

MIT OpenCourseWare
<http://ocw.mit.edu>

12.510 Introduction to Seismology
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Continents: Quick review. Ground Roll/Love waves. Group velocity and phase velocity. Dispersion curves.

Last lecture we looked at...

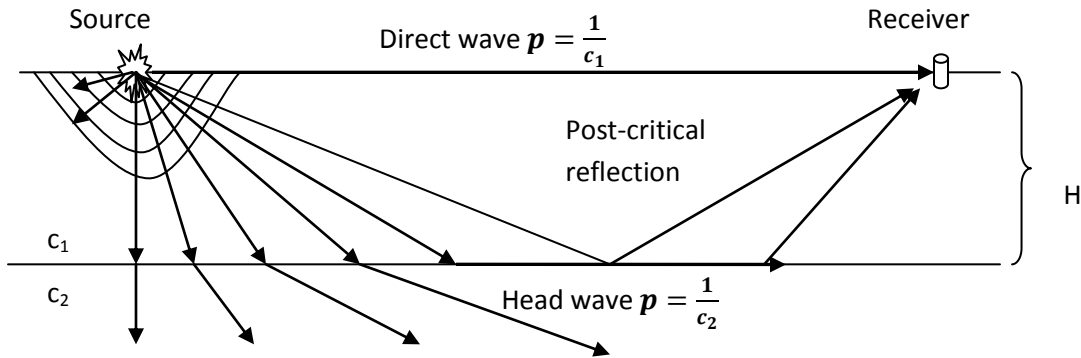


Figure 1: Ray paths for a layer over a half-space model. The head wave exists when $c_2 > c_1$.

Ground roll/Love Waves

The dispersion relationship for the ground roll is given by

$$\tan \left(\omega H \sqrt{\left(\frac{1}{c_1}\right)^2 - p^2} \right) = \frac{\rho_2 \sqrt{\left(\frac{1}{c_2}\right)^2 - p_n^2}}{\rho_1 \sqrt{\left(\frac{1}{c_1}\right)^2 - p_n^2}} \tag{1}$$

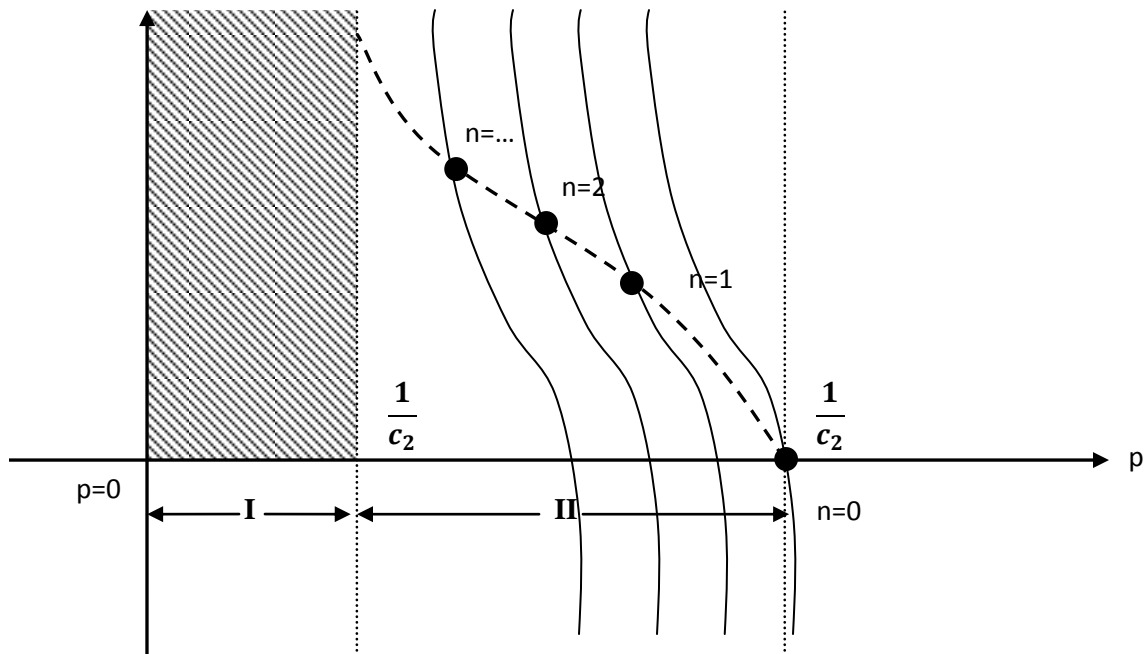


Figure 2: Ground roll dispersion relationship

A finite number of modes have discrete phase velocities. The fundamental mode ($n=0$), represents a direct wave; higher modes depend on the wave frequency. The left hand side of equation (1) shows the frequency relation. The spacing between the curves depends on frequency; the higher the frequency the closer the curves are as a result of the decreasing number of higher modes.

