexity, but at the cost of some loss of insight into the solution. Instead of working with the velocities, u, v, w let us try instead to create a *transport* theory, where the transport is the vertical integral, top to bottom, of the flow. Take the equations (20, 21) and integrate them from the bottom to the top:

$$-fV = -\frac{\partial P}{\partial x} = A\frac{\partial u}{\partial z}\bigg|_{z=0} - A\frac{\partial u}{\partial z}\bigg|_{z=-h}$$
$$fU = -\frac{\partial P}{\partial y} = A\frac{\partial v}{\partial z}\bigg|_{z=0} - A\frac{\partial v}{\partial z}\bigg|_{-h}$$

where we are neglecting any slopes of the sea surface in the pressure terms (that can be justified), and U, V, P are the zonal and meridional transports and pressure integrals. For the terms on the right, note that $A\frac{\partial u}{\partial z}\big|_{z=0} = \tau_x$, $A\frac{\partial v}{\partial z}\big|_{z=0} = \tau_y$. We will set $A\frac{\partial u}{\partial z}\big|_{z=-h}$, $A\frac{\partial v}{\partial z}\big|_{z=-h}$ to zero (which can again, be justified). The integrated equations then become (retaining both components of the wind field),

$$-fV = -\frac{\partial P}{\partial x} + \tau_x \tag{25}$$
$$fU = -\frac{\partial P}{\partial y} + \tau_y$$

Cross-differentiating these two equations to eliminate the pressure produces,

$$-\beta V + f\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = \frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x}. \tag{26}$$

Now take the continuity equation and integrate it vertically:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + w\left(z=0\right) - w\left(z=-h\right) = 0,$$

again neglecting any slope of the sea surface. But then w must be zero at both the top and bottom, and

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{27}$$

This last equations says simply that in an incompressible fluid, the total fluid entering/leaving in the $x-$direction must be balanced by that entering/leaving in the $y-$direction. But then, Eq. (26) reduces to,

$$\beta V = \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} = \hat{\mathbf{k}} \cdot \nabla \times \tau, \tag{28}$$

the famous "Sverdrup relation". The great importance of this relationship is that it tells us that if the various assumptions are correct, the amount of fluid moving meridionally depends only upon the local derivatives of the wind field, not the wind field itself. ($\hat{\mathbf{k}} \cdot \nabla \times \tau$ is read as the "z-component of the curl of the wind-stress").

The reader is reminded that V is the transport, not the velocity. Note particularly, that if the wind is uniform, $V = 0$. This does not mean that the flow is zero! In particular, we have already seen that for a uniform wind, the Ekman transport to the south is in the opposite direction to the geostrophic transport below. Eq. (28) tells us that the two transports actually exactly cancel. They do not cancel if the windfield varies. The Sverdrup relationship tells one why details of the structure of the wind field so obsess oceanographers: the structure of the wind is a major determinant of what the circulation is. By changing the derivatives, one can change the direction of the meridional flow—without actually changing the direction of the wind itself!

We can say a bit more. Eq. (27) permits us to write

$$U\left(x,y\right) = -\int_{x_1}^{x} \frac{\partial V}{\partial y} dx + G\left(y\right)$$

where $G\left(y\right)$ is an arbitrary function of y, determining the zonal transport. But what is the unknown function $G\left(y\right)$? We suppose, trying to be a bit more realistic, that the ocean has zonal boundaries, extending in the meridional direction, say at $x = x_1 = 0, x_2$ (the presence of these boundaries is one of the ways that the ocean differs significantly from the atmosphere).

Physically, it is reasonable to force the zonal flow to vanish at the walls. But there is only one function $G\left(y\right)$ at our disposal, and so only one boundary condition can be imposed. Choosing

Image removed due to copyright restrictions.
Citation: Figure 5. Stommel, H. "The Westward Intensification of Wind-driven Ocean Currents." *Trans Am Geophys Un* 29 (1948): 202-206.

Figure 24: The solution given by Sverdrup (1947) for an analytical wind field (from Stommel, 1948) and which is arbitrarily forced to satisfy a no-flow condition on the right-hand boundary.

arbitrarily (following Sverdrup, 1947) and, as it turns out later, the correct choice,

$$G(y) = \int_{x_1=0}^{x=x_2} \frac{\partial V}{\partial y} dx. \tag{29}$$

An analytical example is helpful. Suppose the wind is purely zonal, but with a $y-$dependence,

$$\tau_x = -\tau_0 \cos(\pi y / L_y), \ \tau_y = 0. \tag{30}$$

The zonal wind is chosen to roughly mimic a region of easterlies (tradewinds) for small y, becoming westerly at larger y somewhat like the actual wind field. L_y is an arbitrary north-south scale. The the Sverdrup relationship produces

$$V = -\frac{\pi}{L_y \beta} \tau_0 \sin\left(\frac{\pi y}{L_y}\right)$$

which is southward for all $0 \leq y \leq L_y$ and Eq. (29) is,

$$U = (x - x_2) \frac{\pi}{L_y \beta} \tau_0 \cos\left(\frac{\pi y}{L_y}\right).$$

U exists because V depends upon y and hence there is more fluid moving north-south at some latitudes than at others, and the fluid can only enter and exit zonally. The so-called streamlines, lines that are parallel to the flow field, can be seen in Fig. 24.

For what follows, it is useful to formally define the streamfunction, ψ, as a function such that $U = -\partial\psi/\partial y$, $V = \partial\psi/\partial x$. Then the curves in Fig. 24 are the lines of constant ψ. Notice that with this definition, U, V satisfy the integrated continuity equation (27) identically. (Whether U or V gets the minus sign is arbitrary.)

