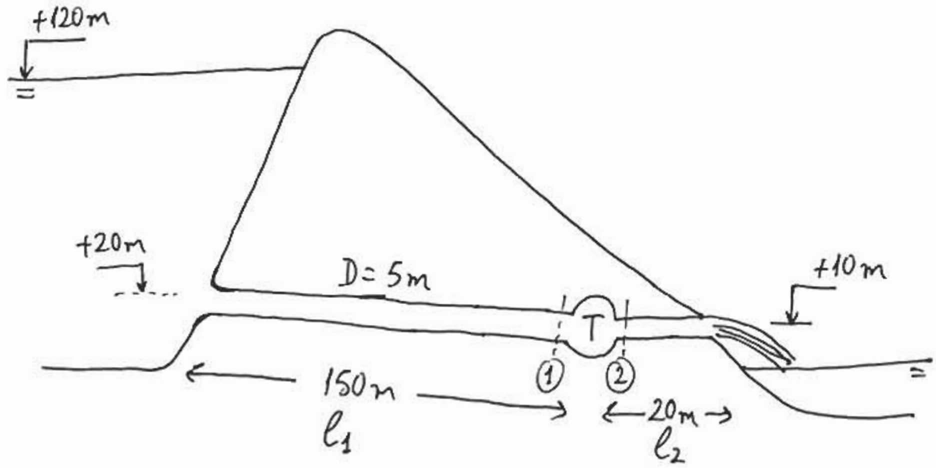
PROBLEM SET 6 - SOLUTIONS

- PROBLEM Nº 1 :



a) Inlet from reservoir has rounded edges, so negligible entrance loss. Applying Bernoulli between the inlet and ①:

$$H_{inlet} = H_1 + \Delta H f_{i\to 1} \Rightarrow H_1 = H_{inlet} - \Delta H f_{i\to 1}$$

Since hydrostatic pressure in the reservoir,

$\frac{p_r}{\rho g} + z_r = 120m$ if p in gauge and $z=0$ at elevation +0. Besides, $V_r = 0$. Thus

$$\underline{\underline{H_1}} = H_{inlet} - f\frac{l_1}{D}\frac{V_1^2}{2g} = H_{inlet} - f\frac{l_1}{D}\frac{\left(\frac{Q}{\pi D^2/4}\right)^2}{2g} =$$

$$= 120 - 0'02 \cdot \frac{150}{5} \cdot \frac{\left(\frac{Q}{\pi \cdot 5^2/4}\right)^2}{2\cdot 9'8} = \underline{\underline{120 - 7'94\cdot 10^{-5} Q^2}} \text{ (S.I.)}$$

b) At the free downstream outflow, the head is

$H_{outlet} = V^2/2g + p_0/\rho g + z_{outlet}$, with $p_0 = 0$ (gauge), so

$$\underline{\underline{H_2}} = H_{outlet} + \Delta H f_{2\to 0} = \frac{\left(\frac{Q}{\pi D^2/4}\right)^2}{2g} + z_{outlet} + f\frac{l_2}{D}\frac{\left(\frac{Q}{\pi D^2/4}\right)^2}{2g} =$$

$$= \underline{\underline{\mathbf{10m + 1.4x10^{-4}Q^2}}} \text{ (S.I.)}$$

(Notice: Tailwater lake level is completely irrelevant)

c) $P = 1'2 \cdot 10^8 = \rho g Q (H_1 - H_2) \eta$

[P is useful electric power generated; $\rho g Q(H_1-H_2)$ is the amount of useful energy entering the turbines, part of which is ending up wasted]. So,

$$Q = \frac{P}{\rho g (H_1 - H_2)\eta} = \frac{1'2 \cdot 10^8}{\rho g (120 - 7'94 \cdot 10^{-5} Q^2 - 10 - \mathbf{1.4x10^{-4}} Q^2) \cdot 0'9} =$$

$$= \frac{123'7}{1 - 8'18 \cdot 10^{-7} Q^2} \quad (\text{S.I.})$$

Solve by first guessing $Q = 123'7$ m³/s, get $1 - 8'18 \cdot 10^{-7} Q^2 = 0'987 \Rightarrow$

$\Rightarrow Q = 123'7 / 0'987 = 125'3$ m³/s $\Rightarrow 1 - 8'18 \cdot 10^{-7} Q^2 = 0'987$, done.