I: Pre-critical ($p < \frac{1}{c_2}$)

This is also known as the “leaky mode” because energy is lost due to wave transmission into the half space.

The reflection coefficient is

$$R = \frac{z_1 \cos(i_1) - z_2 \cos(i_2)}{z_1 \cos(i_1) + z_2 \cos(i_2)} \in \mathbb{R}; \quad (2)$$

where z is the impedance and is given by

$$z = \frac{c}{p}, \quad (3)$$

and the transmission coefficient is

$$T = \frac{2z_1 \cos(i_1)}{z_1 \cos(i_1) + z_2 \cos(i_2)} \in \mathbb{R}. \quad (4)$$

This mode is frequency independent.

II: $i > i_c$ Post-critical $\frac{1}{c_2} < p < \frac{1}{c_1}$ (post critical reflector and “transmission”)

This is also called the “locked mode” because all of the energy stays in the upper layer. $R \in \mathbb{C}$, $T \in \mathbb{C}$. This mode is frequency dependent.

In the layer ($0 < z < H$), the displacement is given by

$$U_n(x, z, t) = A * \cos\left(\omega \left[\sqrt{\left(\frac{1}{c_1}\right)^2 - p_n^2} z \right] \exp\{i(k_n x - \omega t)\}\right). \quad (5)$$

The term p_n can be obtained from the dispersion relation from equation (1). It represents the slowness of n^{th} higher mode (overtone), $p_n = \frac{1}{c_n}$. The exponential term in equation (5) represents

the wave propagation in the x-direction. k_n is the wavenumber, c_1 is the minimum phase velocity and $c_n = \frac{\omega}{k_n}$ is the maximum phase velocity.

The displacement varies with depth as the cosine term of equation (5) show, and so it oscillates. The cosine term depends on the wave frequency, higher mode slowness and depth. (oscillatory wave \rightarrow x-direction)

In the half space ($z > H$), the displacement is given by

$$U_n(x, z, t) = A * \cos\left(\omega \left[\sqrt{\left(\frac{1}{c_1}\right)^2 - p_n^2}\right] H\right) \exp\{i(k_n x - \omega t)\} * A(z), \quad (6)$$

$$A(z) = \exp\left\{-\omega \left[\sqrt{p_n^2 - \left(\frac{1}{c_1}\right)^2}\right] (z - H)\right\}.$$

The wave propagates in the x-direction with wavenumber k_n . The displacement decays exponentially with depth as the first exponential term shows. In the low frequency case, the decay is slow and the amplitude is large at $c = c_2$. In the high frequency case, the decay is fast. If the frequency is infinite, there is no sensitivity. Moreover, as the depth goes to infinity, the displacement or sensitivity goes to zero. Figure 5 describes the pattern of the wave in the layer and in the half space. Higher modes sample deeper and faster (evanescent wave \rightarrow x-direction).