Closing the Circulation

The Sverdrup relation is at best a partial theory of the wind-driven ocean circulation, as it doesn't work in a fully closed basin—having a transport through the western wall (or we could have chosen the eastern wall). The crucial step was taken by Stommel (1948). If one looks at the vertically integrated equations (25), they represent a forcing of the two momentum equations by the wind. But a forced system cannot be closed if there is no dissipation. In effect, the Sverdrup solution is the result of a torque being applied to the ocean circulation by the wind (easterlies and westerlies), but there is no way, in the equations we have used thus far, to dissipate the momentum and energy going into the fluid. Something is missing.

What Stommel (1948) did was to add the simplest rule of friction that is possible to represent dissipation, by rewriting the equations as

$$\begin{aligned} -fV &= -\frac{\partial P}{\partial x} - RU + \tau_x \\ fU &= -\frac{\partial P}{\partial y} - RV + \tau_y \end{aligned} \tag{31}$$

where the "Rayleigh friction" terms RU, RV can be interpreted as representing a drag of the flow field on the bottom. It is possible to rationalize this rule of dissipation in terms of more sophisticated calculations, but for our purposes it can be regarded as simply a way of exploring what would happen if some form of friction were introduced. The integrated continuity equation remains unchanged, and so it is sensible to use a stream function again. Substituting $U = -\partial\psi/\partial y$, $V = \partial\psi/\partial x$, the continuity equation is again automatically taken care of, and the two momentum equations become,

$$\begin{aligned} -f\frac{\partial \psi}{\partial x} &= -\frac{\partial P}{\partial x} + R\frac{\partial \psi}{\partial y} + \tau_x \\ -f\frac{\partial \psi}{\partial y} &= -\frac{\partial P}{\partial y} - R\frac{\partial \psi}{\partial x} + \tau_y \end{aligned} \tag{32}$$

Cross-differentiating to eliminate P, produces

$$R\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \beta\frac{\partial \psi}{\partial x} = \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right) \tag{33}$$

Using the same analytic wind field in Eq. (30), Stommel solved the equation exactly with the result seen in Fig. 25—the circulation closes on the west with an intense current up against the coastline. The equation is actually very easy to solve approximately using what is called singular perturbation theory, but we will not pursue that here.

The Stommel model is very unrealistic, but it isolated the fundamental elements leading to what we know as the Gulf Stream and other "western boundary currents". The essential ingredients are the meridional dependence of f, a curl of the wind stress producing an unbalanced interior mass flux which has somehow to be returned meridionally, and a way to break the geostrophic constraint below the Ekman layer. One should notice that when $R = 0$, the Eq. (33) reduces back to the balance that Sverdrup found. Away from the western wall, Sverdrup's

Image removed due to copyright restrictions.
Citation: Figures 4 and 5. Stommel, H. "The Westward Intensification of Wind-driven Ocean Currents." *Trans Am Geophys Un* 29 (1948): 202-206.

Figure 25: Right panel shows the stream lines of Stommel's (1948) solution closing the Sverdrup interior flow in an intense boundary jet on the west. Left panel shows the same flow in a system in which f is a constant with latitude. There is then no westward intensification.

solution is the same as Stommel's. Note too, that Stommel succeeded in giving a simple analytical solution to his equation by careful selection of the form of the wind stress—one in which $V = 0$ on $y = 0, L_y$ automatically. If the windfield does not produce a Sverdrup flow with $V = 0$ there, one must work much harder to solve the partial differential equation. The intensification on the west is an extremely important zero-order characteristic of the ocean circulation. Here it is manifested as another boundary layer.

In 1950, Munk reanalyzed the problem, but replacing Stommel's Rayleigh friction with a possibly more realistic representation of turbulent processes acting in the horizontal dimension, so that the two equations become

$$
\begin{aligned}
-fV &= -\frac{\partial P}{\partial x} + A_H\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) + \tau_x \\
fU &= -\frac{\partial P}{\partial y} + A_H\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + \tau_y
\end{aligned} \tag{34}
$$

that is, mimicking horizontal molecular viscosity. He also attempted to use observed winds. Because of the presence of horizontal viscosity, the appropriate boundary conditions on the side walls now require that the tangential transport (U or V depending upon the wall) must also vanish. It is not hard to see that this boundary condition can be met by using a stream function and setting ψ to zero on the walls. Munk's (1950) solution is shown in Fig. 26. In this solution, the highest latitudes are one of easterlies again, and which generate another closed "gyre". The near-equatorial circulations are a consequence of the structures built into the wind stress.

In subsequent years, both the Stommel and Munk solutions were greatly extended, and it must be recognized that they represent hugely over-simplified pictures of the ocean circulation (no stratification, flat bottom, linear dynamics, steady winds, no heating or cooling,...). Nonetheless the basic elements generating a western boundary current, a wind-driven circulation

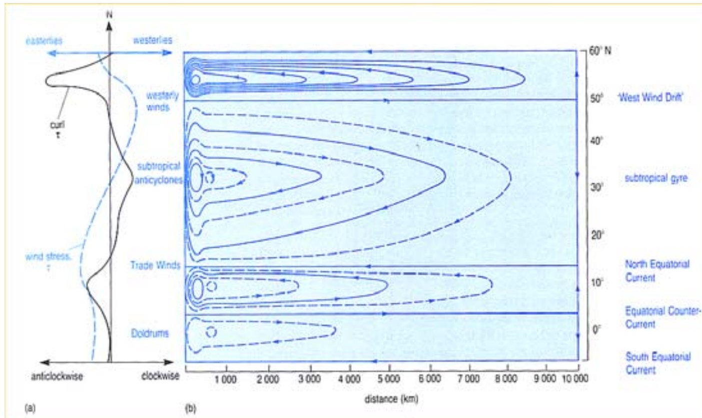

Image courtesy of AMS.