Discharge is $Q =$ **127.0** m³/s.

d)

Rate of energy loss by friction in pipes		Rate of energy loss by power production		Rate of production of internal energy
$\rho g Q \Delta H$	$+$	$\rho g Q (H_1 - H_2)(1-\eta)$	$=$	$\rho Q C_p \Delta T$

$$V = \frac{Q}{\pi D^2/4} = \frac{4 \cdot 125'3}{\pi \cdot 5^2} = 6'38 \text{ m/s}$$

$$\Delta H_{f_{i \to 1}} = f \frac{l_1}{D} \frac{V^2}{2g} = 0'02 \; \frac{150}{5} \; \frac{6'38^2}{2 \cdot 9'8} = 1'25 \text{ m}$$

$$\Delta H_{f_{2 \to 0}} = 0'02 \cdot \frac{20}{5} \cdot \frac{6'38^2}{2 \cdot 9'8} = 0'17 \text{ m}$$

$H_1 = 120 - 1'25 = 118'75$ m, $H_2 = 10 + 0'17 = 10'17$ m

$$1000 \cdot 9'8 \cdot 125'3 \cdot (1'25 + 0'17) + 1000 \cdot 9'8 \cdot 125'3 \cdot (118'75 - 10'17) \cdot 0'1 = 1000 \cdot 125'3 \cdot 4210 \cdot \Delta T \Rightarrow$$

$$\Rightarrow \Delta T = \frac{1'7 \cdot 10^6 + 13'3 \cdot 10^6}{5'28 \cdot 10^8} = 0'028 \text{ K} = 0'028 \,°\text{C} \Rightarrow T_{out} = 12'028 \,°\text{C}$$

(Very small change)

Assumption: All energy lost is carried by Q as internal energy.

- PROBLEM N°2:

a) Apply Bernoulli between A and B

$$H_A + \sum H_p = H_B + \sum \Delta H_{losses}$$

Therefore, the total pump head we need is

$$\sum H_p = (H_B - H_A) + \sum \Delta H_{losses}$$

where

$$\sum \Delta H_{losses} = \Delta H_{m_A} + \Delta H_{m_B} + \Delta H_{f\, A \to B}$$

$$V = \frac{Q}{(\pi D^2/4)} = \frac{0'5}{(\pi \cdot 0'8^2/4)} = 0'995 \text{ m/s}$$

$$\left.\begin{array}{l} Re = \frac{VD}{\nu} = 0'995 \cdot 0'8 \cdot 10^6 = 8'0 \cdot 10^5 \\ \varepsilon/D = 2 \cdot 10^{-3}/0'8 \quad = 2'5 \cdot 10^{-3} \end{array}\right\} \xrightarrow{\text{MOODY}} f = 0'025$$

$$\Delta H_{m_A} = K_{LA} \frac{V^2}{2g} = 0'8 \cdot \frac{0'995^2}{2 \cdot 9'8} = 0'04 \text{ m}$$

$$\Delta H_{m_B} = K_{LB} \frac{V^2}{2g} = 1 \cdot \frac{0'995^2}{2 \cdot 9'8} = 0'05 \text{ m}$$

$$\Delta H_f = f \frac{L}{D} \frac{V^2}{2g} = 0'025 \cdot \frac{5000}{0'8} \cdot \frac{0'995^2}{2 \cdot 9'8} = 7'89 \text{ m}$$