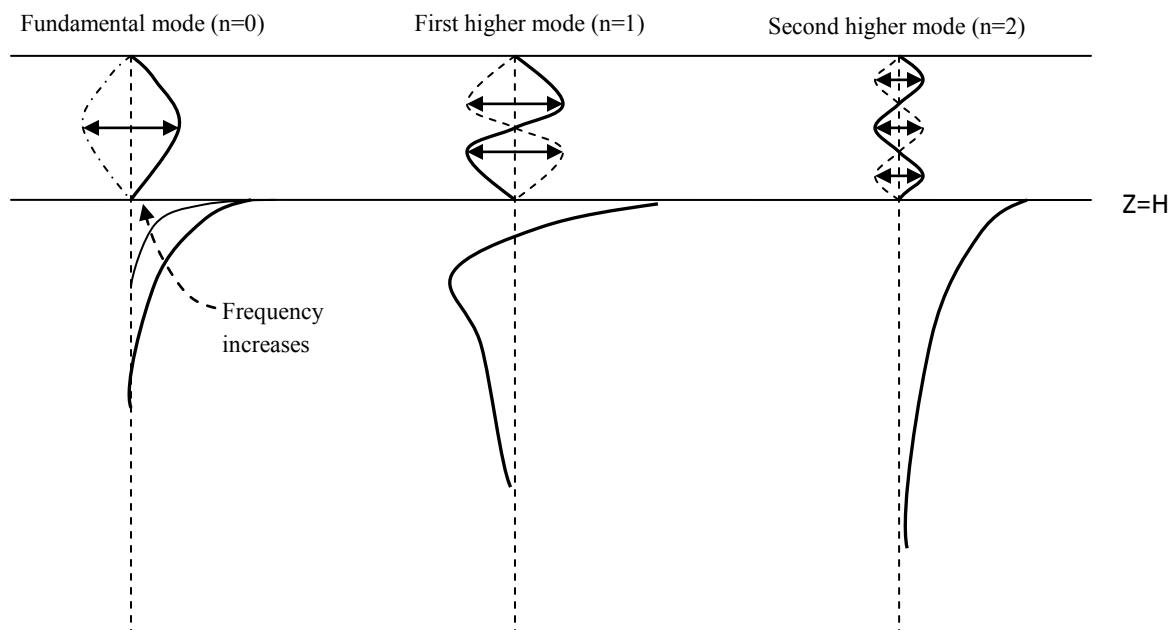


Figure 3: Wave patterns in a layer over half space

The higher the mode, the better the sensitivity is to the deeper structure. There are distinct modes, which are understood through interference boundary conditions, thus constraining which combinations can be used. Some of the modes will not propagate as a wave because there is no constructive interference.

Frequency-Wavenumber domain (ω - k)

$\omega - k$ domain is obtained by taking the Fourier transform from space-time (x,t) domain. In the $\omega - k$ space, we can analyze the previous study of surface wave dispersion relation in equation (1). A constant phase velocity can be represented by a straight line $c = \frac{\omega}{k}$. The fundamental mode and higher modes can also be represented.

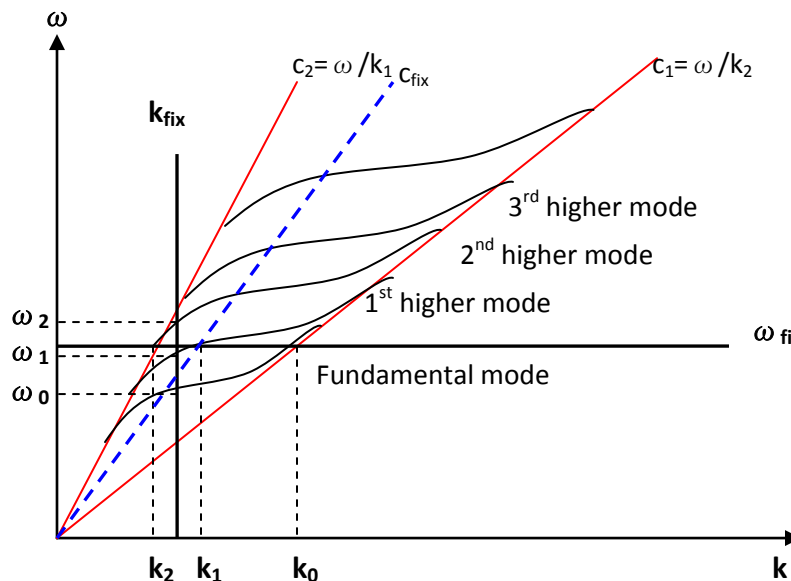


Figure 4: Surface wave dispersion in the $\omega - k$ domain

Higher frequency makes the mode approach c_1 asymptotically (i.e. no sensitivity). Conversely, smaller frequency makes the mode approach c_2 asymptotically, which means low frequency waves are more sensitive to half space and evanescent waves at low frequency sample deep in the half space. We can consider three cases: fixed frequency, fixed wavenumber, and fixed wave speed.

(1) At fixed frequency, waves propagate along different directions consistent with each discrete wavenumber. Discrete wavenumber represents discrete phase speed, $c_n = \frac{\omega}{k_n}$. The smaller wavenumber has the higher phase velocity. The lowest velocity corresponds to fundamental mode and high velocity comes from higher modes.

