

Application Example 17 (Reliability Analysis)

STABILITY OF NATURAL SLOPES

The possible instability of natural slopes is a major geologic hazard in many regions of the world. The degree of stability of a slope may be expressed through the factor of safety F , which is the ratio between a soil resistance measure and the associated applied load. Hence values of F larger than 1 are associated with stability and values smaller than 1 indicate unstable conditions.

For natural slopes with a shallow soil layer (so-called infinite slopes; see Figure 1), the factor of safety is given by (Skempton and DeLory, 1957):

$$F = \frac{c' + [\gamma z - (z - z_w) \gamma_w] \cos^2 \alpha \tan \phi'}{\gamma z \sin \alpha \cos \alpha} \quad (1)$$

Eq. 1 can be simplified to:

$$F = \frac{c'}{\gamma z \sin \alpha \cos \alpha} + \frac{\tan \phi'}{\tan \alpha} - m \left(\frac{\gamma_w \tan \phi'}{\gamma \tan \alpha} \right) \quad (2)$$

where

c' is the cohesion of the soil

ϕ' is the angle of internal friction of the soil

γ is the unit weight of the soil

γ_w is the unit weight of water

α is the inclination of the slope to the horizontal

z is the depth below the ground surface

z_w is the depth of the water table below the ground surface

m is the degree of saturation of the soil, $m = \left[\frac{z_w - z}{z} \right]$

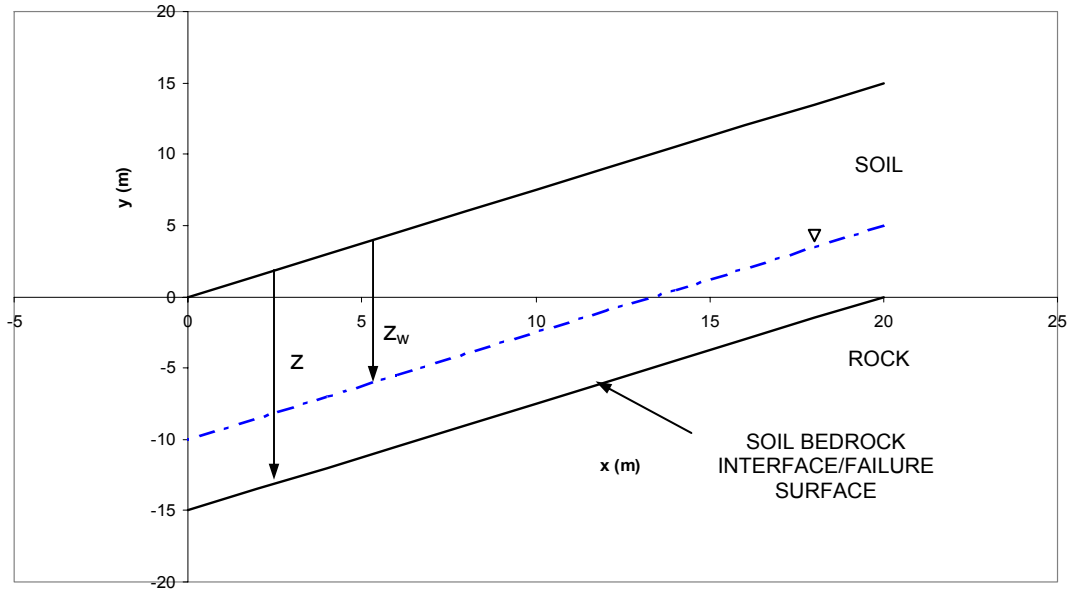


Figure 1. Infinite Slope Geometry and Definition of Parameters

As indicated above, the failure criterion is $F < 1$. For example, a slope with the parameters in Table 1 has $F = 1.15$ and is stable.

z (m)	5
z_w (m)	2.5
m	0.5
c' (kN/m ²)	25
ϕ'	30
α	35
γ (kN/m ²)	20
γ_w (kN/m ²)	9.81

Table 1. Hypothetical slope parameters

However, if the parameters are uncertain, there may be a nonzero probability of slope failure. Of the variables in Table 1, the most uncertain ones are the strength parameters c' and ϕ' , with typical coefficients of variation $V_{c'} = 0.20$ and $V_{\phi'} = 0.25$. Experience shows that they may be assumed to have normal distribution.