Thus

$$\sum H_p = (100 - 0) + (0'04 + 0'05 + 7'89) = 107'98 \text{ m}$$

Maximum H_p in a single pump:

$$H_{p,max} = H_{out,max} - H_{in,min}$$

$$H_{out,max} = z_{pump} + \frac{p_{out,max}}{\rho g} + \frac{V^2}{2g} = z_p + 100 + \frac{0'995^2}{2 \cdot 9'8} =$$
$$= z_p + 100'05 \text{ m}$$

$$H_{in,min} = z_p + \left(\frac{p_{in,min}}{\rho g} + \frac{V^2}{2g}\right) = z_p + \left(NPSH_{req} + \frac{p_{vapor,gauge}}{\rho g}\right) =$$
$$= z_p + 6 + \frac{(2300 - 101300)}{9800} = z_p - 4'1 \text{ m}$$

$$H_{p,max} = (z_p + 100'05) - (z_p - 4'1) = 104'15 \text{ m}$$

Since $H_{p,max} = 104'15 < 107'98 = \sum H_p$, we need two pumps, each of $H_p = \frac{107'98}{2} \approx 54 \text{ m}$

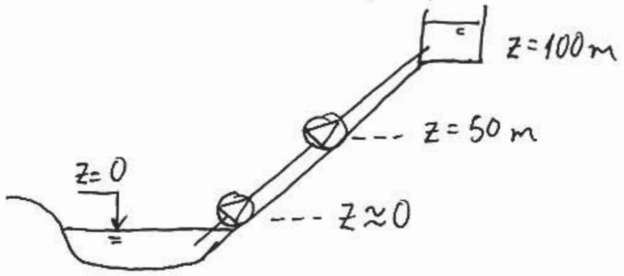
There are many possible locations for the pumps that satisfy the pressure constraints,

$$p_{min} < p < p_{max}$$

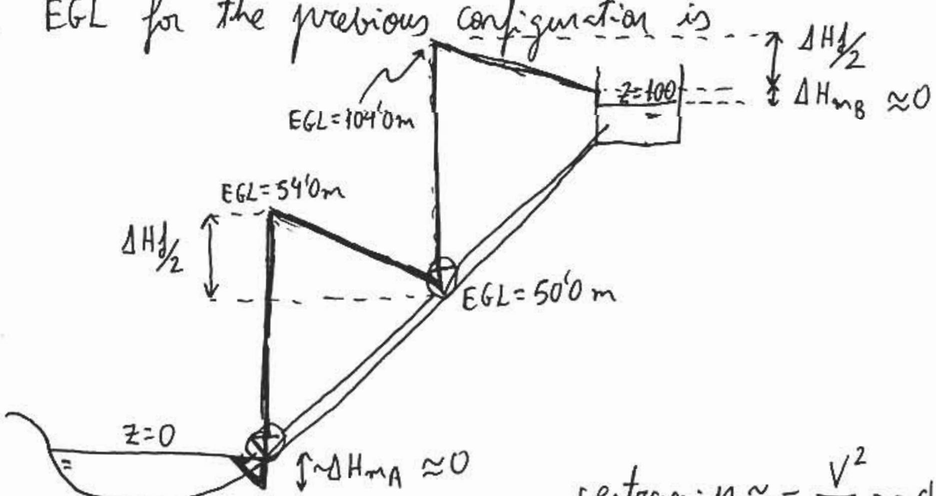
$$p_{min} = H_{in,min} - z_p - \frac{V^2}{2g} = -4'1 - \frac{0'995^2}{2 \cdot 9'8} = -4'15 \text{ mwc}$$

$$p_{max} = 100 \text{ mwc}$$

One possibility is to place the first pump next to the river and the second pump half way to the reservoir:



b) The EGL for the previous configuration is



Check conditions:

PUMP 1 $\begin{cases} \text{entrance: } p \approx -\frac{V^2}{2g} = -0'05 \text{ mwc} \checkmark \\ \text{exit: } p = 54 - 0'05 = 53'95 \text{ mwc} \checkmark \end{cases}$

