## Problem Set \#4

1.224 Fall 2003

Due: December 5, 2003

## QUESTION 1: Transit Crew Scheduling Case Study

In this case study, you are provided with the vehicle blocks for the Mattapan High Speed Line of the MBTA (Massachusetts Bay Transportation Authority) along with possible relief points in an Excel file. The following is an example of a vehicle block.

| B3 | $6: 40$ | $6: 45$ | $7: 15$ | $7: 45$ | $8: 15$ | $8: 45$ | $9: 13$ | $9: 43$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $6: 57$ | $7: 27$ | $7: 57$ | $8: 27$ | $8: 57$ | $9: 27$ |  |

The first time shown for a vehicle block is the pull-out time from the depot, and the last time is the lay-up time at the depot. Reliefs are only allowed at one terminus. Therefore, a round trip is the smallest scheduling unit. The upper time shown in each column is the departure time at the relief terminal. You may assume that any cut occurs 5 minutes before the corresponding terminal departure time. For example, if a cut is made before the trip departing at $8: 15$, the duty time of the crew being relieved ends at $8: 10$, and the work time of the relieving crew begins at $8: 10$ as well. The work rules are attached in the Appendix. The wage rate is $\$ 25$ for full-time operators and $\$ 30$ for trippers. Work rules must be complied with fully.

In addition to the given work rules, there are the following policies.
P1. Reliefs should not occur between 7-9 AM and 4-6PM.
P2. No piece of work should be shorter than 3 hours. For an 8-hour day, 5-day week duty, no piece should be longer than 5 hours. A tolerance of 6 minutes beyond these requirements is taken as acceptable.
A. For this part assume that no tripper runs are allowed.
(1) On sheet " 8 -hour" in the Excel file provided, you have been given all possible partitions complying with the stated work rules and policies for all vehicle blocks. A set of legal runs for 8 -hour day, 5-day week operators are provided on sheet "Form 8". Calculate the work time, spread time, and cost for each run in Excel.
(2) The trip-run matrix is also provided on sheet "Form 8". Implement the set partition model in AMPL with the provided runs, trips and trip-run matrix.
(3) Solve the problem as an IP with only these full-timer runs.
B. We now permit some tripper runs.
(4) The unique pieces from the block partitions are provided on sheet "Piece8". For your convenience, the time is denoted by a number. Generate all tripper runs - a tripper run is
simply a single piece of work. Calculate the cost for each tripper run and augment the triprun matrix to include tripper runs.
(5) Suppose the transit agency is now allowed to use trippers, but the number of tripper runs cannot exceed $20 \%$ of the number of full-timer runs. Write the corresponding constraint. Implement the model with tripper runs under this constraint. Solve the IP and comment on the difference between this solution and the one to (3).
(6) Consider a different form of tripper constraint which specifies that total tripper work time can be no more than $20 \%$ of the total timetable time. Write this constraint and solve the corresponding IP. Is this constraint preferred from the agency's perspective to the constraint given in (5) above? Explain briefly this result.
(7) Can the transit agency further improve the solution in (6) above if it is allowed to use more tripper runs? Provide some quantitative evidence without re-solving the problem.

## C. Alternative work rules:

(8) Suppose instead of using trippers, the transit agency has the option to use operators working up to 10 hours a day but just 4 days per week. The work rules for 10 -hour operators are also included in the Appendix. In addition, for 10-hour runs only, a piece of work should be no shorter than 4 hours and no longer than 6 hours. Remember: NO trippers are allowed.
(i) On Sheet " 10 -Hour", you are provided with all possible additional partitions for all vehicle blocks except block B8, for which some additional partitions may exist. Possible runs for 8 -hour and 10 -hour operators associated with the pieces from the provided partitions are on Sheet "Form 10". Enumerate any possible additional partitions for block B8 and generate all possible runs associated with the pieces from the additional partitions for both 8 -hour and 10 -hour duties. Please mark your partition(s) and runs generated.
(ii) Generate the trip-run matrix, and implement the set partition model and solve the IP to get the crew schedule for a single day. Is any 10-hour run used? Does it provide much benefit?
(iii) Are there any additional problems in introducing 10-hour day 4-day week runs?

## D. Further Improvement

(9) What is your best solution? How would you estimate the maximum possible further cost reduction beyond your current best solution, assuming that the wage rates do not change?

## QUESTION 2:

In the case study discussed in class, we presented and compared four approaches to solving the maintenance routing and crew pairing problems. These approaches were based on the premise that maintenance routing is strictly a feasibility problem. How would each of these four approaches be impacted if we were to modify the problem to incorporate an objective function in the maintenance routing problem? For example, assume each route string $r$ had a cost $c_{r}$. How would you compare the four approaches now?

## Appendix Work Rules

A. For an 8 -hour day 5 -day week operator
(1) Each operator is guaranteed 8 hours pay per day.
(2) Work hours can be no more than 8 hours 30 minutes and any time over 8 hours is paid at an overtime premium of $50 \%$ of the wage rate.
(3) Spread penalties: If a driver's run requires clocking off at the end of the day more than a specific number of hours after clocking on at the start, a bonus known as a spread penalty is paid. The driver is paid a premium of the basic wage rate for time worked beyond 9 hours after the run begins. No run can have spread time more than 12 hours.

For example, suppose a driver starts working at 6:00 and finishes at 17:30 and the work time is 7.5 hours. The spread time in this case is 11.5 hours. The cost of this run is $8 \times 25+2.5 \times 25=262.5$
(4) No run may have more than two pieces of work.
B. For a 10-hour day 4-day week operator:
(1) Each operator is guaranteed 10 hours pay per day.
(2) Work hours can be no more than 10 hours 30 minutes and any time over 10 hours is paid at an overtime premium of $50 \%$ of the wage rate.
(3) Spread penalties: The driver is paid a premium of the basic wage rate for time worked beyond 11 hours after the run begins. No run can have spread time more than 14 hours.
(4) No run may have more than two pieces of work.
C. For trippers:
(1) Tripper runs can be up to 5 hours per day in a single piece of work.