In the following, we investigate how uncertainty on c' and ϕ' affects the reliability of infinite slopes. We start by using a first-order second-moment (FOSM) approach and then compare results with a second-moment (SM) analysis and Monte Carlo simulation.

17.1. First-Order Second-Moment (FOSM) Reliability Index

A first approximation to the reliability of slopes is obtained by linearizing the function in Eq. 2 around the mean value of the strength parameters. This gives

$$F(c', \phi') = F(m_{c'}, m_{\phi'}) + \frac{\partial F(c', \phi')}{\partial c'} (c' - m_{c'}) + \frac{\partial F(c', \phi')}{\partial \phi'} (\phi' - m_{\phi'}) \quad (3)$$

where

$$\frac{\partial F(c', \phi')}{\partial c'} = \frac{1}{\gamma z \sin \beta \cos \beta}, \quad \frac{\partial F(c', \phi')}{\partial \phi'} = \left[\frac{\gamma - m_{\gamma_w}}{\gamma \tan \beta \cos^2 \phi'} \right]$$

The first order reliability index is then given by:

$$\beta_{\text{FOSM}} = \frac{E[F(c', \phi')] - 1}{\sqrt{\text{Var}[F(c', \phi')]}} \quad (4)$$

Assuming that the parameters in Table 1 are mean values and that c' and ϕ' are the only random quantities, are independent, and have coefficients of variation $V_{c'} = 0.20$ and $V_{\phi'} = 0.25$, one finds $E[F(c', \phi')] = 1.15$, $\text{Var}[F(c', \phi')] = 0.0467$, and $\beta_{\text{FOSM}} = 0.714$. Under the assumption of normality, the probability of slope failure is $P_f = \Phi(-0.714) = 0.237$, where Φ is the standard normal cumulative distribution function.

17.2. Second-Moment (SM) Reliability Index

A more accurate evaluation of slope reliability is obtained by using the actual nonlinear expression of F in Eq. 2 and calculating the second moment reliability index. Figure 2a shows the failure boundary (obtained by setting $F = 1$) in the space of the variables c' and ϕ' , and the largest dispersion ellipse contained in the safe region. Figure 2b displays the same information in the space of the variables $c^* = \frac{(c' - m_{c'})}{\sigma_{c'}}$ and $\phi^* = \frac{(\phi' - m_{\phi'})}{\sigma_{\phi'}}$, linearly transformed to have zero mean and unit variance. The second moment reliability index β corresponds to the radius of the tangent circle in Figure 2b. Its value $\beta = 0.739$ has been obtained using the numerical procedure described in class. The associated probability of failure is $P_f = \Phi(-\beta) = 0.230$. Due to the small nonlinearity of the function $F(c', \phi')$, these results are close to those from FOSM analysis.

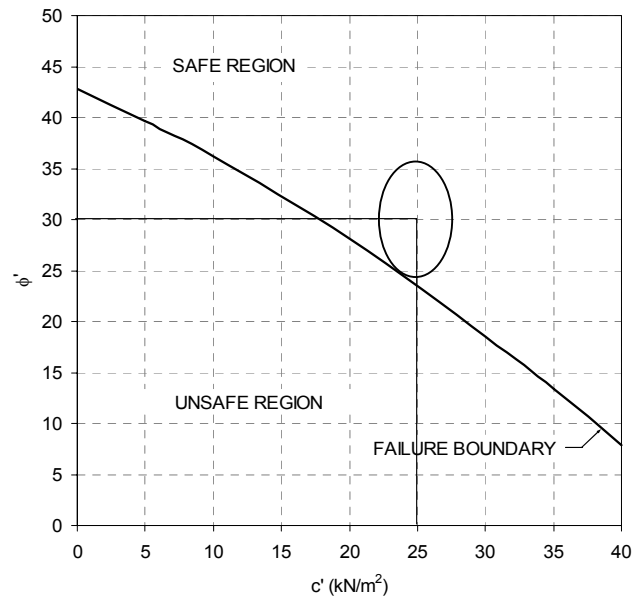


Figure 2a. Failure boundary and critical dispersion ellipse in the space of the original variables

