12.005 Lecture Notes 19

Stress and strain from a screw dislocation

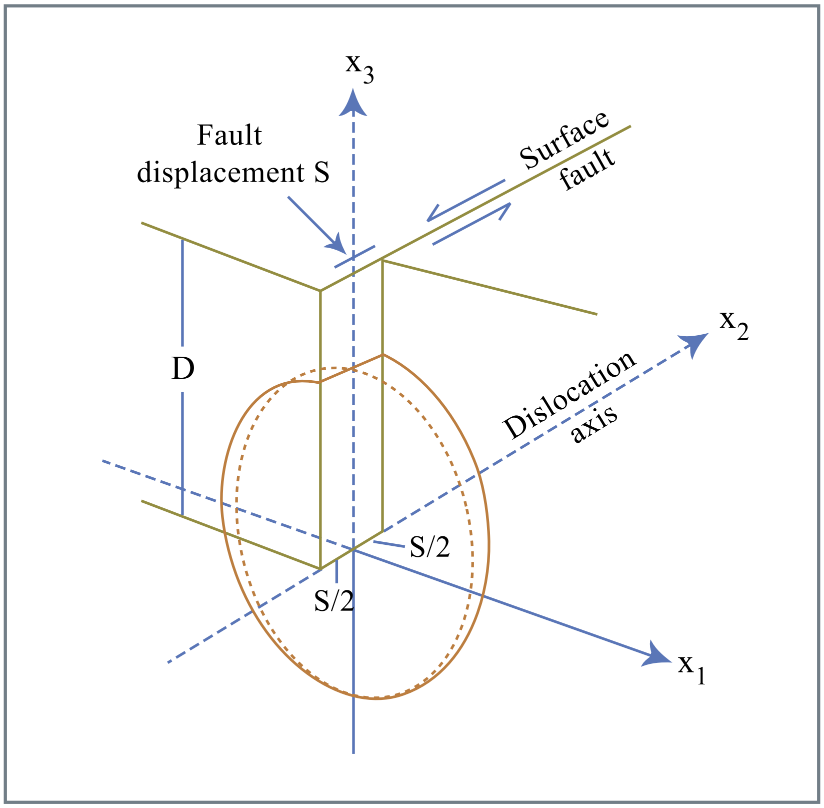

Figure 19.1

Figure by MIT OCW.

Need to get traction = 0 at surface. First consider ∞ medium.

Assume:

$$u_1 = u_3 = 0$$

$$u_2 = \frac{S\theta}{2\pi}$$

$\nabla \cdot u = 0 \;\Rightarrow\;$ no compression, only shear

Symmetry $\Rightarrow$ cylindrical coordinates, r, θ, z

with z parallel to x_2 axis; $\quad u_r = u_\theta = 0$

Solution is $\sigma_{\theta z} = \dfrac{\mu}{r}\dfrac{\partial u_z}{\partial \theta} = \dfrac{\mu S}{2\pi r}$

We can get σ_{ij} by coordinate transformation.

How to get traction on surface?

Trick – image dislocation

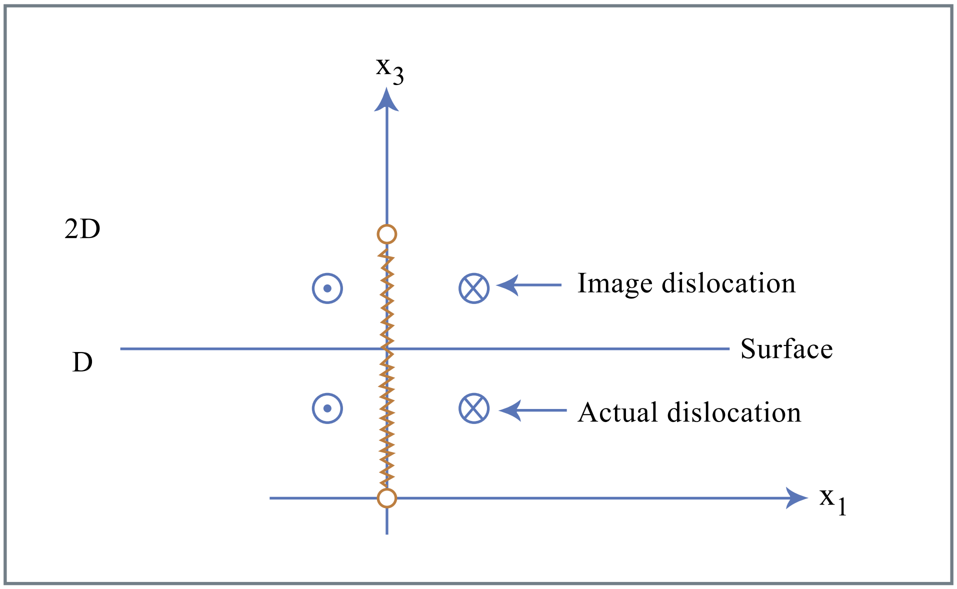

Figure 19.2

Figure by MIT OCW.

