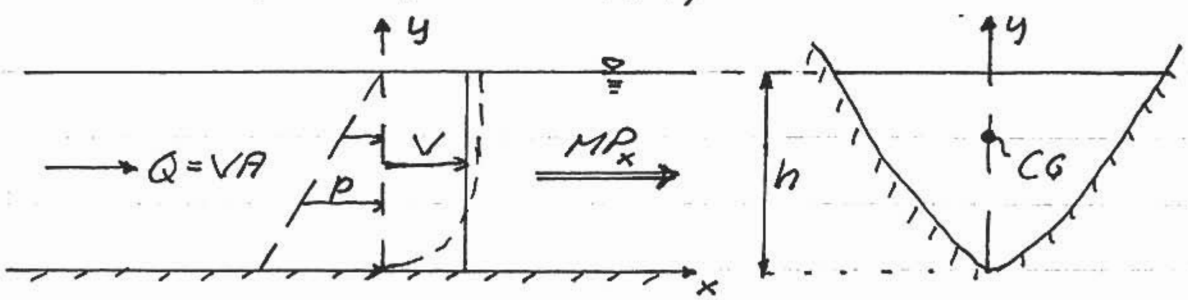
LECTURE # 27

1.060 ENGINEERING MECHANICS II

THE MOMENTUM PRINCIPLE FOR OPEN CHANNELS

For well behaved flow - just as in closed conduits - we have in the direction parallel to the bottom (the x-direction)



$$MP_x = MP = \left(\rho V^2 + p_{CG}\right) A = \rho \frac{Q^2}{A} + \rho g (h - y_{CG}) A \underbrace{\cos\beta}_{\simeq 1}$$

$$\frac{MP}{\rho g} = \frac{Q^2}{gA} + (h - y_{CG}) A$$

For given Q and cross-section :

$MP/\rho g \rightarrow \infty$ as $h \rightarrow 0$: $Q^2/gA \rightarrow \infty$

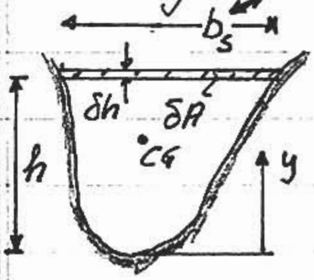
$MP/\rho g \rightarrow \infty$ as $h \rightarrow \infty$: $(h - y_{CG}) A \rightarrow \infty$

Thus, just as we found for the specific energy, E, the Thrust $MP/\rho g$ must have a minimum value for given Q and channel cross-section. This minimum is obtained from

$$\frac{\partial (MP/\rho g)}{\partial h} = 0$$

$$\frac{\partial(MP/\rho g)}{\partial h} = \frac{\partial}{\partial h}\left[\frac{Q^2}{gA} + hA - y_{CG}A\right] =$$

$$-\frac{Q^2}{gA^2}\frac{\partial A}{\partial h} + A + h\underbrace{\left(\frac{\partial A}{\partial h}\right)}_{= b_s} - \frac{\partial(y_{CG}A)}{\partial h}$$



$y_{CG}A$ = moment of A around $y=0$

$\delta(y_{CG}A) = (\delta h \cdot b_s)\cdot h$

$\partial(y_{CG}A)/\partial h = b_s h = h\dfrac{\partial A}{\partial h} = b_s h$

Thus,

$$\frac{\partial}{\partial h}\left(\frac{MP}{\rho g}\right) = -\frac{Q^2 b_s}{gA^2} + A = 0 \quad \text{or}$$

