## Transit Vehicle Scheduling: Problem Description

## Outline

- Problem Characteristics
- Service Planning Hierarchy (revisited)
- Vehicle Scheduling


## Problem Characteristics

- Consolidated Operations (vs. Direct Operations)
- Passengers (vs freight) being moved
- Urban vs. intercity (short vs. long trip lengths)
- Relatively high service frequency (several trips per hour vs one trip per day)
- High temporal variation in demand within day
- Feasible speeds vary by time of day if vehicles affected by traffic congestion
- Operations/service plan is stable over a period of months
- Different type of competition
- May be a public agency or a private company
- Crew costs are significant fraction of total costs
- Routes have many nodes


## Temporal Variation in Vehicle Requirements and Vehicle Blocks

Vehicles
in service


## Service Planning Hierarchy

## Input

Function
Output


## Service Planning Hierarchy

| Infrequent Decisions | Service and Lost Considerations both significant | Incremental Analysis Methods Dominate |
| :---: | :---: | :---: |
| Network Design |  |  |
| Frequency Setting |  |  |
| Timetable Development |  |  |
| Vehicle Scheduling |  |  |
| Crew Scheduling |  |  |
| Frequent | Cost Considerations | Computer Dominates |
| Decisions | Dominate | Optimization-Based |
|  |  |  |

## Vehicle Scheduling Problem

## Input:

- The timetable: a set of vehicle revenue trips to be operated, each characterized by:
-- starting point and time
-- ending point and time
- Possible layover/recovery arcs between the end of a trip and the start of a (later) trip at the same location
- Possible deadhead arcs connecting:
-- depot to trip starting points
-- trip ending points to depot
-- trip ending points to trips starting at a different point


## Vehicle Scheduling Problem

Observations:

- there are many feasible but unattractive deadhead and layover arcs, generate only plausible non-revenue arcs
- layover time affects service reliability, set minimum layover (recovery) time

Objective:

- define vehicle blocks (sequences of revenue and non-revenue activities for each vehicle) covering all trips so as to:
-- minimize fleet size (i.e. minimize \#crews)
-- minimize non-revenue time (i.e. minimize extra crew time)
Observation:
- these are proxies for cost, but a large portion of cost will depend on crew duties which are unknown at this stage of solution.


## Vehicle Scheduling Problem (continued)

## Constraints:

- Minimum vehicle block length
- Maximum vehicle block length


## Variations:

- Each vehicle restricted to a single line vs. interlining permitted
- Single depot vs multi-depot
- Vehicle fleet size constrained at depot level
- Routes (trips) assigned to specific depot
- Multiple vehicle types


## Example: Single Route AB



Results of earlier planning and scheduling analysis:

|  | AM Peak Period |  |
| :--- | :---: | :---: |
| $(6-9 \mathrm{AM})$ | Base Period <br> (after 9 AM) |  |
| Headways | 20 min | 30 min |
| Scheduled trip time | 40 min | 35 min |
| $(\mathbf{A} \Rightarrow \mathrm{~B}$ or $\mathrm{B} \Rightarrow \mathrm{A})$ | 10 min | 10 min |
| Minimum layover time | 10 |  |
| Dominant direction of travel in $\mathbf{A M}$ is $\mathbf{A} \Rightarrow \mathbf{B}$ |  |  |

## Timetable and Vehicle Block Development

| Depart A | Arrive B |
| :---: | :---: |
| $6: 00$ | $6: 40$ |
| $6: 20$ | $7: 00$ |
| $6: 40$ | $7: 20$ |
| $7: 00$ | $7: 40$ |
| $7: 20$ | $8: 00$ |
| $7: 40$ | $8: 20$ |
| $8: 00$ | $8: 40$ |
| $8: 20$ | $9: 00$ |
| $8: 40$ | $9: 20$ |
| $9: 00$ | $9: 35$ |
| $9: 30$ | $10: 05$ |
| $10: 00$ | $10: 25$ |
| $10: 30$ | $11: 05$ |
| $11: 00$ | $11: 35$ |