Figure 26: Munk's (1950) solution using horizontal eddy friction in a form analogous to the molecular behavior, and using realistic time-mean winds. The "gyres" that are a consequence of the windstress curl emerge clearly in this solution.

generally, and more widely, dependence on the derivatives of the wind field, and a tendency for all large scale oceanic motions to be strongest on the west have survived.

6 The Meridional Overturning Circulation

As outlined here, the circulation is basically two-dimensional, with no vertical structure (although it is important to recall that the vertical integrals hide the distinction between the Ekman layer and the geostrophic circulation below). The vertical integrations are best justified if the fluid does not have any density variations—which is clearly false in the ocean. Nonetheless, the basic physical mechanisms that emerge from the simplest wind-driven theories do describe qualitatively much of what the ocean does. (It is perhaps useful to recall an old physics dictum that if some physical process can operate, it surely does. Demonstrating that it dominates a particular situation, however, is a completely different matter that is often forgotten in discussions of the ocean in climate.)

The recognition that in the deep ocean the water is much colder than near the surface is very ancient (see especially Chapter 1 by B. A. Warren in Warren and Wunsch, 1981), as is visible in the figures shown earlier. It was also realized very early on that the only place this cold water could come from is the high latitudes (Assuming, however, that the fluid was not just sitting there with only molecular diffusion tending to change the temperature. That is, if one postulated the fluid was simply sitting there, having been injected many thousands of years ago, the very low rate of molecular processes would permit the temperatures to remain unchanged

Image removed due to copyright restrictions.
Citation: Figures 3 and 4. "Östlund, H. G., and C. G. H. Rooth. "The North Atlantic Tritium and Radiocarbon Transients 1972-1983." *J Geophys Res* 95 (1990): 20147-20165.

Figure 27: Form Östlund and Rooth (1990) showing the tritium $\left[{}^{3}H\right]$ concentration in the North Atlantic about 1971 and then again about 10 years later.

for extremely long times. If, however, the temperature changes imply density changes (they do), and the density changes imply pressure gradients, the fluid must be moving—and one then needs to know what is maintaining the observed structure. The rates of movement and of turbulent diffusion remain controversial to this day.) See figure 27.

By the 1950s, it had been noticed that there were only a very few, very small in area, regions of the world ocean where cold, dense water appeared at the surface (L. H. N. Cooper, and H. Stommel). It was inferred that these were regions of "high latitude convection" where fluid became dense enough through cooling and evaporation (leading to salinity increases) to sink to the sea floor. It is from these regions that the cold water seen everywhere in the deep ocean must arise. (Keep in mind, however, that there are many more regions in the world ocean where fluid sinks, but only to intermediate depths. These regions have received much less attention than the ones leading to the bottom water properties.)

Henry Stommel, with his collaborator Arnold Arons, set out to understand the zero-order implications of feeding a deep layer of the ocean with a dense source of water. The result, which is quite counter-intuitive, is often known as Stommel-Arons theory. We need only look at the same geostrophic moment equations we have already been using, along with the continuity equation repeated here:

$$-fv = -\frac{\partial p}{\partial x}, \tag{35}$$

$$fu = -\frac{\partial p}{\partial y}, \tag{36}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{37}$$

with the proviso that we will again regard f as a function of y. Solving the first two for u, v and substituting into the continuity equation produces,

$$\beta v = f\frac{\partial w}{\partial z}, \tag{38}$$

best known as the "geostrophic vorticity equation".[5] Note that these are not transports, but unintegrated velocities.

The significance of Eq. (38) is that it tells us that for a fluid on a rotating sphere, *any north-south geostrophic motion must be associated with a vertical velocity.* One cannot move fluid north-south without having a vertical motion as well (this is a consequence of the need to conserve angular momentum in a system in which the effective rate of rotation is a function of position). There is no such restriction on zonal motions!

Think of the ocean as being made up of two layers, the deeper one having a density greater than the upper one, so that the system is statically stable. We then postulate a fluid source pumping mass into this layer from above (the convective injection of new fluid) at a very localized high latitude position. Let the rate of pumping be Q (in m^3/s). Again, assuming a steady-state is reached, this fluid must move away from the source to prevent a build-up of fluid there.

If we are adding fluid to the deep layer, it's top surface, h_1, must be moving upward. (It may be objected that this violates the steady-state assumption, and it does. Accept it for the moment, and we will find a way of preventing an arbitrary filling up by the deep layer a bit later on.) But Eq. (38) says that w is then positive at the layer's upper surface. w however, must vanish at the bottom, and therefore we conclude $\partial w/\partial z > 0$ in the bottom layer (it is not hard to show that w must be linear with depth in this layer). If fluid is coming into the layer at a rate Q, then h_1 must be moving upwards on average over the ocean at a rate

$$a\frac{\partial h_1}{\partial t} = Q,$$

[5] Unfortunately some authors call it "Sverdrup balance" leading to confusion with the Sverdrup relation. To my knowledge, Sverdrup never used such an equation. The justification for using the terminology appears to be that it underlies the Sverdrup relationship, but it is much more generally applicable than in that context.

where a is the area of the ocean where the uplift is taking place. Because $\partial h_1/\partial t = w$ at $z = -h_1$,

$$\frac{\partial w}{\partial z} = \frac{Q}{a h_1}$$

on average. So far, it's straightforward. But, notice that if $\partial w/\partial z > 0$, Eq. (38) shows that $v > 0$. Here's the trouble: if the source is in the northern part of the North Atlantic (as Fig. 27 suggests), then the fluid in the deep layer must be moving northward—that is toward the source! How then can the fluid escape the source region?