(2) At fixed wavenumber, waves propagate along a fixed direction but with a discrete number of wave frequencies. The higher frequency has the higher phase velocity. The lowest velocity comes from fundamental mode and higher modes give higher velocities.

(3) At fixed wave speed, there are infinite numbers of modes. For a given distance, waves of low frequency of the fundamental mode can arrive at the same time as higher modes at a higher wave frequency. Note that the phase velocity is described by a straight line.

Dispersion: Phase Velocity and Group velocity

Consider two harmonic waves with particular wavenumbers k_1 and k_2 , and with particular wave frequencies ω_1 and ω_2 . The two waves can be represented by the following relations:

$$\begin{aligned}
 u(x, t) &= \cos(k_1x - \omega_1 t) + \cos(k_2x - \omega_2 t) \\
 \delta\omega &= \omega - \omega_1 = \omega_2 - \omega \text{ where } (\omega_1 < \omega < \omega_2) \\
 \delta k &= k - k_1 = k_2 - k \text{ where } (k_1 < k < k_2),
 \end{aligned}
 \tag{7}$$

using the above relations we can add the cosines and simplify to

$$\begin{aligned}
 u(x, t) &= \cos(k_1x - \omega_1 t) + \cos(k_2x - \omega_2 t) \\
 e^{i(kx - \omega t)} &= \cos(kx - \omega t) - i\sin(kx - \omega t) \\
 u(x, t) &= 2 * \cos(kx - \omega t) \cos(\delta kx - \delta\omega t).
 \end{aligned}
 \tag{8}$$

The first cosine term (carrier) travels with phase velocity $c = \frac{\omega}{k}$, and the second (envelope or beat pattern) travels with group velocity, $u = \frac{\delta\omega}{\delta k}$.

Figure 5 shows the ‘beating’ effect.

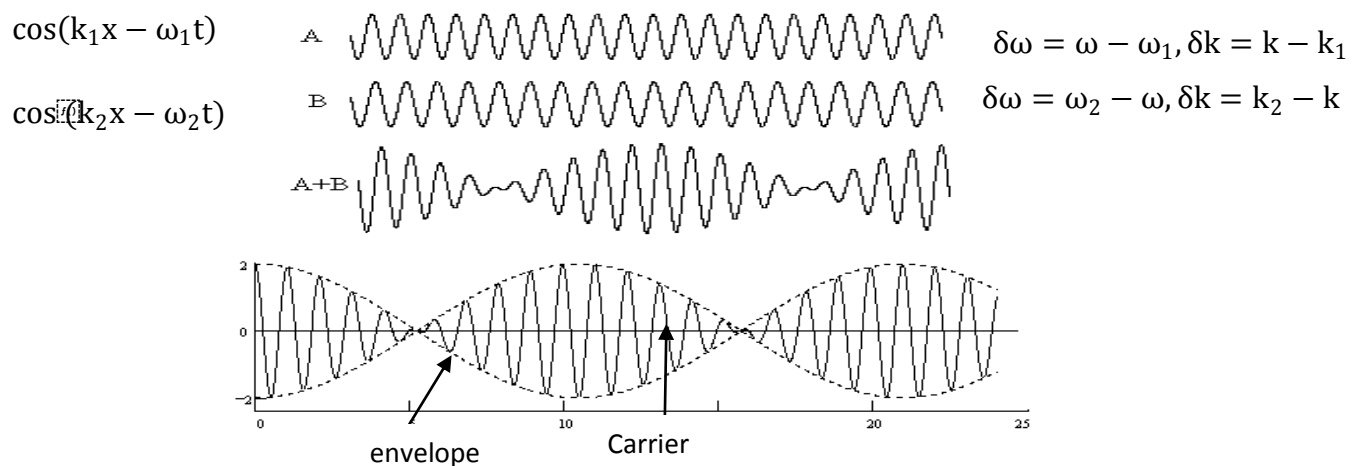


Figure 5: Two waves with different frequencies and wave numbers; their sum yields to a beating pattern (long period envelope) that propagates at the group velocity. The high frequency oscillation (carrier) propagates at the phase velocity c . The dotted line represents the group velocity and the solid line represents the phase velocity.