## Timetable and Vehicle Block Development

| Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: |
| $6: 00$ | $6: 40$ | $6: 50$ | $7: 30$ |
| $6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
| $6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
| $7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
| $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
| $8: 00$ | $8: 40$ | $8: 50$ | $9: 30$ |
| $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
| $8: 40$ | $9: 20$ |  |  |
| $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
| $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
| $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
| $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
| $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | x-->6:00 | 6:40 | 6:50 | 7:30---> |
|  | 6:20 | 7:00 | 7:10 | 7:50 |
|  | 6:40 | 7:20 | 7:30 | 8:10 |
|  | 7:00 | 7:40 | 7:50 | 8:30 |
|  | 7:20 | 8:00 | 8:10 | 8:50 |
|  | 7:40 | 8:20 | 8:30 | 9:10 |
|  | 8:00 | 8:40 | 8:50 | 9:30 |
|  | 8:20 | 9:00 | 9:15 | 9:50 |
|  | 8:40 | 9:20 |  |  |
|  | 9:00 | 9:35 | 9:45 | 10:20 |
|  | 9:30 | 10:05 | 10:15 | 10:50 |
|  | 10:00 | 10:25 | 10:45 | 11:20 |
|  | 10:30 | 11:05 | 11:15 | 11:50 |
|  | 11:00 | 11:35 | 11:45 | 12:20 |

$x=$ from depot

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | x-->6:00 | 6:40 | 6:50 | 7:30---> |
|  | 6:20 | 7:00 | 7:10 | 7:50 |
|  | 6:40 | 7:20 | 7:30 | 8:10 |
|  | 7:00 | 7:40 | 7:50 | 8:30 |
|  | 7:20 | 8:00 | 8:10 | 8:50 |
| 1 | 7:40 | 8:20 | 8:30 | 9:10 |
|  | 8:00 | 8:40 | 8:50 | 9:30 |
|  | 8:20 | 9:00 | 9:15 | 9:50 |
|  | 8:40 | 9:20 |  |  |
|  | 9:00 | 9:35 | 9:45 | 10:20 |
|  | 9:30 | 10:05 | 10:15 | 10:50 |
|  | 10:00 | 10:25 | 10:45 | 11:20 |
|  | 10:30 | 11:05 | 11:15 | 11:50 |
|  | 11:00 | 11:35 | 11:45 | 12:20 |
| $\mathrm{x}=$ from depot |  |  |  |  |

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{x}-->6: 00$ | $6: 40$ | $6: 50$ | $7: 30-->$ |
|  | $6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
|  | $6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
|  | $7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
|  | $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
|  | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30$ |
|  | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
|  | $8: 40$ | $9: 20$ |  |  |
|  | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
|  | $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
|  | $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
|  | $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
|  | $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

$x=$ from depot

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{x}-\mathrm{-} 6: 00$ | $6: 40$ | $6: 50$ | $7: 30-->$ |
|  | $6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
|  | $6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
|  | $7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
|  | $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
|  | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30$ |
|  | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
|  | $8: 40$ | $9: 20$ |  |  |
|  | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
|  | $10: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
|  | $10: 30$ | $10: 25$ | $10: 45$ | $11: 20$ |
| 1 | $11: 00$ | $11: 05$ | $11: 15$ | $11: 50$ |
|  | $\mathrm{x}=$ from depot |  |  |  |
|  |  |  | $12: 20$ |  |

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{x}-->6: 00$ | $6: 40$ | $6: 50$ | $7: 30--->$ |
| 2 | $\mathrm{x}-->6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
|  | $6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
|  | $7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
|  | $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
| 2 | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30->y$ |
|  | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
|  | $8: 40$ | $9: 20$ |  |  |
|  | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
| 1 | $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
|  | $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
|  | $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
| 1 | $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

$$
x=\text { from depot }
$$

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $x-->6: 00$ | $6: 40$ | $6: 50$ | $7: 30--->$ |
| 2 | $x-->6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
| 3 | $x-->6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
|  | $7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
|  | $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
| 2 | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30-->y$ |
| 3 | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
|  | $8: 40$ | $9: 20$ |  |  |
|  | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
| 1 | $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
| 3 | $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
|  | $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
| 1 | $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

$$
x=\text { from depot }
$$

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $x-->6: 00$ | $6: 40$ | $6: 50$ | $7: 30--->$ |
| 2 | $x-->6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
| 3 | $x-->6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
| 4 | $x-->7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
|  | $7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
| 2 | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30-->y$ |
| 3 | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
| 4 | $8: 40$ | $9: 20->y$ |  |  |
|  | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
| 1 | $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
| 3 | $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
|  | $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
| 1 | $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