PUMP 2 $\begin{cases} \text{entrance: } p = 50 - 0'05 - 50 = -0'05 \text{ mwc} \checkmark \\ \text{exit: } p = 104 - 0'05 - 50 = 53'95 \text{ mwc} \checkmark \end{cases}$

c) The total BHP of both pumps is

$$BHP = \frac{\rho g Q \Sigma H_p}{\eta} = \frac{9800 \cdot 0'5 \cdot (2.54)}{0'8} = 6'62 \cdot 10^5 \ W = 662 \ kW$$

Cost per hour $= 662 \ \frac{kWh}{h} \cdot 0'1 \ \frac{\$}{kWh} = 66'2 \ \$/hour$

Volume lifted per hour $= 0'5 \ m^3/s \cdot 3600 \ s/h = 1800 \ m^3$

Therefore, it costs $\underline{\underline{37 \ ¢/m^3}}$ or $0'14 \ ¢/gallon$.

That is, it costs only 1 cent to lift 7 gallons of water a height of 100 m and a distance of 5000 m!

- PROBLEM N°3 :

a) Bernoulli from A to G:

$$H_A + H_p = H_G + \sum \Delta H_m + \Delta H_f$$

$$Q = 0'1\ m^3/s \Rightarrow V = \frac{Q}{\pi D^2/4} = 1'41\ m/s \Rightarrow \frac{V^2}{2g} = 0'102\,m$$

$$\left.\begin{aligned} Re &= \frac{VD}{\nu} = 4'2\cdot 10^5 \\ \varepsilon/D &= 10^{-3} \end{aligned}\right\} \xrightarrow{\text{MOODY}} f = 0'0205$$

$$\underline{\underline{H_p}} = H_G - H_A + \sum \Delta H_m + \Delta H_f =$$

$$= 25 - 5 + \Big\{ \underbrace{(0'8 + 1'5 + 1'5 + 1'5 + 1)}_{K_{LA}\ K_{LC}\ K_{LE}\ K_{LF}\ K_{LG}} + \underbrace{0'0205}_{f} \cdot \underbrace{\frac{100+600}{0'3}}_{\frac{L_{AB}+L_{BG}}{D}} \Big\} \cdot \underbrace{0'102}_{\frac{V^2}{2g}} = \underline{\underline{25'5\,m}}$$

b) Bernoulli from A to the pump inflow:

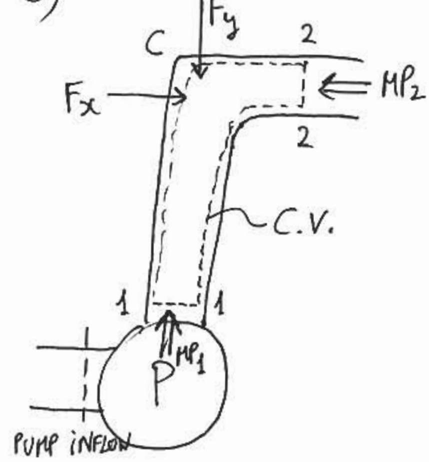
$$H_A = 5 = H_{in} + \Delta H_m + \Delta H_f = H_{in} + \left\{ 0'8 + 0'0205 \cdot \frac{100}{0'3} \right\} \cdot 0'102 \Rightarrow$$

$$\Rightarrow H_{in} = 4'22\,m$$

$$H_{in} = z_{in} + \frac{p_{in}}{\rho g} + \frac{V^2}{2g} = 0 + \frac{p_{in}}{\rho g} + 0'102 = 4'22 \Rightarrow$$

$\Rightarrow p_{in} = 40'2\ kPa$ (gauge) $> 0 \Rightarrow$ NO CAVITATION, regardless of the NPSH of the pump, since $p_{in} > p_{ATM}$.

c)



Fx, Fy: Forces on fluid.

Note: Flow is well behaved at 1-1 & 2-2.