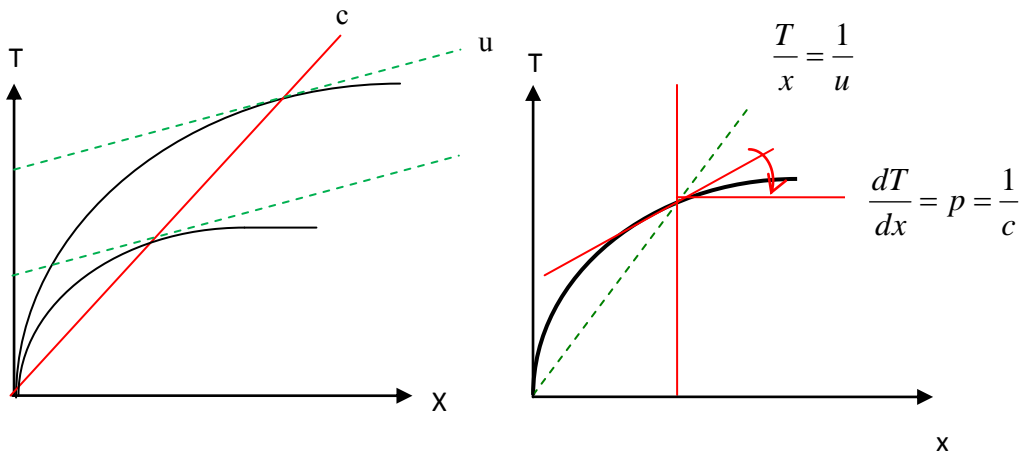


Figure 6: Group and phase velocity in the space-time (x,t) domain. The tangent red line is the phase velocity and the green dashed line is the group velocity.

The phase velocity is given by

$$\frac{dT}{dx} = p = \frac{1}{c} = \frac{k}{\omega}, \quad (9)$$

and the group velocity is represented by

$$u = \frac{\delta\omega}{\delta k} = \frac{X}{T}. \quad (10)$$

The phase velocity is faster than the group velocity.

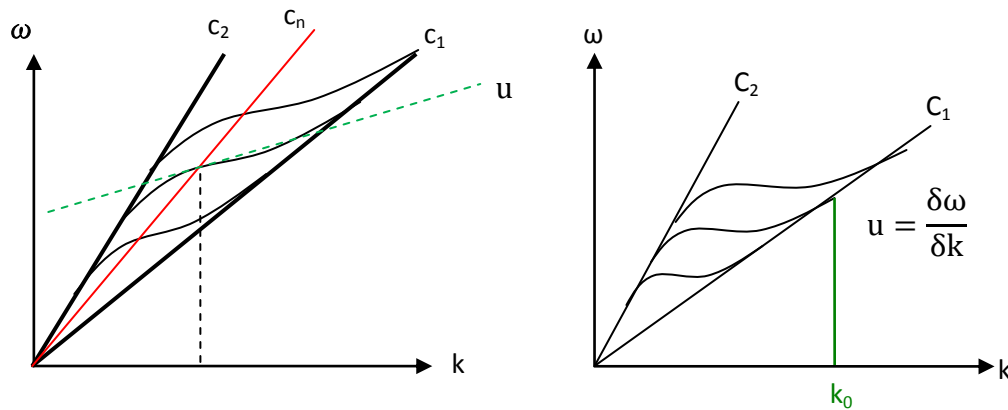


Figure 7: Group and phase velocity in the frequency-wavenumber domain (ω, k).

As frequency goes to infinity, group velocity is equal to phase velocity, this relation can be written as

$$\lim_{n \rightarrow \infty} u = \lim_{n \rightarrow \infty} \frac{\delta \omega}{\delta k} = \frac{\omega}{k} \quad (11)$$

In this case, group velocity will be equal to c_1 for high frequency waves.

$$\lim_{\omega \rightarrow \infty} u = \lim_{\omega \rightarrow \infty} \frac{\delta \omega}{\delta k} = \frac{\omega}{k} = c_1 \quad (12)$$

Arrival time

From previous relations we know: $T = \frac{x}{c_1}$; $u = \frac{x}{T}$; $u = \frac{\delta \omega}{\delta k}$; $k = \frac{\omega}{c}$; $\omega = ck$

$$\Rightarrow u = c + k \frac{\delta c}{\delta k}$$

$$k = \frac{2\pi}{\lambda} \quad (13)$$

$$\Rightarrow u = c - \lambda \frac{\delta c}{\delta \lambda} = \text{group velocity}$$

