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12.002 Physics and Chemistry of the Earth and Terrestrial Planets
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Geochronology

Types of radioactive decay

1. Alpha Decay.

Ejection of a ${}^4\text{He}$ nucleus: two protons (p), two neutrons (n)

2. Beta Decay

Ejection of an electron or a positron

a. Electron emission (β^- decay): $n \rightarrow p + \text{electron} + \text{antineutrino}$

b. Positron emission (β^+ decay): $p \rightarrow n + \text{positron} + \text{neutrino}$

c. Electron capture: $p + \text{orbital electron} \rightarrow \text{neutron} + \text{neutrino}$

Derive some basic equations about radioactive decay.

The decay rate of a parent nuclide (N) is proportional to its abundance:

$$dN/dt = -\lambda N$$

where λ is the decay constant.

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\ln N = -\lambda t + C$$

$$(C = \ln N_0)$$

$$N(t) = N_0 e^{-\lambda t} \quad N_0 = N(t = 0) \tag{1}$$

But N_0 usually not known by experimentalist.

Concentration of the daughter $D - D_0 = N_0 - N$. Substituting this into (1) gives

$$D - D_0 = N_0 - N e^{-\lambda t} = N_0(1 - e^{-\lambda t})$$

Dividing by (1) gives

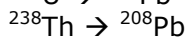
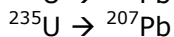
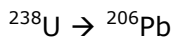
$$(D - D_0)/N = N_0(1 - e^{-\lambda t})/N_0 e^{-\lambda t} = (1 - e^{-\lambda t})/e^{-\lambda t}$$

$$(D - D_0)/N = e^{\lambda t} - 1$$

$$\text{So } D = D_0 + N(e^{\lambda t} - 1)$$

U/Pb Dating:

3 U radioactive isotopes:



Th dating rarely used.

$$\lambda_{238} = 1.55 \times 10^{-10} \text{ y}^{-1}$$

$$\lambda_{235} = 9.85 \times 10^{-10} \text{ y}^{-1}$$

$$\lambda_{232} = 4.95 \times 10^{-11} \text{ y}^{-1}$$

$$\text{half life } (T_{1/2}) = 4.5 \text{ billion years}$$

$$T_{1/2} = 0.750 \text{ billion years}$$

$$T_{1/2} = 14 \text{ billion years}$$

^{235}U relatively more abundant than ^{238}U in early solar system due to its higher decay constant.

Lead has four main stable isotopes:
 ^{204}Pb (not radiogenic)
 $^{206}, ^{207}, ^{208}\text{Pb}$ (radiogenic)

Pb/Pb Dating:

Write decay equation $D = D_0 + N(e^{\lambda t} - 1)$ in terms of the two radioactive U isotopic systems:

$$\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}} = \left(\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}}\right)_0 + \frac{{}^{238}\text{U}}{{}^{204}\text{Pb}} (e^{\lambda_{238}t} - 1)$$

$$\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}} = \left(\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}}\right)_0 + \frac{{}^{235}\text{U}}{{}^{204}\text{Pb}} (e^{\lambda_{235}t} - 1)$$

Divide two equations:

$$\frac{\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}} - \left(\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}}\right)_0}{\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}} - \left(\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}}\right)_0} = \frac{{}^{235}\text{U}(e^{\lambda_{235}t} - 1)}{{}^{238}\text{U}(e^{\lambda_{238}t} - 1)}$$

Multiply both sides by $\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}} - \left(\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}}\right)_0$:

$$\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}} = \frac{{}^{235}\text{U}(e^{\lambda_{235}t} - 1)}{{}^{238}\text{U}(e^{\lambda_{238}t} - 1)} \frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}} + \left(\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}}\right)_0 - \frac{{}^{235}\text{U}(e^{\lambda_{235}t} - 1)}{{}^{238}\text{U}(e^{\lambda_{238}t} - 1)} \left(\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}}\right)_0$$

Assume $\frac{{}^{235}\text{U}}{{}^{238}\text{U}}$ is constant everywhere in the solar system today $\sim 1/137$

This is then the equation for a straight line!

$$y = mx + b$$

ordinate: $y = {}^{207}\text{Pb}/{}^{204}\text{Pb}$

abscissa: $x = {}^{206}\text{Pb}/{}^{204}\text{Pb}$

$$\text{slope: } m = \frac{{}^{235}\text{U} (e^{\lambda_{235}t} - 1)}{{}^{238}\text{U} (e^{\lambda_{238}t} - 1)}$$

$$y\text{-intercept: } b = \left(\frac{{}^{207}\text{Pb}}{{}^{204}\text{Pb}}\right)_0 - \frac{{}^{235}\text{U} (e^{\lambda_{235}t} - 1)}{{}^{238}\text{U} (e^{\lambda_{238}t} - 1)} \left(\frac{{}^{206}\text{Pb}}{{}^{204}\text{Pb}}\right)_0$$