Bernoulli between pump inflow and 1:

$$H_{in} + H_p = H_1 \Rightarrow H_1 = 4'22 + 25'5 = 29'7\,m$$

$$H_1 = \cancel{z_1}^{0} + \frac{p_1}{\rho g} + \frac{V^2}{2g} \Rightarrow p_1 = 2'90 \cdot 10^5\ Pa$$

Bernoulli between pump inflow and 2:

$H_{in} + H_p = H_2 + \Delta H_{m_C}$ (ΔH_f negligible because short distance)

$$4'22 + 25'5 = H_2 + 1'5 \cdot 0'102 \Rightarrow H_2 = 29'6\,m \Rightarrow p_2 = 2'89 \cdot 10^5\,Pa$$

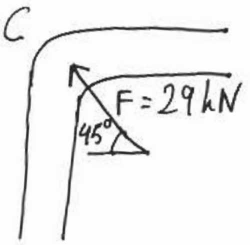
Conservation of momentum on the C.V.:

$$\left.\begin{aligned} F_x &= MP_2 = (\rho V^2 + p_2)\frac{\pi D^2}{4} = 20'6\,kN \\ F_y &= MP_1 = (\rho V^2 + p_1)\frac{\pi D^2}{4} = 20'6\,kN \end{aligned}\right\} \text{forces on the fluid}$$

The total force on the elbow is

$$F = \sqrt{20'6^2 + 20'6^2} = 20'6 \cdot \sqrt{2} \simeq 29\,kN$$

acting on the opposite direction than the force on the fluid, i.e.:



Force from the fluid on the elbow.

d)

(i) H_A, H_G and the headlosses (minor losses & friction) between A and G would be the same as in (a), so to provide the flowrate $Q = 0'1\, m^3/s$, <u>H_P would be the same</u>

(ii) The pressure at the inflow to the pump would be smaller, because the headloss due to the elbow would happen now before the pump. The pump would be then <u>more susceptible</u> to cavitation. However, the difference in pressure would be small, and the pump won't experience cavitation anyway (it was very far from having cavitation problems to start with).

(iii) The forces on the elbow would be <u>much smaller</u> now. The large force on the elbow in (c) was due to the big pressures at 1 and 2, consequence of the increase in head due to the pump. If we placed the pump after the elbow, the pressure in the elbow would be much smaller and the forces would significantly decrease.

(iv) The only relevant difference is the much smaller force on the elbow in the new configuration. This is desirable, because supporting a larger force on the elbow would imply more expensive anchoring systems. Therefore, <u>it is better to place the pump after the elbow.</u>

- PROBLEM Nº 4:

a) Bernoulli between (1) and (2):

$$H_1 + H_p = H_2 + \sum \Delta H_m + \Delta H_f \Rightarrow H_p = (H_2 - H_1) + \sum \Delta H_m + \Delta H_f$$

$$H_2 - H_1 = 3\,m$$

$$\sum \Delta H_m = (\underset{\uparrow \atop \text{INFLOW}}{0'8} + \underset{\uparrow \atop \text{ELBOW}}{0'3} + \underset{\uparrow \atop \text{OUTFLOW}}{1})\frac{V^2}{2g} = 2'1 \cdot \frac{Q^2}{2g\left(\frac{\pi \cdot 0'15^2}{4}\right)^2} = 343\,Q^2$$

$$\Delta H_f = f \frac{L}{D} \frac{V^2}{2g} = 0'02 \cdot \frac{50}{0'15} \cdot \frac{Q^2}{2g\left(\frac{\pi \cdot 0'15^2}{4}\right)^2} = 1089\,Q^2$$

$$\varepsilon/D = 10^{-3} \Rightarrow f = 0'020 \quad (\text{R.T. flow})$$

$$\underline{\underline{H_p = 3 + 1432\,Q^2 \quad (\text{S.I.})}} \qquad (\underline{\text{PARABOLA}})$$

