State Variable Equations

$$\varepsilon_{ij} = \varepsilon^{E}{}_{ij} + \int_0^t \dot{\varepsilon}_{ij}^{I} d\tau + \varepsilon^{TH}{}_{ij}$$ ***Kinematic***

$$\dot{\varepsilon}_{ij}^{I} = \left| \dot{\varepsilon}_{o}^{I} \left(\sigma_{kl}, T, P, f_p, \varsigma_r, \ldots \right) \right|_{ij}$$ ***Kinetic***

$$\varsigma_m = \varsigma_m \left(\dot{\varepsilon}_{ij}^{I}, \sigma_{kl}, P, T, f_p, \varsigma_s, \ldots \right)$$ ***Evolution***

Stouffer and Dame, “Inelastic deformation of metals”, 1996

Apparent (Empirical) Activation Parameters

$$\dot{\varepsilon} = \dot{\varepsilon}_o \sigma^n \exp(-\frac{\Delta H}{RT})$$

$$n = \left.\frac{\partial \ln \dot{\varepsilon}}{\partial \ln \sigma}\right|_{T,P,\ldots}$$ Stress exponent

$$V^* \triangleq \left.\frac{\partial \ln \dot{\varepsilon}}{\partial P}\right|_{T,\sigma,\ldots}$$ Activation Volume

$$H^* \triangleq \left.\frac{\partial \ln \dot{\varepsilon}}{\partial (1/T)}\right|_{\sigma,P,,\ldots}$$ Activation $\begin{cases}\text{Energy}\\ \text{Enthalpy}\end{cases}$

$$A^* \triangleq \left.\frac{1}{b}\frac{\partial \ln \dot{\varepsilon}}{\partial (\sigma)}\right|_{P,T,\ldots}$$ Activation $\begin{cases}\text{Volume}\\ \text{Area}\end{cases}$

Orowan's Equation
(Kinetic Equation)

Egon Orowan 1966 as a Visiting Scientist at the Boeing Scientific Research Laboratory in Seattle, Washington.