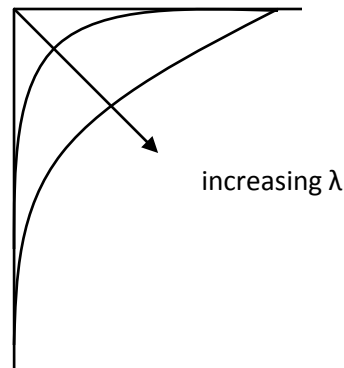


Figure 8: Evanescence of the wave

$e^{-\eta\omega z}$, $e^{-k_z z} = e^{-\frac{2\pi}{\lambda} z}$. Low frequency wave is more sensitive to deep structure. Therefore, low frequency wave should arrive earlier than high frequency wave.

Looking at the fundamental mode will give us some information about shallow depths. Combining with higher modes will give even more information about what is happening at depth (see Figure 3).

Principle of the stationary phase

Only certain frequencies and directions will interfere constructively to create arrivals. If at a particular time you have wave propagation they add up to give the seismogram amplitude.

Seismogram:

$$u(x, t) = \int_{\omega k} A(\omega, k) e^{i(kx - \omega t)} d\omega dk \quad (14)$$

Equation 14 shows plane wave superposition.

Building interference in means many combinations of ω & k will not result in displacement.

$$u(x, t) = \int_{\omega k} A(\omega, k) e^{i(kx - \omega t)} d\omega dk$$

$$\frac{d}{d\omega} (kx - \omega t) = 0 \rightarrow \text{expressions of stationary phase}$$

$$\frac{d}{dk} (kx - \omega t) = 0 \quad (15)$$

$$\frac{d\omega}{dk} = \frac{x}{T} = u \text{ group velocity}$$

has a beating affect – interference of waves. Highest amplitude where many arrive at the same time, and the other ω 's & k 's exist, but do not produce the amplitude.

(note: In S & W: Read 2.7 and 2.8, 2.8.2 does not call it the stationary phase, but they look at $f(\omega,k)=0$, which is the stationary phase approach.)

$$\frac{d\left(\omega\left(\frac{x}{c}-t\right)\right)}{d\omega} \rightarrow u = \frac{c}{\left(1 - \frac{\omega}{c} \frac{\partial c}{\partial \omega}\right)} \quad (16)$$

as a consequence of stationary phase principle

Airy Phase

Wave that arises if the phase and the change in group velocity are stationary. That is, more energy arriving at once.

$\frac{du}{d\omega} = 0 \rightarrow$ gives the highest amplitude in terms of group velocity and are prominent on the seismogram.

Surface arrival period $T \sim 20$ -30sec. This can be used to filter the surface waves from the seismogram.

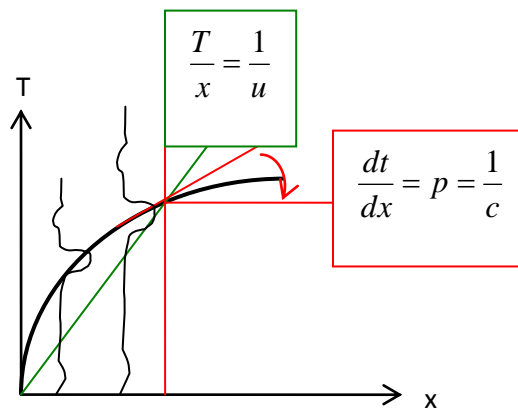


Figure 9: Part of the wave that propagates with the group velocity is not same part of the seismogram. The peaks and troughs are related to the phase velocity (e.g. first onset). Group velocity is related to the frequency band.

With greater propagation distances the arrivals spread out more and shift to lower frequencies. Looking at fundamental and higher modes becomes easier at higher distances because they are more spread out.

Rayleigh waves are more complicated than Love waves.

