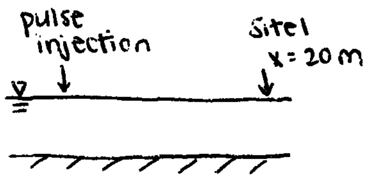
Problem set #3   2-1, 4, 20, 21, 30   Solutions

1.

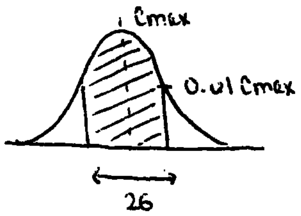


a) use travel time for the center of mass (max. concentration)

$$v = \frac{\text{distance}}{\text{time}} = \frac{20\,m}{750\,s} = \boxed{0.027\,m/s}$$

b) Strategy: use properties of a Gaussian curve (even though this isn't... we're approximating) to find σ_t, then convert to σ_x

for a Gaussian.



shaded area = 68% of total area
(95% for 2σ in each direction)

$C_{max} = 4700 - 300 = 4400\,\mu\Omega$
↑ correct for background salt level

$0.61\,C_{max} = 2684 \approx 2700$

this happens at t = 500s and t = 1100s

$2\sigma_t = 600\,s$

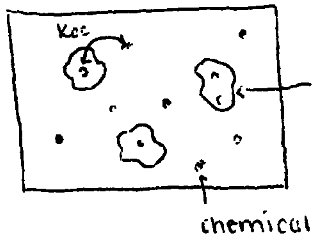
$\sigma_t = 300\,s$

convert to spatial: $300\,s \times 0.027\,\frac{m}{s} = 8.1\,m$

$$D = \frac{\sigma^2}{2t} = \frac{(8.1\,m)^2}{2(750\,s)} = \boxed{0.04\,m^2/s}$$

The time vs. concentration curve is not expected to be exactly Gaussian. There are several possible reasons – most importantly, the plume is still spreading out as it passes the measuring site, so we'd expect the "tailing" seen in the plot. (So even if the concentration vs. distance plot is a perfect Gaussian, the concentration vs. time plot will not be.)

4.



$$K_{oc} \equiv \frac{C \text{ of chemical in org. carbon} \quad (mg/kg)}{C \text{ of chemical in water} \quad (mg/L)} \rightarrow \frac{L}{kg}$$

from lecture: $C_{total} = C_w + \underbrace{(C_w K_d)}_{C_{sed}}(\text{conc. of sediment})$ where C_{phase} refers to conc. of chemical in that phase

in this case, $K_d = K_{oc}$, and conc. of sediment = suspended solid content (TSS, total suspended solids)

$$\text{so } C_{total} = C_w + C_w \cdot K_{oc} \cdot TSS$$

$$= C_w(1 + K_{oc} \cdot TSS)$$

$$= C_w\left(1 + \frac{4000\,L}{kg} \times \frac{20\,mg}{L} \times \frac{kg}{10^6\,mg}\right) = \boxed{1.08\,C_w}$$

20. a) (When the lake becomes anoxic, Fe(III) is reduced to Fe(II), which is soluble. This causes phosphate to be released from the sediment.)

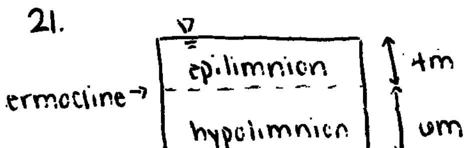
$$y = \sqrt{D \cdot t}$$

$$t = 4\,mo \times \frac{30\,d}{mo} \times \frac{86{,}400\,s}{d} = 1.04 \times 10^7\,s$$

$$y = \sqrt{(10^{-3}\,cm^2/s)(1.04 \times 10^7\,s)} = \boxed{102\,cm}$$

b) The lowest value would be $D \approx 10^{-5}\,cm^2/s$, when molecular diffusion dominates (i.e. very little turbulent diffusion). This would be the case if there was very little mixing due to turbulence, such as when the lake is strongly stratified, so the hypolimnion is not affected by wind mixing very much.

21.



- assume hypolimnion is fairly well mixed
- assume no O_2 flux across thermocline

To draw oxygen profiles, we need to find $[O_2]_{hypo}$ at t = 1 week, 1 month, 3 months.