Generation

Recovery

Recrystallization

$$\dot{\varepsilon} = \rho_m \left|\mathbf{b}\right| v$$

Lattice Friction

Precipitates

Dislocation Interactions

Source/Sink

Frank-Read Source

- Force balance gives

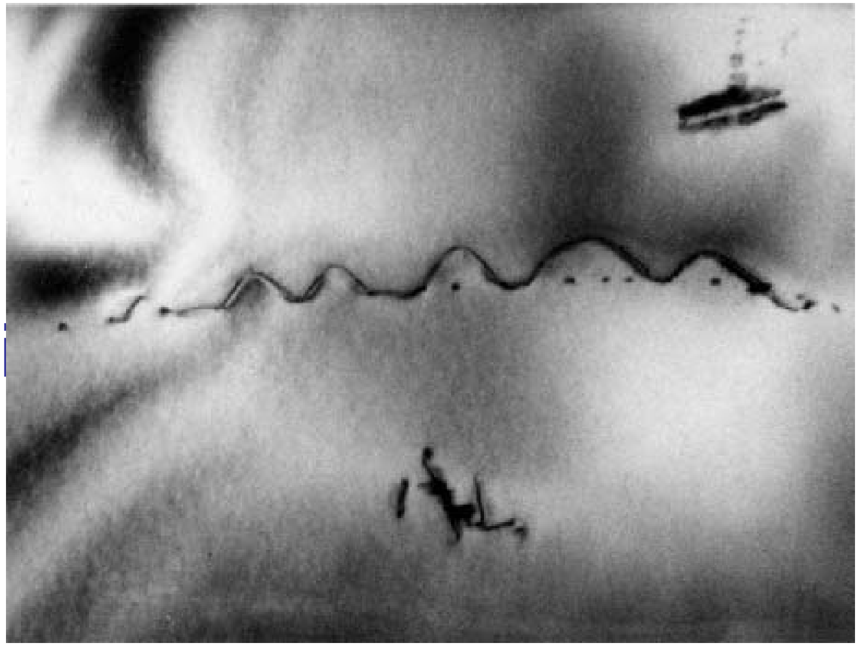
$$\boxed{\sigma = \frac{\mu b}{2R}}$$

- The dislocation density i

$$L = \rho^{-1/2}$$

- Then

$$\sigma_{crit} = \frac{\mu b \rho^{1/2}}{2}$$



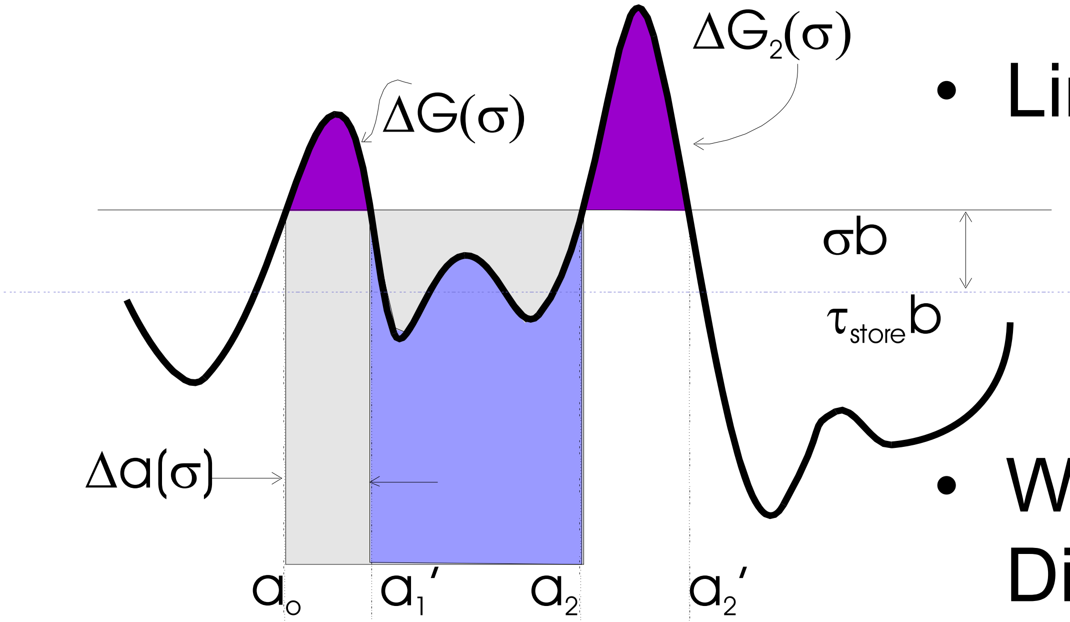
Work Hardening and Recovery

$$\frac{d\sigma}{dt} = \frac{d}{dt}\left[\sigma\left(\varepsilon\left(t\right), t\right)\right] = \left.\frac{\partial\sigma}{\partial\varepsilon}\right|_t \dot{\varepsilon} + \left.\frac{\partial\sigma}{\partial t}\right|_\varepsilon$$

$$\frac{d\sigma}{dt} = h\dot{\varepsilon} + r$$ Bailey-Orowan Eq.

- At steady state, hardening = recovery
- If square root of disl. dens. proportional to mean free path, $\lambda = a\rho^{1/2}$
  and if density is a function of stress $,\sigma^2 = \alpha\mu b\rho^{1/2}$
  then can show that $\frac{h}{\mu} = \frac{\alpha}{a}$
Qtz	Olivine	W	FCC	Easy Glide
1	1-3	50	350	10^4

Thermodynamics of Glide and Climb



- Line glide resistance
  - Thermal activation
  - Applied Stress
- Work done on mat'l.
  - Dissipated energy
  - Force on dislocation

Kochs et al., 1975

Velocity Kinetics

$$\bar{v} = \frac{\Delta L}{t_g + t_o} \quad where \begin{cases} \Delta L \text{ is distance to next obstacle} \\ t_g \text{ is time to glide} \\ t_o \text{ is time to overcome obstacle} \end{cases}$$

- If obstacle can be overcome by thermal activation (e.g. lattice friction, cutting of soft ppts., unraveling attractive junctions)
  - $\Delta L >> a_1 a_1'$ then $\Delta L = \lambda$ (where λ is the dislocation spacing) $t_g << t_o$ e.g. *Cross slip* in FCC metals
  - $\Delta L \cong a_1 a_1'$, obstacle met as soon as overcome, *Glide-controlled Creep*
- Obstacles can't be overcome but may be avoided by climb,

  Recovery-controlled Creep

Recovery Processes

- Recovery-removal of defects
  - Recovery processes
    - Collapse of dipoles
    - Loop collapse
    - Annihilation
    - Sub-grain boundary annihilation
    - Climb and glide to grain boundary, surface
    - Sub-grain boundary coarsening
- Recrystallization-creation and motion of high angle grain boundaries
  - Recrystallization Processes
    - Grain growth
    - Static recrystallization
    - Dynamic recrystallization
    - Chemically induced gb migration

Power Law Creep

$$\dot{\varepsilon} = \varepsilon_o \upsilon_o \frac{\mu\Omega}{kT} f(\mu_i)^{p_i} \left(\frac{\sigma}{\mu_{shear}} \right)^n \exp\left(-\frac{\Delta H}{kT} \right)$$

- n combined effect of density and mobility terms
- Fugacity and chemical potential of phases may affect dislocation mobility
- Debye frequency, shear modulus and molecular volume
- For specific models see *Kohlstedt et al.*, and *Al et Kohlstedt* 95.

Competition between Diffusion and Dislocation Creep

- Dislocation Creep:

$$\dot{\varepsilon} = A_{xs}\sigma^2 \exp\text{-}\left[\frac{Q_{xs}\left[\left(1 - C_{xs}\sigma\right)\right]}{RT}\right]$$

- Diffusion Creep

$$\dot{\varepsilon} = A_{diff}\,\frac{\sigma}{d^m}\cdot \exp\left(-\frac{Q_{diff}}{RT}\right)$$

- Composite Flow (Ter Heege et al.)

$$\dot{\varepsilon} = \dot{\varepsilon}_{cs} V_{cs} + \dot{\varepsilon}_{diff} V_{diff}$$

- State Variable: Grain size

Low Temperature High Stress Laws (Other Creep)

$$\dot{\varepsilon} = \dot{\varepsilon}_o \exp\left[-\frac{\Delta g}{kT} \left(1 - \frac{\sigma}{\tau} \right)^p \right]^q$$