$$
x=\text { from depot }
$$

## Timetable and Vehicle Block Development

| Veh \# | Depart A | Arrive B | Depart B | Arrive A |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $x-->6: 00$ | $6: 40$ | $6: 50$ | $7: 30--->$ |
| 2 | $x-->6: 20$ | $7: 00$ | $7: 10$ | $7: 50$ |
| 3 | $x-->6: 40$ | $7: 20$ | $7: 30$ | $8: 10$ |
| 4 | $x-->7: 00$ | $7: 40$ | $7: 50$ | $8: 30$ |
| 5 | $x->7: 20$ | $8: 00$ | $8: 10$ | $8: 50$ |
| 1 | $7: 40$ | $8: 20$ | $8: 30$ | $9: 10$ |
| 2 | $8: 00$ | $8: 40$ | $8: 50$ | $9: 30-->y$ |
| 3 | $8: 20$ | $9: 00$ | $9: 15$ | $9: 50$ |
| 4 | $8: 40$ | $9: 20->y$ |  |  |
| 5 | $9: 00$ | $9: 35$ | $9: 45$ | $10: 20$ |
| 1 | $9: 30$ | $10: 05$ | $10: 15$ | $10: 50$ |
| 3 | $10: 00$ | $10: 25$ | $10: 45$ | $11: 20$ |
| 5 | $10: 30$ | $11: 05$ | $11: 15$ | $11: 50$ |
| 1 | $11: 00$ | $11: 35$ | $11: 45$ | $12: 20$ |

$$
x=\text { from depot }
$$

## Example: Vehicle Blocks

Block 1: Depot - A (6:00) - B (6:50) - A (7:40) - B (8:30) - A (9:30) B (10:15) - A (11:00) - B (11:45) - ...

Block 2: Depot - A (6:20) - B (7:10) - A (8:00) - B (8:50) - Depot

Block 3: Depot - A (6:40) - B (7:30) - A (8:20) - B (9:15) - A (10:00) B (10:45) - . .

Block 4: Depot - A (7:00) - B (7:50) - A (8:40) - Depot

Block 5: Depot - A (7:20) - B (8:10) - A (9:00) - B (9:45) - A (10:30) B (11:15) - ...

## Vehicle Scheduling Model Approaches

Heuristic approaches:

1. Define compatible trips at same terminal $k$ such that trips $i$ and $j$ are compatible iff :

$$
\begin{aligned}
& t_{s j}-t_{\mathrm{e} i}>M_{k} \\
& t_{s j}-t_{\mathrm{e} i}<2 D_{k}
\end{aligned}
$$

where $\quad t_{s j}=$ starting time for trip $j$
$t_{\mathrm{e}_{j}}=$ ending time for trip $\boldsymbol{i}$
$M_{k}=$ minimum recovery/layover time at terminal $\boldsymbol{k}$
$D_{k}=$ deadhead time from terminal $k$ to depot

## Vehicle Scheduling Model Approaches

2. Apply Restricted First-in-First-out rules at each terminal
(a) Order arrivals and departures at the terminal chronologically
(b) Start with (next) earliest arrival at terminal; if none, go to step (e)
(c) Link to earliest compatible trip departure; if none, return vehicle to depot and return to step (b)
(d) Check vehicle block length against constraint: if constraining, return vehicle to depot and return to step (b); otherwise return to step (c) with new trip arrival time
(e) Serve all remaining unlinked departures from depot

## Time-Space Network Representation

## Route $1---------------->$ Route $\mathbf{N}$



## Time-Space Network Representation



## Time-Space Network Representation

Route 1 --------------->> Route N


## Time Space Network Representation Detail


$\left(I_{i j}, u_{i j}, c_{i j}\right) \quad$ (minimum flow, maximum flow, cost per unit of flow)
$\bigcirc$ correspond to revenue trips
..................... deadhead trip to or from the depot or between routes, or layovers between revenue trips on same route

## Minimum Cost Network Flow Formulation

$$
\begin{aligned}
& \operatorname{Minimize} \sum_{(i, j) \in A} c_{i j} x_{i j} \\
& \text { s.t. }
\end{aligned}
$$