$$\frac{Q^2 b_{sc}}{g A_c^3} = Fr^2 = 1$$

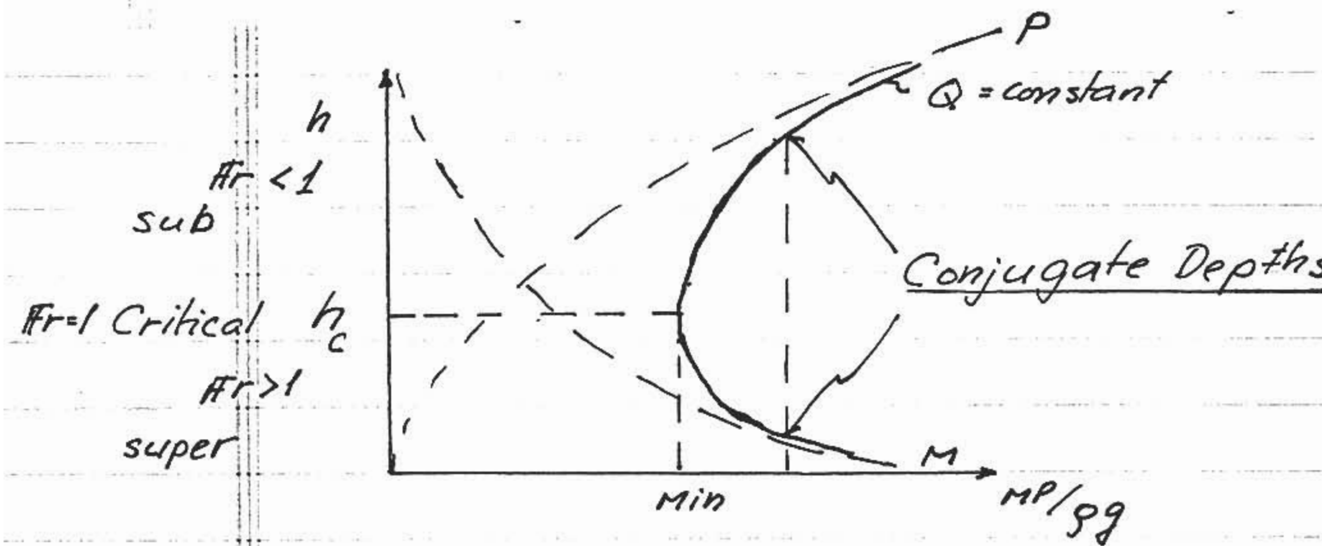
How about that! MP is minimum for a given channel carrying a certain discharge Q when the Froude Number is unity, i.e. when the flow is critical, just as we obtained for the minimum of specific energy, E.

The corresponding minimum value is given by

$$\left(\frac{MP}{\rho g}\right)_{min} = \left[\frac{Q^2}{gA_c^2} + (h_c - y_{CG,c})\right]A_c = \left(\frac{A_c}{b_{sc}} + (h_c - y_{CG,c})\right)A_c =$$

$$(h_{mc} + h_c - y_{CG,c})A_c .$$

which, for a rectangular channel ($h_{mc} = h_c$, $y_{CG,c} = \frac{1}{2}h_c$, $A_c = bh_c$) is $[(MP/b)/\rho g]_{min} = (3/2)h_c^2$



Momentum-Depth Diagram

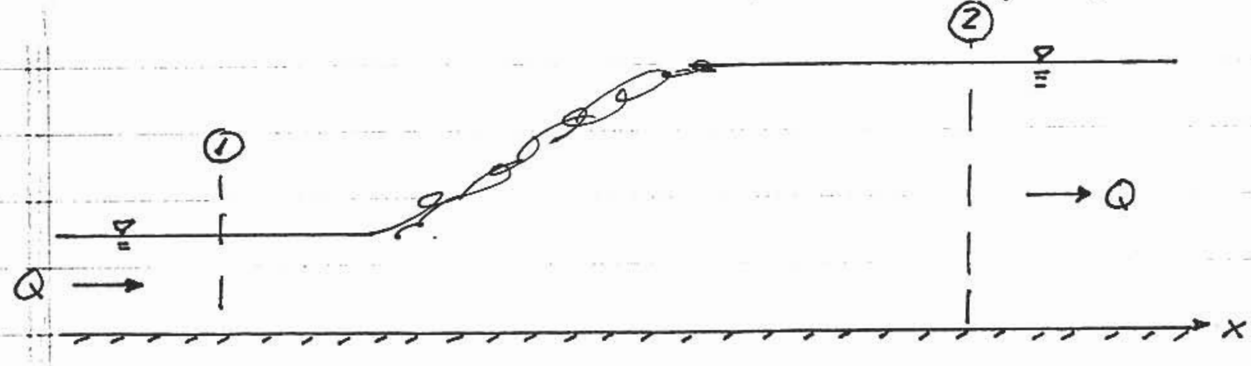
Thus, analogous to the E vs h diagram we have