What Stommel proposed was based on his earlier work on the Gulf Stream—that the geostrophic assumption had to somewhere breakdown so that Eq. (38) would not apply. He proposed that it broke down in the same way—that a dissipative boundary layer would appear on the west, carrying the fluid away from the source and not subject to Eq. (38). Supposing that is true (without even working out the dynamics of the boundary layer), we can close the solution purely kinematically:

Two things have to be true: fluid has to be moving southward away from the source at a rate Q minus whatever has already upwelled poleward of that point, and fluid has to be moving northward over almost the whole basin at a rate

$$v = \frac{f}{\beta}\frac{\partial w}{\partial z} = \frac{f}{\beta}\frac{Q}{a h_1},$$

which if integrated vertically over the layer, and across the basin to a wall at L_x, gives a northward transport of

$$V = \frac{f}{\beta}\frac{Q}{a} L_x. \tag{39}$$

By adding together the amount that has not yet upwelled to the amount moving northward in Eq. (39), we know how much the western boundary current must be carrying southward at any latitude. The details are left to the reader (one needs to specify L_x as a function of y). Stommel and Arons postulated that w would be uniformly upward everywhere over the ocean. There is no particular justification for this assumption other than that there was no information to the contrary, and it is the simplest possible choice.

Fig. 28 depicts the origin region for the North Atlantic deep western boundary current, and Fig. 29 shows a complete construct that Stommel produced under the assumption that there are two sources of abyssal water, one in the northern North Atlantic, the other in the Weddell Sea, and that w is spatially constant at $z = h_1$. (Note that the unrestricted zonal flow in the Southern Ocean is used to make the volume fluxes balance globally. The equator, surprisingly, does not seem to require any special treatment.)

The Stommel-Arons theory is very a beautiful one, being based upon very simple equations and ideas, and producing a quite counter-intuitive solution (so counter-intuitive that it's authors

Image removed due to copyright restrictions.
Citation: Worthington, V. "The Norwegian Sea as a Mediteranean Basin." *Deep-Sea Res* 17 (1970): 77-84.

Figure 28: Worthington's depiction of the formation region of the deep western boundary current in the northern North Atlantic.

Image removed due to copyright restrictions.
Citation: Stommel, H. "The Abyssal Circulation." *Deep-Sea Res* 5 (1958): 80-82.

Figure 29: The Stommel and Arons flow field computed under the assumption that only two deep layer mass sources exist, one in the North Atlantic, and one in the Weddell Sea.

tried it out in the laboratory. See the Chapter by Faller in Warren and Wunsch, 1981). It also predicted the presence of intense deep western boundary currents (DWBC) sometimes flowing counter to the known directions of the surface boundary currents, e.g. and especially, in the North Atlantic. Almost immediately, an effort was made to test that prediction, and was beautifully vindicated by the movement of the then new Swallow floats at the predicted location of the DWBC in the North Atlantic.

As with the wind-driven circulation theories, the basic building blocks going into the Stommel-Arons theory have survived 50 years of added complexity and realism—that the interior is geostrophic obeying the geostrophic vorticity equation, and that DWBCs are important. As will be seen later, however, the actual flow in the deep ocean actually looks very little like what is depicted in Fig. 29, despite its reproduction in all textbooks.

7 Vertical Diffusion

One of the objections to the Stommel-Arons picture, at least as presented here, is that it predicts the indefinite rising of the interface between the two fluid layers, and this violates the presumption of a steady-state. How can that objection be circumvented? The picture in the minds of physical oceanographers is that a "mixing process" at the top of the layer converts the fluid there from the density of the abyssal layer into that of the overlying lighter layer (which is providing water to the sinking region). To understand how that might work, it is convenient to make our discussion slightly more realistic and to acknowledge that the ocean does not have layers with discontinuous densities, but rather has a continuous variation. Fig. 30 is taken from Munk (1966) and shows the distribution of temperature and salinity in the deep central Pacific Ocean. Munk pointed out that both temperature and salinity both very closely followed the formula $T, S = T_0 e^{\alpha z}, S_0 e^{\alpha z}$. Now the equation of state of sea water, although complicated, is not far from linear

$$\rho = \bar{\rho}\,(1 - b_1 T + b_2 S) \tag{40}$$

where $\bar{\rho}$ is a constant not far from $1029\text{kg/m}^3 \approx 1\text{gm/cm}^3$. Thus the density field will also be simply exponential with depth:

$$\rho = \bar{\rho}\exp\left(\alpha z\right).$$

We need to ask how "scalars" such as temperature, salinity, or some kind of dye, call any concentration C, are carried about in a fluid. On the laboratory scale, it is not hard to convince oneself that the appropriate equation is

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \kappa_C \nabla^2 C \tag{41}$$

Image removed due to copyright restrictions.
Citation: Munk, W. H. "Abyssal Recipes." *Deep-Sea Res* 13 (1966): 707-730.

Figure 30: From Munk (1966) showing the observed temperature and salinity measurements in the central Pacific Ocean, as well as several curves fit to the observations, for different values of w/K.

where ∇^2 is as defined above. The left hand terms, apart from the time-derivative, represent the "advection" of the property by the flow. The right hand term represents ordinary diffusion of the same type as is seen in a solid. Like molecular viscosity, molecular diffusion for $C = T$, for temperature, is very weak, with $\kappa \approx 1.4 \times 10^{-3}\mathrm{cm}^2/\mathrm{s} = 1.4 \times 10^{-7}\mathrm{m}^2/\mathrm{s}$ (diffusion time over a vertical distance of h goes as h^2/κ).