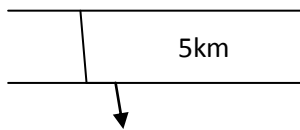
Dispersion

Phase velocity $c(\omega)$

→ phase dispersion curves

→ group dispersion curves

Oceanic crust



Continental

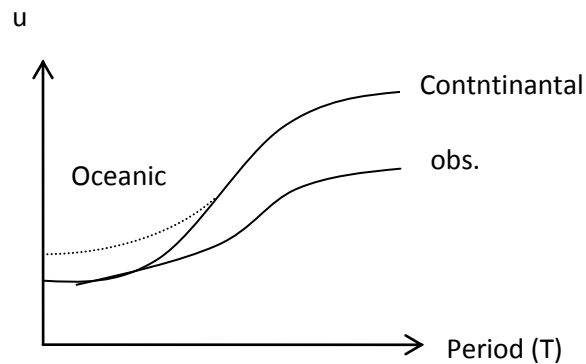
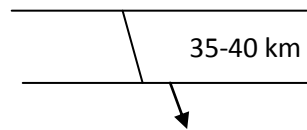


Figure 10: Group velocity vs period

For evanescent waves such as Rayleigh and Love waves we have seen that long wavelength waves penetrate deeper into the half space than short-wavelength waves.

How exactly structure in a certain depth interval influences a wave of a particular frequency is described by a sensitivity kernel (Figure 11). They represent the maximum particle motion at a certain depth as a function of frequency, which can be computed from a reference Earth model.

Excitation of surface waves

Generally, the position of the earthquake (i.e. the depth in our case of depth-dependent media) determines which modes can be excited. A fundamental mode has no displacement deeper than a certain depth; by reciprocity, a source (assume a white spectrum of the source so that it can, in principle, excite all frequencies) that is located at those large depths will not cause displacement of that fundamental mode at the surface.

Figure removed due to copyright restrictions.

Figure 11: Phase speed sensitivity kernels. (From "An Introduction to Seismology, Earthquakes, And Earth Structure" by Stein & Wysession (2007).)

Dispersion curves

We have seen that the radial variation of shear wave speed causes dispersion of the surface waves. This means that the observed surface wave dispersion contains structural information about the radial variation of seismic properties. A plot of the group or phase velocity as a function of frequency is called a dispersion curve. Their diagnostic value of 1D structure has been explored in great detail. Typically, the curves produced from observed records are matched with standard curves computed from an assumed reference Earth model that can have a structure that is characteristic for a certain type of upper mantle (e.g., old/young continents, old/young oceans, etc.). Such analyses have produced the first maps of the thickness of oceanic lithosphere which revealed the increase in thickness with increasing age of the lithosphere (or distance from the ridge. Figure 12 shows a variety of typical dispersion curves for different tectonic provinces.