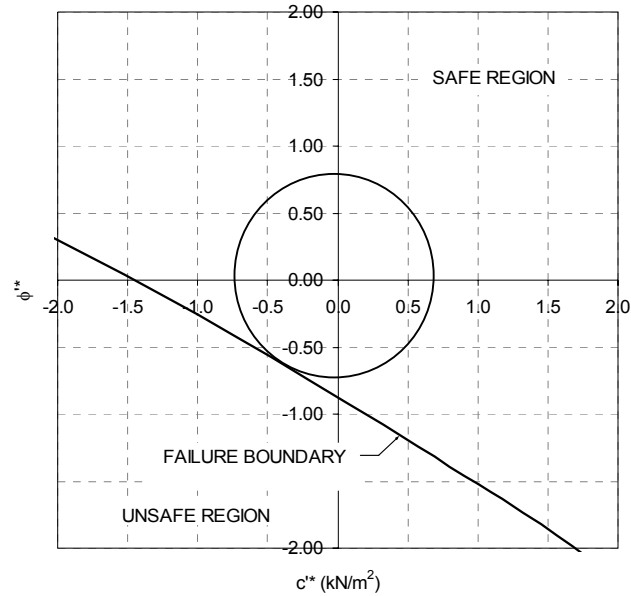


Figure 2b. Failure boundary and critical dispersion disc in the space of the transformed variables

17.3. Monte Carlo Simulation

The exact probability of failure can be obtained by either numerical integration of the joint density of c' and ϕ' over the unsafe region in Figure 2a, or through Monte Carlo simulation. Using the latter approach, we have simulated 20,000 (c', ϕ') pairs and for each pair calculated the value of F using Eq. 2. The empirical probability density function of F is shown in Figure 3. The empirical probability of failure is $P_f = P[F < 1] = 0.223$. Again due to near-linearity of the function $F(c', \phi')$, this probability is close to those obtained through FOSM and SM analysis.

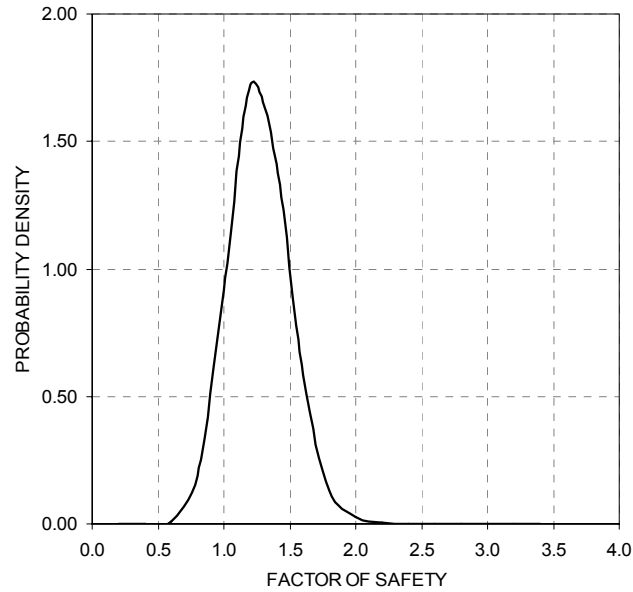


Figure 3. Empirical probability density function of the factor of safety

17.4. Discussion

Suppose that, in a given geological setting, the slope parameters (including c' and ϕ') are similar, but that the slope angle α may vary. It is then of interest to determine how the probability of failure depends on α . By repeating the Monte Carlo analysis for different slope angles α , one obtains the results in Figure 4.