<u>If $MP > MP_{min}$</u> there are <u>two solutions</u> for the depth. One corresponds to supercritical flow, the other to sub-critical flow. These depths that correspond to the same value of MP (for given channel geometry and discharge Q) are <u>CONJUGATE DEPTHS</u>

<u>If $MP = MP_{min}$</u> there is only <u>one solution</u> corresponding to <u>CRITICAL DEPTH</u>

<u>If $MP < MP_{min}$</u> there is <u>no solution</u>. The specified combination of MP and Q is <u>physically impossible</u> in the given channel.

The Unassisted (Free) Hydraulic Jump



$$\underset{\rightarrow}{MP_1} + \underset{\rightarrow}{\text{Gravity in } x} = \underset{\leftarrow}{MP_2} + \underset{\leftarrow}{\text{Friction force from bottom}} + \underset{\leftarrow}{\text{Other Forces in } x}$$

1) Channel must be prismatic and have a plane bottom. If this were not the case, there would be pressure forces from the sidewalls and the bottom that would not be $\perp x$, i.e. there would be a contribution of unknown magnitude from "Other Forces in x".

2) If the plane bottom slope β is small the gravity component $g_x = g \sin\beta$ would be small. Furthermore, if channel is sloping in the direction of the flow, $\beta > 0$, the Friction Force from the bottom would counteract the gravity force, so that the net result would be smaller than either of the two contributions.

3) The transition from ① to ② is "short"

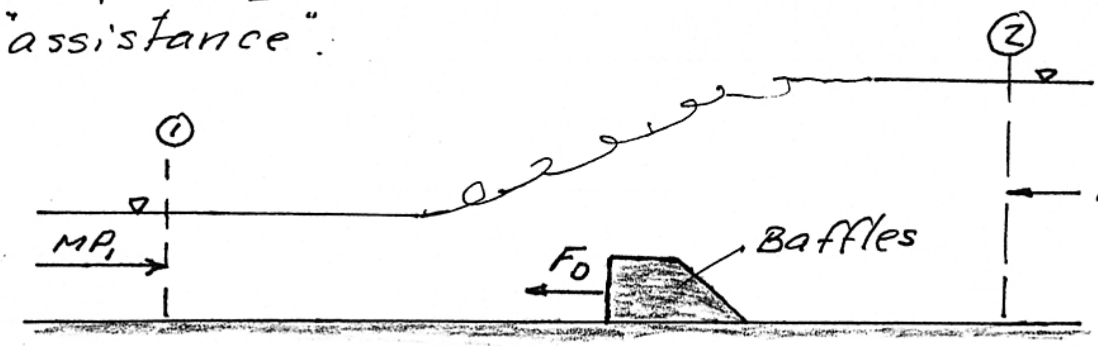
The result is that

$MP_1 = MP_2 \Rightarrow h_1$ & h_2 are CONJUGATE DEPTHS

which is known a the condition for a Free (or Unassisted) Hydraulic jump across which the flow changes from supercritical (h_1) to subcritical (h_2) [or vice versa - but this turns out to violate energy conservation]

The Assisted Hydraulic Jump

Keeping the channel prismatic of negligible slope and friction (gravity + friction ≈ 0) there may be cases, e.g. in the Stilling Basin downstream of a dam, when the balance of $MP_1 = MP_2$ can not be achieved without some "assistance".



An example of this assistance is baffles, i.e. flow obstructions that produce a drag force on the fluid, that helps to achieve the necessary force equilibrium for the jump:

$$MP_1 = MP_2 + F_D$$

Since F_D would vary depending on the position of the baffles within the jump [large approach velocity = large F_D if close to start of jump, low velocity = small F_D if towards end of jump] the baffles can, within a range, adjust F_D so that the jump takes place across the baffles, i.e. we may control the location of the hydraulic jump; for a range of flow conditions.