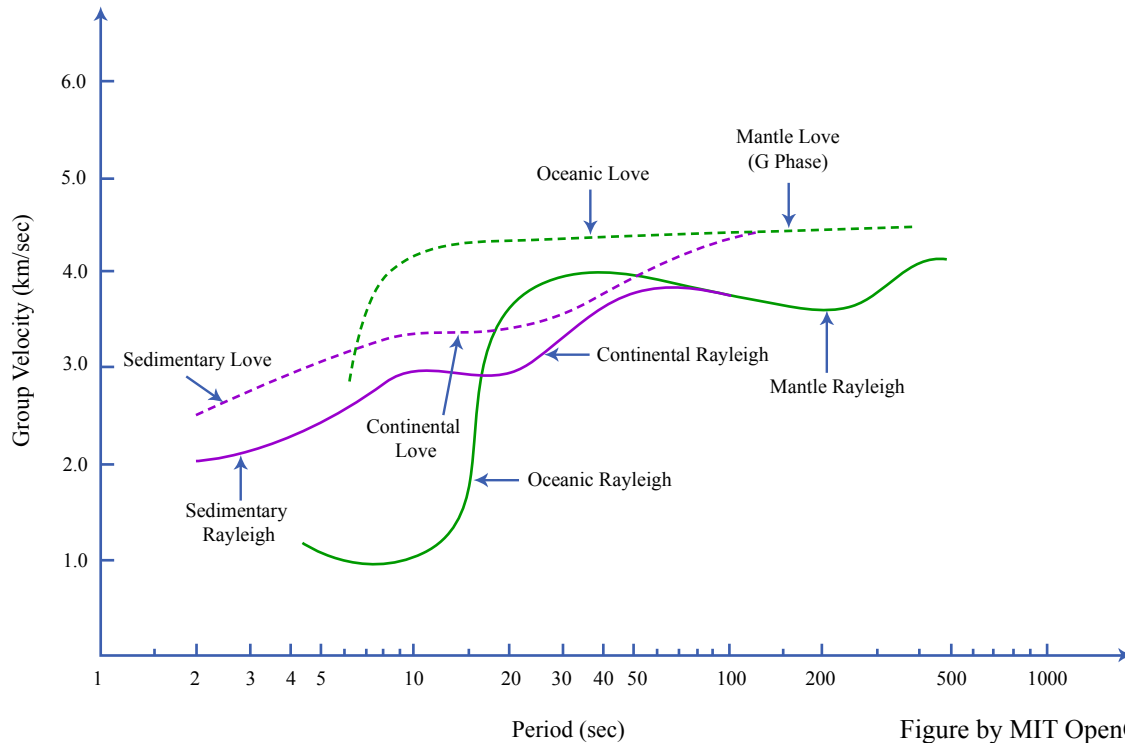


Figure by MIT OpenCourseWare.

Figure 12: the dashed lines are travel time curves for those S/SS/SSS phases. But note that the frequency of those phases change with distance, so that the waveform changes. For instance, with increasing distance, the first arriving phase is composed of waves with larger frequencies (because they sample deeper).

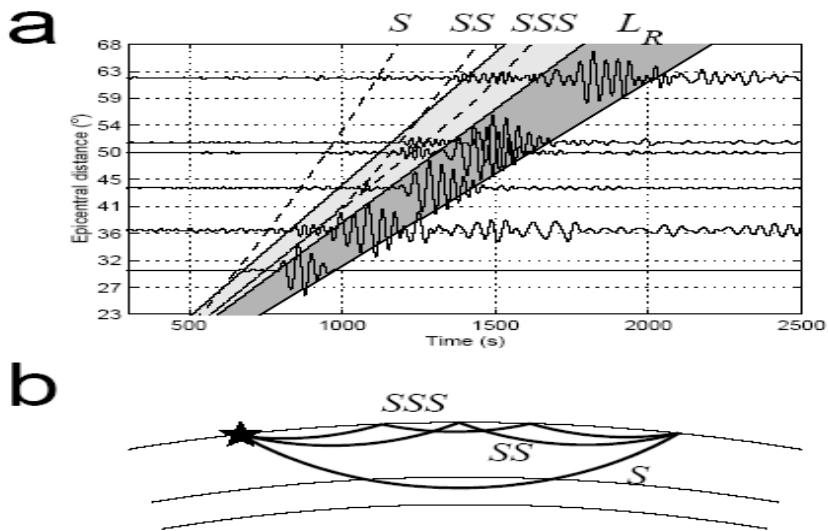


Figure 13: Group velocity windows and phase velocity curves

The group velocity of surface waves of a particular frequency defines a straight line through the origin and through the signal of that particular frequency on records of ground motion at different distances. The group velocity decreases as the frequency increases. As a result, high frequency phases become less and less pronounced with increasing distance from the source (or time in the seismogram). The group velocity is very important: the energy in surface waves

propagates mainly in the constructively interfering wave packets, which move with the group velocity. Narrow-band filtering can isolate the wave packets with specific central frequencies (see Figure 13), and the group velocity for that frequency can then be determined by simply dividing the path length along the surface by the observed travel time. This technique can be used for the construction of dispersion curves (Figure 12).

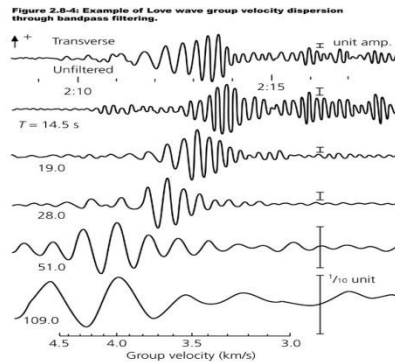


Figure 14: Group velocity dispersion through bandpass filtering (Figure 13 and 14 are adapted from Stein and Wysession, p97)

Updated by: Sami Alsaadan

Sources: April 11, 2005 by Kang Hyeun Ji. April 13, 2005 by Patricia Gregg. April 20, 2005 by Sophie Michelet. April, 04, 2008 lecture.

“An Introduction to Seismology, Earthquakes, And Earth Structure” by Stein & Wysession (2007).