Solution for matched image dislocation is whole space gives $\sigma_{i3} = 0$ on surface of ½ space!

Shear strain at surface:

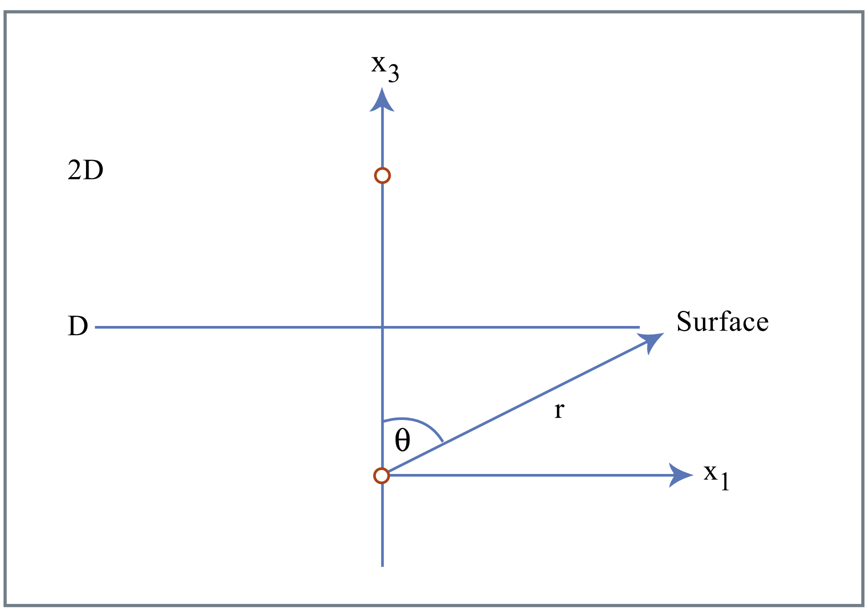

Figure 19.3

Figure by MIT OCW.

$$r^2 = x_1^{\;2} + x_3^{\;2}$$

From each dislocation $\varepsilon_{z\theta} = \dfrac{S}{2\pi r}$

Rotating strain tensor

$$\varepsilon_{12} = \varepsilon_{z\theta}\cos\theta = \varepsilon_{z\theta}\frac{x_3}{r}$$

At surface

$$\varepsilon_{12}^{D} = \frac{S}{2\pi}\left[\frac{x_3}{x_1^{\;2}+x_3^{\;2}} + \frac{2D - x_3}{\left(2D - x_3\right)^2 + x_1^{\;2}}\right] = \frac{SD}{\pi(D^2 + x_1^{\;2})}$$

where

$\dfrac{x_3}{x_1^{\;2}+x_3^{\;2}}$ is actual dislocation

$\dfrac{2D - x_3}{\left(2D - x_3\right)^2 + x_1^{\;2}}$ is image dislocation

Displacement

$$u_2 = \int_{\infty}^{x_1} \varepsilon_{12}^{D}\,dx_1 = \frac{S}{2}\left(1 - \frac{2}{\pi}\tan^{-1}\frac{x_1}{D}\right)$$

Aside – slip discontinuity objectionable?

$\sigma_{12} \to \infty$ along $x_1 = 0$ as $x_3 \to 0$

(stress singularity at tip of fault)

Alternative model

Apply uniform σ_{12}^{0}

Cut $0 \le x_3 \le D$, set $\sigma_{12} = 0$

$$u_2 = \frac{S}{2}\left(\left(1+\frac{x_1^{\,2}}{D^2}\right)^{1/2} - \frac{x_1}{D}\right)$$