Based on Figure 4, agencies can prioritize where action needs to be taken. For example, one may decide to intervene on slopes with a probability of failure greater than a given value. Intervention may be active, such as providing drainage or tie-backs, or passive, such as building protective galleries. Drainage has the effect of reducing the degree of saturation in the slope, hence reducing the weight of water and increasing stability.

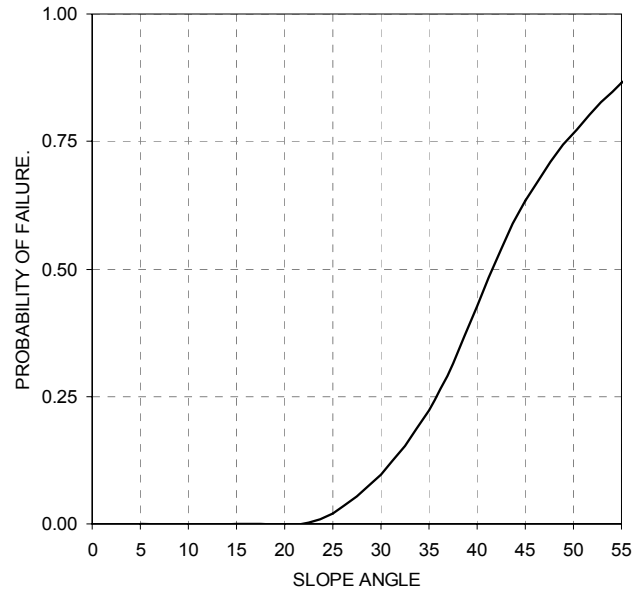


Figure 4. Probability of slope failure as a function of slope angle

Figure 5 shows the probability of failure as a function of degree of saturation, all other parameters having the mean values in Table 1. Plots of this type allow decision makers to determine optimal intervention levels.

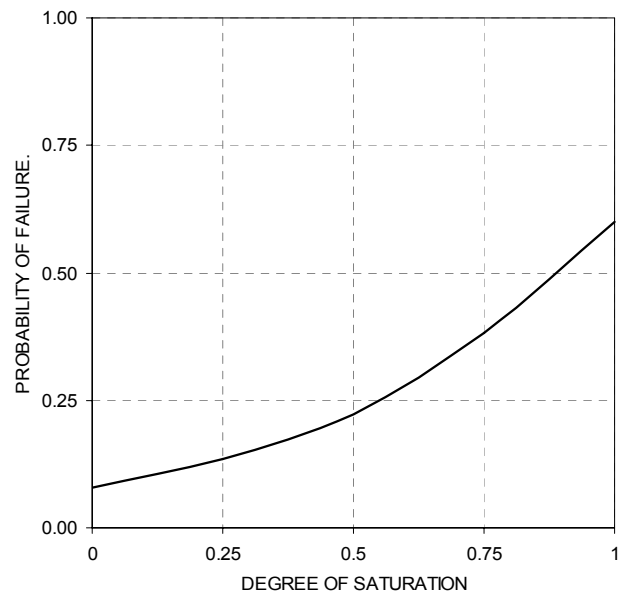


Figure 5. Probability of slope failure against degree of saturation

Problem 17.1

The FOSM analysis that produced $\beta_{\text{FOSM}} = 0.714$ and (under the assumption of normality) $P_f = \Phi(-0.714) = 0.237$ assumed that the strength parameters c' and ϕ' are independent. Now suppose that c' and ϕ' are correlated with correlation coefficient ρ . Find expressions for and plot β_{FOSM} and P_f as a function of ρ . Many soils typically exhibit negative correlation between c' and ϕ' . Comment on the implications of negative correlation on slope stability.

Problem 17.2

Suppose that extensive soil testing shows that c' and ϕ' have lognormal distribution. Assuming that the mean values and coefficients of variation of c' and ϕ' are as in the previous analysis, re-estimate the second-moment reliability index. Start by transforming c' and ϕ' to standard normal variables, and apply the iterative procedure discussed in class to obtain β .