12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

Geochronology

Types of radioactive decay

1. Alpha Decay.
   Ejection of a ^4He nucleus: two protons (p), two neutrons (n)
2. Beta Decay
   Ejection of an electron or a positron
   a. Electron emission (β^- decay): n → p + electron + antineutrino
   b. Positron emission (β^+ decay): p → n + positron + neutrino
   c. Electron capture: p + orbital electron → neutron + neutrino

Derive some basic equations about radioactive decay.

The decay rate of a parent nuclide (N) is proportional to its abundance:

$dN/dt = -\lambda N$

where λ is the decay constant.

$$\int \frac{dN}{N} = \int -\lambda dt$$

$\ln N = -\lambda t + C$
($C = \ln N_0$)

$N(t) = N_0 e^{-\lambda t}$ $\quad N_0 = N(t = 0)$ (1)

But N_0 usually not known by experimentalist.

Concentration of the daughter $D - D_0 = N_0 - N$. Substituting this into (1) gives

$D - D_0 = N_0 - Ne^{-\lambda t} = N_0(1 - e^{-\lambda t})$

Dividing by (1) gives

$(D-D_0)/N = N_0(1-e^{-\lambda t})/N_0 e^{-\lambda t} = (1-e^{-\lambda t})/e^{-\lambda t}$

$(D-D_0)/N = e^{\lambda t}-1$

So $D = D_0 + N(e^{\lambda t} - 1)$

U/Pb Dating:

3 U radioactive isotopes:

$^{238}U \rightarrow {}^{206}Pb$
$^{235}U \rightarrow {}^{207}Pb$
$^{238}Th \rightarrow {}^{208}Pb$

Th dating rarely used.
$\lambda_{238} = 1.55 \times 10^{-10}\ y^{-1}$ half life ($T_{1/2}$) = 4.5 billion years
$\lambda_{235} = 9.85 \times 10^{-10}\ y^{-1}$ $T_{1/2}$ = 0.750 billion years
$\lambda_{232} = 4.95 \times 10^{-11}\ y^{-1}$ $T_{1/2}$ = 14 billion years

^{235}U relatively more abundant than ^{238}U in early solar system due to its higher decay constant.

Lead has four main stable isotopes:

204 (not radiogenic)

206, 207, 208 (radiogenic)

Pb/Pb Dating:

Write decay equation $D = D_0 + N(e^{\lambda t} - 1)$ in terms of the two radioactive U isotopic systems:

$$\frac{^{206}Pb}{^{204}Pb} = \left(\frac{^{206}Pb}{^{204}Pb}\right)_0 + \frac{^{238}U}{^{204}Pb}\left(e^{\lambda 238t} - 1\right)$$

$$\frac{^{207}Pb}{^{204}Pb} = \left(\frac{^{207}Pb}{^{204}Pb}\right)_0 + \frac{^{235}U}{^{204}Pb}\left(e^{\lambda 235t} - 1\right)$$

Divide two equations:

$$\frac{\frac{^{207}Pb}{^{204}Pb} - \left(\frac{^{207}Pb}{^{204}Pb}\right)_0}{\frac{^{206}Pb}{^{204}Pb} - \left(\frac{^{206}Pb}{^{204}Pb}\right)_0} = \frac{^{235}U\left(e^{\lambda 235t} - 1\right)}{^{238}U\left(e^{\lambda 238t} - 1\right)}$$

Multiply both sides by $\frac{^{206}Pb}{^{204}Pb} - \left(\frac{^{206}Pb}{^{204}Pb}\right)_0$:

$$\frac{^{207}Pb}{^{204}Pb} = \frac{^{235}U\left(e^{\lambda 235t} - 1\right)}{^{238}U\left(e^{\lambda 238t} - 1\right)}\frac{^{206}Pb}{^{204}Pb} + \left(\frac{^{207}Pb}{^{204}Pb}\right)_0 - \frac{^{235}U\left(e^{\lambda 235t} - 1\right)}{^{238}U\left(e^{\lambda 238t} - 1\right)}\left(\frac{^{206}Pb}{^{204}Pb}\right)_0$$

Assume $\frac{^{235}U}{^{238}U}$ is constant everywhere in the solar system today ~ 1/137

This is then the equation for a straight line!

$y = mx + b$

ordinate: $y = {}^{207}Pb/{}^{204}Pb$

abscissa: $x = {}^{206}Pb/{}^{204}Pb$

slope: $m = \frac{^{235}U}{^{238}U}\frac{\left(e^{\lambda_{235}t} - 1\right)}{\left(e^{\lambda_{238}t} - 1\right)}$

y-intercept: $b = \left(\frac{^{207}Pb}{^{204}Pb}\right)_0 - \frac{^{235}U}{^{238}U}\frac{\left(e^{\lambda_{235}t} - 1\right)}{\left(e^{\lambda_{238}t} - 1\right)}\left(\frac{^{207}Pb}{^{204}Pb}\right)_0$