Munk argued in Eq. (41) that to a reasonable first approximation, one had a steady state, and that the horizontal derivatives would be much smaller than the vertical ones, reducing the equation to,

$$w\frac{\partial C}{\partial z} = \kappa_C \frac{\partial^2 C}{\partial z^2},$$

but that as with the argument about molecular viscosity, that one should replace κ_C by a much larger value K, representing the effects of turbulence on the larger-scale flows which are our present interest, thus rendering the equation in the form,

$$w\frac{\partial C}{\partial z} = K\frac{\partial^2 C}{\partial z^2}. \tag{42}$$

His primary goal was to determine K (more or less in the same way that we determined A from the nominal Ekman layer depth).

To proceed, note that Eq. (42) has the very simple solution (if w, K are constants)

$$C = B\exp\left(wz/K\right) \tag{43}$$

noticing that K/w has the units of a length (called the "scale height"). Because it is observed that both temperature and salinity, and hence density, had the same scale height, he could set $w/K \approx \alpha$ taken from observations. Notice that if one estimates w from Q, the rate of bottom water formation, one can then separately find K, which proves to be $K \approx 10^{-4}\mathrm{m}^2/\mathrm{s} = 1\mathrm{cm}^2/\mathrm{s}$

and which is notably easy to remember! (The area of the ocean is about $3\times10^{14}\mathrm{m}^2$, and thus if $Q \approx 30 \times 10^6\mathrm{m}^3/\mathrm{s}$, $w \approx 10^{-7}\mathrm{m/y}$, or about 3m/y.)[6]

This makes a nice simple story. Setting $C = T$ for temperature, Eq. (42) and its solution, Eq. (43) represent a steady-state balance in which the upward advection of cold water, is balanced in temperature by the downward diffusion from above of heat. The same argument would be applied to salt.

Reverting then to the Stommel-Arons picture, we interpret their lower layer as being stratified in density, with Q/a representing the rate at which cold water is being moved upward. At the top of the abyssal layer, mixing maintains the temperature and salinity in a steady-state by bringing down warmth and salt by turbulent diffusion at just the right rate to prevent a change in density.

8 Difficulties

The combination of the wind-driven theory with the Stommel/Arons/Munk abyssal circulation discussion gives a quite pretty and intriguingly complete picture of how the ocean circulation works. Furthermore, it remains true, that the physics embodied in these theories is extremely important in any discussion of the ocean circulation and how it works. Coupled with the large-scale property distributions shown above, and much more mathematically sophisticated extensions of the theory, by about 1970, it seemed that a reasonably comprehensive understanding was at hand. Then the pretty picture started to come apart in a number of ways that we now sketch, but which is not generally reflected in the textbooks you may read.

8.1 The Reference Level Problem

The necessity of choosing a level-of-no-motion had bothered oceanographers over the years, primarily because no one was quite sure how to select it and different scientists working with the same data produced circulation schemes that differed in a number of ways, although qualitatively, they did not differ by much (everyone had a Gulf Stream or Kuroshio for example), and there was no way to tell which, if any, were more correct than others.

There is little doubt that in general, deep flows (say, arbitrarily, below 1000m) are weaker than near-surface ones (see Fig. 17). Measurements by ship-lowered instruments on cables are difficult, expensive and slow at depth, and there was a very powerful incentive to simply assume

[6] Munk (1966) did not separate w, K this way. Rather he employed radiocarbon data which satisfies a slightly different equation (one with an extra decay term), and produced the memorable canonical value of $1\mathrm{cm}^2/\mathrm{s}$. The method based on Q was used by Munk and Wunsch (1998).

that the deep ocean did nothing of any importance. Measurements to the sea floor had been made notably by the German vessel R/V Meteor in the 1920s (leading to some of the charts displayed above), an immense amount of water exists at depth, water which even if moving slowly, is capable of transporting large quantities of various properties like temperature, salt, carbon, oxygen. Thus a handful of oceanographers, perhaps most notably L. V. Worthington of Woods Hole, had begun making measurements in the North Atlantic to the sea floor, leading initially to the important sections obtained across the North Atlantic by the UK and the US during the International Geophysical Year (1957-58 nominally). Worthington took these data and additional ones collected subsequently and made the first serious attempt since the 1930s (Defant and Wüst especially), to describe the flow over the whole water column. He made various choices of levels-of-no-motion and attempted to account in the North Atlantic for the budgets of mass, salt, oxygen and nutrients in the water. Having worked on the problem for more than 20 years, he produced a well-known book (Worthington, 1976) in which he explained (but obscurely in a footnote) that he had concluded the circulation of the North Atlantic did not make sense if one adhered to geostrophic balance, and therefore his description of the top-to-bottom circulation did not obey Eqs. (8, 9).[7]

Worthington's inference was a kind of crisis in the subject—geostrophic balance is the fluid representation of Newton's Laws of motion on a rotating sphere. Moving water generates a Coriolis force, and if there is no pressure force to balance it, one cannot have a steady-state—unless some other force is acting. Other forces (e.g. viscous ones) are possible. But no one could find a convincing substitute for the pressure force. If Worthington was right, there was no zero-order understanding of how the ocean works!

The problem was resolved, almost immediately, by the invention of what have become known as "inverse methods (see Wunsch, 1996). Worthington's puzzle was removed by throwing out the completely unjustifiable idea that there had to be simple, quasi horizontal levels of no motion. The assumption that they existed had become so ingrained in the field, that it had been forgotten that they had been assumed and not deduced. Space and time preclude a discussion of how one solves the problem of determining not the level-of-no-motion, but a level of known motion (see Wunsch, 1996 for an extended discussion; it boils down to recognizing that equations such as Eq. (42) carry enough information to determine enough about the velocity field to proceed).