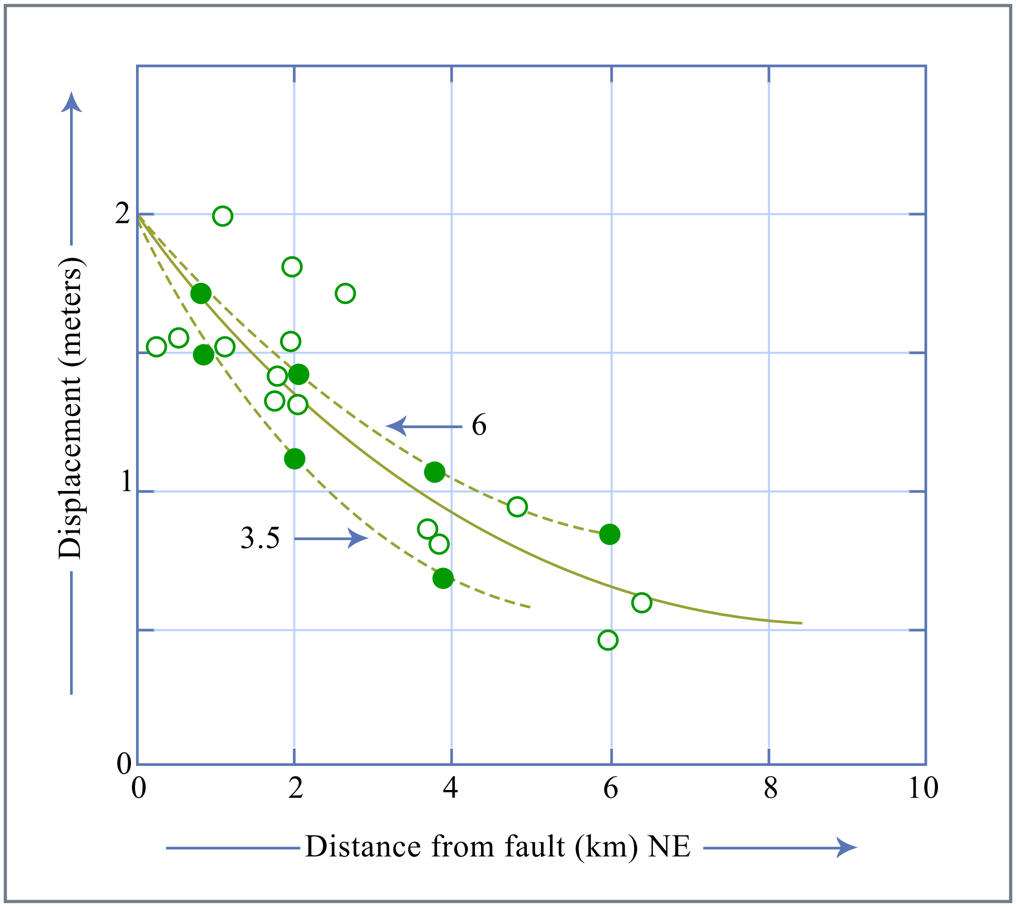

Figure 19.4

Figure by MIT OCW.

Virtually indistinguishable!

St. Venant's principle

Elastostatics – if boundary tractions on a part S_1 of the boundary of S are replaced by statically equivalent traction dist, effects on stress dist are negligible at pts whose distance from S_1 is large compare to size of S.

Usual context – long beam under end load (non-uniform)

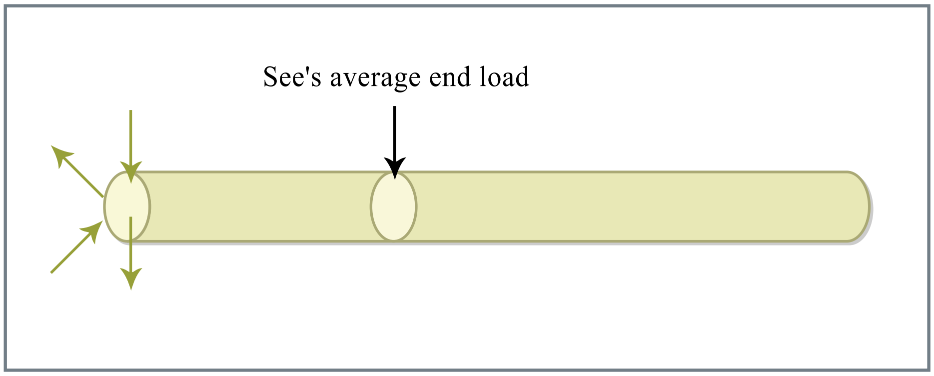

Figure 19.5

Figure by MIT OCW.

Apply to loading 1/2 space

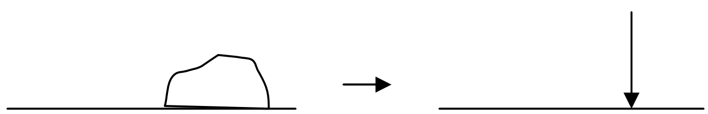

Figure 19.6

Pt source approximates in seismology.