- how much O_2 do we start with?

volume (of hypolimnion) = A × 6m

$$6m \cdot A \times \frac{1000 L}{m^3} \times \frac{10.8 mg}{L} = 64{,}800 A \frac{mg}{m^2}$$

- depletion in 1 week (168 hr).

$$= \frac{60 mg}{m^2 \cdot hr} \times A \times 168 hr = 10{,}080 A \frac{mg}{m^2}$$

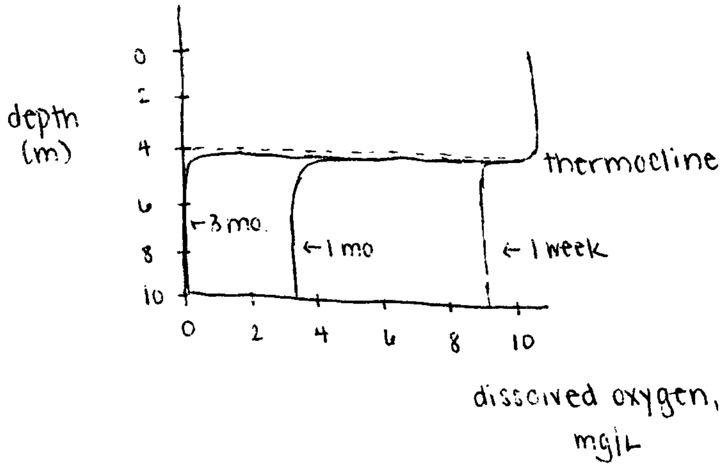
- O_2 remaining = $64{,}800A - 10{,}080A = 54{,}720A \frac{mg}{m^2}$

$$[O_2]_{hypo} = \frac{\text{amt of } O_2}{\text{volume}} = \frac{54{,}720 A\ mg/m^2}{6m \cdot A} = 9120 \frac{mg}{m^3} \times \frac{m^3}{1000 L} = \boxed{9.12 mg/L}$$

repeat calculation.

1 month = 720 hr → 3.6 mg/L

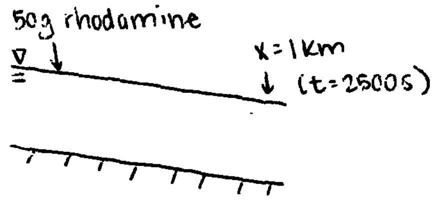
3 months = 2160 hr → 129,600A mg/m² depletion → anoxic
(more than originally present)



Above the thermocline, there is contact with the atmosphere and mixing due to wind, so $[O_2]$ remains at 10.8 mg/L (assuming constant temperature).

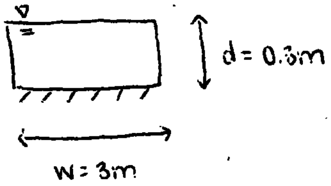
b) Cold-water fish need 7-10 mg/L of dissolved oxygen, so they would not survive in this lake.

30.

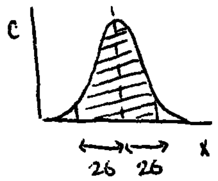


slope = 0.001
n = 0.03
V = 0.4 m/s

cross-section:



a) 95% of mass is within 2σ of C_{max}



(shaded part contains 95% of area under curve)

so σ = 100 m

$$D_L = \frac{\sigma^2}{2t} = \frac{(100\,m)^2}{2(2500\,s)} = \boxed{2\,m^2/s}$$

b) degassing can be described by $C = C_0 e^{-k_r \tau}$

$$V = \frac{1000\,m}{2500\,s} = 0.4\,m/s \qquad \tau = \frac{1500\,m - 200\,m}{0.4\,m/s} = 3250\,s$$

$$\frac{C}{C_0} = e^{-k_r \tau} \qquad \frac{52}{75} = e^{-k_r(3250\,s)} \Rightarrow k_r = 1.13 \times 10^{-4}\,s^{-1}$$

This is k_r for propane, which is being used as a tracer. We are looking for the reaeration coefficient, which is k_r for oxygen (with wastewater treatment plants, it's important to consider how quickly the water can become re-oxygenated).

river → surface renewal model

$$\frac{k_{O_2}}{k_{pro}} = \sqrt[4]{\frac{MW_{pro}}{MW_{O_2}}} = \sqrt[4]{\frac{44}{32}} = 1.08$$

$$k_r(O_2) = 1.08(1.13 \times 10^{-4}\,s^{-1}) = \boxed{1.2 \times 10^{-4}\,s^{-1}}$$

useful molecular weights:
C = 12, H = 1, O = 16
N = 14, S = 32
C_3H_8: 44 g/mol
O_2: 32 g/mol

c) length of transverse mixing zone:

$$L \approx \frac{W^2 V}{2D_t}$$

estimate D_t from river characteristics

$$u^* = \sqrt{gds} = \sqrt{(9.8\,m/s^2)(0.3\,m)(0.001)} = 0.054\,m/s$$

$$D_t \approx 0.15\,d u^* = 0.15(0.3\,m)(0.054\,m/s) = 2.44 \times 10^{-3}\,m^2/s$$

(for straight channel)

$$L = \frac{(3\,m)^2(0.4\,m/s)}{2(2.44 \times 10^{-3}\,m^2/s)} = \boxed{740\,m}$$