Figs. 31, 32 show the estimated absolute velocity across the Indian and North Atlantic Oceans as determined by Ganachaud (1999) using the so-called geostrophic or box inverse method. The primarily vertical orientation of the zero lines is striking; this inference—that the water flow does not take place in simple layers (as the text books claim)—has come only

[7] A more extended discussion of the history of this problem can be found in Chapter 2 of Wunsch (1996).

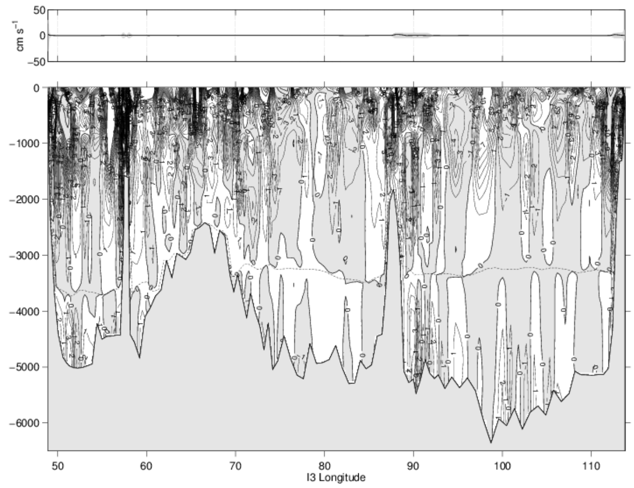

Figure 31: Estimated north-south velocity across 18° S in the Indian Ocean as calculated by Ganachaud (1999). Dotted line is the reference surface. Light shaded areas denote southward flow, clear is northward. Division of the ocean into a moving upper layer and a quiescent deep one does not appear to be very realistic. Upper ocean velocities are, however, stronger than deep ones.

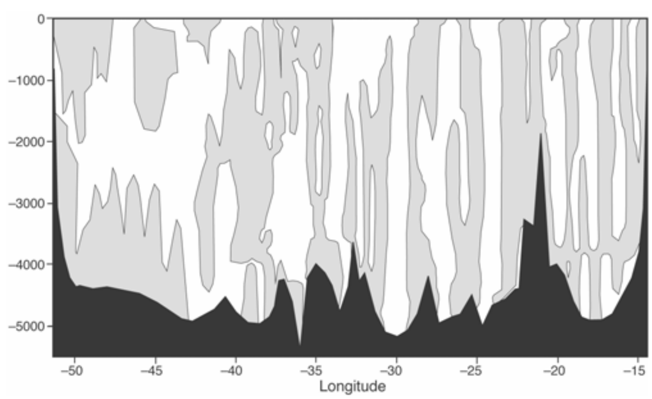

Figure 32: Same as Fig. 31 except at 7.5N in the North Atlantic, and with the numerical values omitted to more clearly show the zero contour lines.

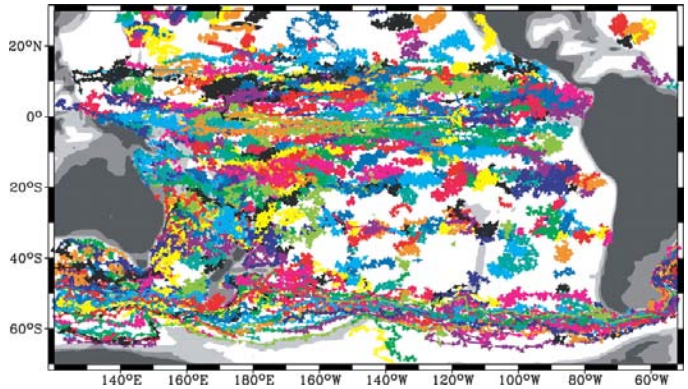

Figure 33: From Davis (2005) showing nominal 1000m float trajectories over several years in the South Pacific. Note the dominantly zonal motion and the lack of obvious connection to the Stommel-Arons theory.

gradually to be recognized. (The paper Wunsch, C., 2006 called The Past and Future Ocean Circulation from a Contemporary Perspective and available at http://ocean.mit.edu/~cwunsch shows more such figures and further discusses the implications.)

8.2 The Abyssal Flow

As noted above, the Stommel-Arons solution suggests two major features—the presence of strong deep western boundary currents in most oceans, with directions dictated by the geography of the deep-water sources, and an interior circulation generally directed poleward everywhere. Diligent searching has turned up evidence for the DWBCs almost always where sought, although there has been some difficulty with rationalizing the directions. The movement in the interior is, however, a quite different story.

Figs. 33, 34 display the multiyear trajectories of neutrally buoyant floats. The ones in the South Atlantic exist within the great property tongue seen in Fig. 13 where one might have anticipated southward motion. Both figures suggest a predominantly zonal flow, with little or no indication of the meridional flow predicted by the theory. Indeed, there appears to be no supporting evidence anywhere in the ocean for an abyssal flow in conformity with the theory—at least in its textbook form.

8.3 Time-Dependence

When the only means to observe the ocean lay with ship-board observations, the ocean was treated as though it was in a steady-state, and this dovetailed nicely with the theories that

Image removed due to copyright restrictions.
Citation: Hogg, N. G., and W. B. Owens. "Direct Measurement of the Deep Circulation within the Brazil Basin." *Deep-Sea Res II - Topical Studies in Oceanog* 46 (1999): 335-353.

Figure 34: From Hogg and Owens (1999) depicting the movement of deep floats in the Brazil Basin. Motions are primarily east-west even over very long periods.

emerged beginning about 1950. As solid state electronic devices gradually became available in the 1960s and 1970s, *time series* measurements in the open ocean, not requiring the constant presence of the ship began to emerge. In contrast to the basic hypotheses of the large-scale circulation, it gradually became clear that the ocean is in a constant state of change—on all time and space scales. Records such as those shown in Figs. 4, 5 were obtained by the thousands, with some extending over many years. Gradually too, the spatial scale of these motions emerged, as being of the order of a few hundred kilometers. Perhaps surprisingly, they too were very close to geostrophic balance, and there was no way that one could claim that oceanic flows were dominated by the basin scale geostrophic motions.

By the middle 1990s, the community had available satellite altimeters which are able to measure the ocean globally, and to detect that part of the circulation manifested in the surface geostrophic flow (satellite altimetry and its results are discussed extensively in Fu and Cazenave, 2001). One could separate surface slope features (as in Fig. 19) that emerged from the record mean, from those showing time variability over the entire record. Given the surface slope, the geostrophic relationship permits direct calculation of the flow. Fig. 35 shows the kinetic energy in the time-average slope and that in the time variability over several years. The lowest panel shows the ratio of time-variable kinetic energy to that in the time-mean. About 99.9% of the kinetic energy in the ocean is in the fluctuations!

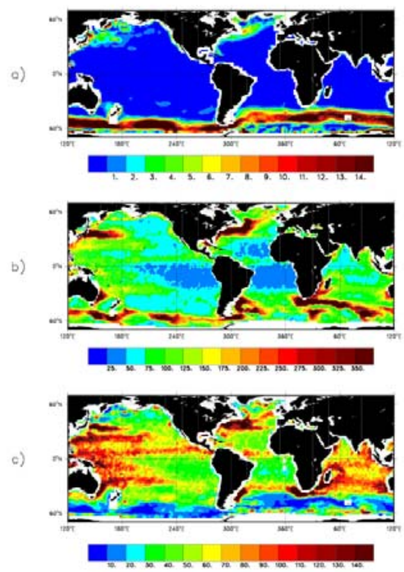

Figure 35: Kinetic energy (KE) of the mean geostrophic flow (upper panel), and of its time variability (middle panel) from seven years of altimetry. The ratio of the time-dependent kinetic to the mean kinetic energy is shown in the lowest panel. Both plots were multiplied by $\sin^2\phi$, where ϕ is the latitude, so as to suppress the equatorial singularity of geostophic balance. The variability is generally 100 times larger than the mean as measured by their KE. KE of the mean is generally somewhat over estimated because the geoid employed is too smooth at short scales, leaving structures in the surface elevation that are present because of gravity field variations rather than the presence of oceanic flow. The ratio, on average, is thus a lower bound. Strong spatial variations in the ratio and its generally large amplitude put the burden of proof on modelers to show that the eddy flux effects do not introduce large systematic errors in long integrations. (Wunsch, 2006).

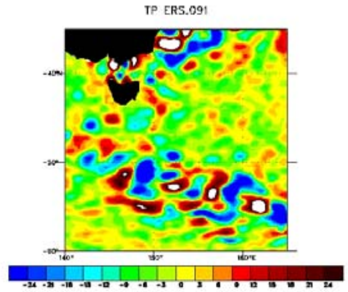

Figure 36: The eddy field as seen south of Tasmania by mutliple satellite altimeters. (An animation of this field exists.) The scale is in cms.

Figure 36 shows the surface elevation in centimeters as observed by multiple altimeters. These small scales dominate the oceanic flows, and carry most of the kinetic energy. (The time mean does however, dominate the *potential* energy.) Although it could be argued that these motions "average out" over long enough time spans, enough theory exists to suggest that such a conclusion cannot be generally supported.

It also became clear that the ocean was varying at arbitrarily long periods, in all components. These variations are not so large that on a human lifetime they very noticeably affect the large-scale temperature and salinity distributions, but over extended time period, they do affect many of the important elements of the ocean circulation, thus influencing the atmosphere, and hence climate.

8.4 The Meridional Overturning Circulation

In the late 1990s, questions began to be asked about some of the fundamental elements of the physics connected with what we have called the meridional overturning circulation. Consider in particular, the balance Eq. (43) involving the turbulent mixing coefficient K. Although it is easy to simply use a large value, either in an analytical solution, or in much fancier ones on computers, it was noted that K is the result of a turbulent flow superimposed upon the larger scale that more people were interested in. But the turbulence has to come from somewhere, and in particular as it is doing work against the buoyancy forces in the fluid, it requires energy to sustain it. How much energy is that and how does it get into the deep ocean to mix the fluid as required? It turns out that finding the turbulence at the levels required, and then explaining it is quite difficult, raising questions about the fundamental physics.

The invocation of "global conveyor belts" as a description of the meridional overturning circulation had evidently led to many scientists thinking of the dominant elements of the ocean circulation as a kind of heat engine, in which the circulation was being driven in the overturning mode by the convective sinking at high latitudes. What had generally been forgotten, or waved away as irrelevant, was a very old paper of Sandström (1908; this paper is in German, but Defant, 1961 has a good explanation of it in English) in which he pointed out that the motions induced, in a steady-state, in a fluid heated and cooled at the surface would be extremely weak. (That is, weak in contrast to a fluid heated at the bottom and cooled at the top, like a pan of water on a stove, or the atmosphere, which is heated by re-radiation from the surface below.)

Although there are some subtleties, such convective motions as heating and cooling at the sea surface can generate, do not appear to be capable of providing the turbulent mixing required in Eq. (43). It is true that water becomes heavy at high latitudes and sinks. If K is very small, the vertical motion of the deep layers will carry the surfaces of constant temperature upward in the ocean, as the abyss gradually fills with the properties of the high latitude sinking waters (one might argue that we see this filling because there is so much cold water present in the ocean). Eventually, the deep isotherms will become so close to the sea surface that molecular diffusion will suffice to bring down enough heat to produce a steady state. (It is instructive to calculate how thick that near-surface boundary layer is, using the molecular value, κ.)

So where does the energy come from to support the turbulence? At almost the same time, new technologies made it possible to directly measure K all the way to the sea floor (many of the references can be found in the recent paper by St. Laurent and Simmons, 2006). It was found that far from being spatially uniform as almost all models had it, there were two order of magnitude variations horizontally, with very large values appearing over deep rough topography.

The consequences of weak mixing over large areas of the ocean are immediately apparent from the equations we already have. The balance (43) shows that absent mixing, $w \to 0$. If w is small, then $\partial w/\partial z \to 0$, and from the geostrophic vorticity balance, $v \to 0$—no meridional flow. (Recall the deep float movements shown in Figs. 33, 34, which are dominantly zonal). Thus one cannot have a Stommel-Arons interior.[8]

Munk and Wunsch (1996) argued that the spatial average value of K could not be too far from the old value of $10^{-4}\mathrm{m}^2/\mathrm{s}$ (and see St. Laurent and Simmons, 2006), but nothing much could be said about its lateral variations. They then calculated how much energy would be required to sustain the turbulence implied by such an average value and estimated it as about $2{\times}10^{12}$W (2 Terrawatts) for the region below about 1000m (nothing could be said about the

[8] One should note that the Stommel-Arons solution so widely disseminated made the assumption of a globally uniform w, implying a globally uniform K. There is nothing in their theory requiring uniformity, and with w, K known, one could construct a new kinematic picture. No one has yet had the courage to try that.

upper ocean energy requirement). Their inference was that the tides, forcing stratified fluid over deep topography could account for about 1/2 the energy, and that the wind field generating what are called internal waves would account for the other half. The most startling conclusion was that the tides were important to the ocean circulation. These results are comparatively recent, are controversial, and large literature has now collected around it that we cannot go into here. Describing the energetics of the ocean circulation remains an unfinished business.

9 Some Summary Comments

In the climate context, the main message to retain is that the ocean circulation is much more interesting than widely circulated schematic diagrams such as conveyor belts would suggest. In particular, the movement of water is often quite counter-intuitive (such as the interior flow towards sources implied by geostrophic balance) and it is difficult to take seriously arguments that ignore the dynamics and are simply based upon ill-founded intuition. Because of the small scale energetic, time-dependent structures, there are serious issues of space/time sampling and of understanding of the interactions of the smaller scales to produce the larger-scale structures. Undiscussed here is the entire question of whether the numerical models that now exist of the ocean circulation have sufficient realism to be integrated over the thousands of years necessary to depict past and future climate?

In ending here, note that a wide range of very important oceanographic elements of climate have not even been mentioned including El Niño, the physics of mixed-layers, why and how convection occurs, feedbacks to the atmosphere, the so-called thermocline theories explaining the mean temperature and salinity structure of the ocean, formation and effects, of sea ice, the physics of the near-equator and of the Southern Ocean, boundary upwelling, the physics of the eddy field, etc. The references can give on a start on most of these subjects.

As a last word, note that it is difficult to escape discussions of the ocean as a "global conveyor" as depicted in the memorable cartoon by W. Broecker implying that the ocean circulation is one-dimensional and basically simple. Because of its ubiquitous nature, and its impressively wrong implications, I list here what is right, what is missing, and what is conceptually wrong.

What's right? There is a net meridional circulation involving sinking at high latitudes in the North Atlantic, and which does transport some heat. The water reaching the near-bottom does eventually have to return to the seasurface to close the mass/volume budget.

What's Missing? Water sinks at high southern latitudes in roughly equal amounts to that sinking in the North Atlantic. A lot of water sinks to intermediate depths all over the world ocean, including the high latitude North Pacific. Much of the water sinking in the high latitude

North Atlantic returns to near the surface in the Southern Ocean (some of it under the ice) before eventually sinking again, and moving on. The largest current on earth is the Antarctic Circumpolar Current carrying about $150\times10^6\mathrm{m}^3/\mathrm{s}$ and it has a profound climate impact. Much of the surface water entering the South Atlantic comes from the Circumpolar Current. The North Pacific appears to be the region of *minimal* deep water upwelling. Water moving from the North Pacific into the Indian Ocean between New Guinea and Australia recirculates primarily around Australia. A major component of the time average circulation is the intense western boundary currents that at the surface are primarily wind-driven. In the Southern Ocean, heat fluxes are dominated by eddy processes. Kinetic energy of the ocean is 99.9% in the time-dependent rather than in the steady components.

Conceptual Problems. The mean circulation involves an integral of many small-scale, extremely energetic time-varying components. A change in high latitude sinking rates need not simply propagate upstream in the one-dimensional conveyor. Stoppage of North Atlantic sinking does not preclude its appearance somewhere else. Sinking motions cannot drive the meridional overturning circulation—they can only influence its properties (Sandström's ideas).

The reduction of the complex turbulent flow of the real ocean to a one-dimensional steady flow if useful would represent an astonishing breakthrough in the physics of turbulent fluids that would be landmark in the history of fluid dynamics. Purely verbal arguments about how the ocean circulation *must* change in the climate system should be regarded as science fiction.

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