Change to H_p in ft and Q in $\frac{gal}{min}$

$$H_p(ft) \cdot 0'305 \frac{m}{ft} = 3\,m + 1432 \frac{m}{(m^3/s)^2}\left(Q\left(\frac{gal}{min}\right) \cdot \frac{3'875 \cdot 10^{-3}\, m^3/gal}{60\ s/min}\right)^2$$

$$H_p(ft) = 9'84 + 1'96 \cdot 10^{-5}\left[Q\left(\frac{gal}{min}\right)\right]^2$$

b) The previous relationship is represented on the graph on next page ("NEW PIPE"). From the graph, the point N is the point where the pump works and

$$\underline{\underline{Q}} \approx 1650\ gal/min \approx \underline{\underline{0'11\ m^3/s}}$$

$$\underline{\underline{H_p}} \approx 63\ ft \approx \underline{\underline{19\ m}}$$

$$\eta \approx 0'87$$

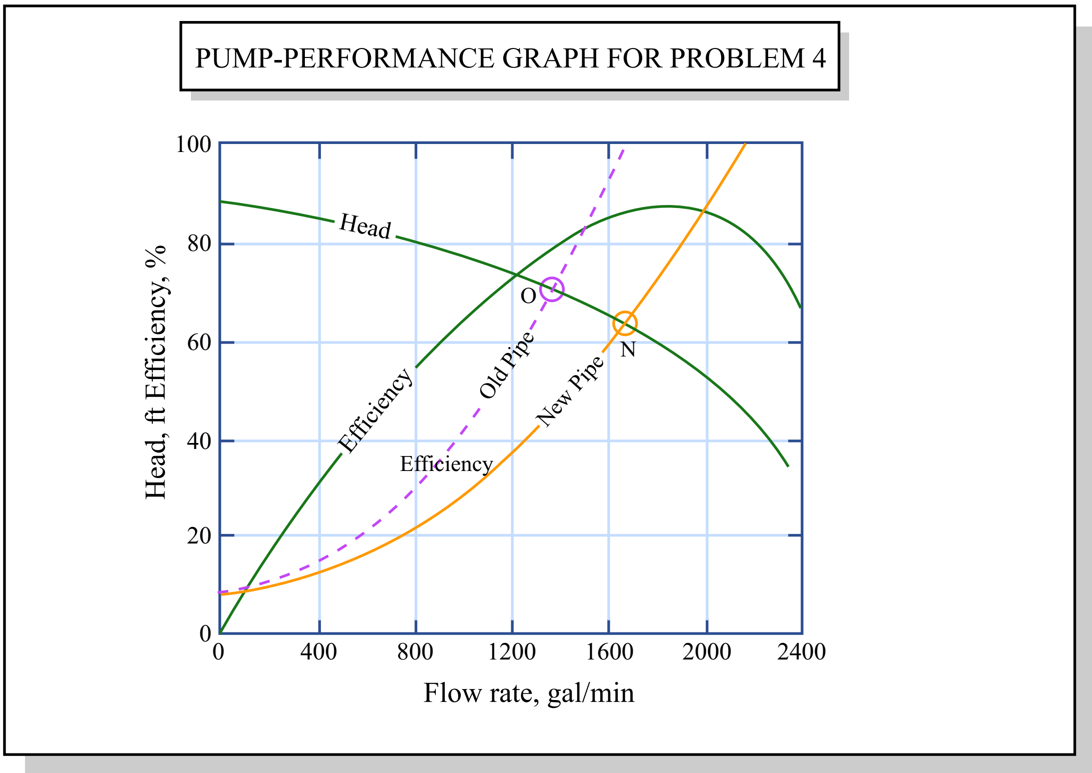


Figure by MIT OCW.

Adapted from:
Figure E11.3 in Young, Donald F., Bruce R. Munson, and Theodore H. Okiishi.
A Brief Introduction to Fluid Mechanics. 2nd ed. New York, NY: John Wiley & Sons, Inc., 2001, pp. 461.

c) $\underline{\underline{BHP}} = \dfrac{\rho g Q H_p}{\eta} = \dfrac{1000 \cdot 9{,}8 \cdot 0{,}11 \cdot 19}{0{,}87} = \underline{\underline{23{,}5\ kW = 31{,}6\ HP}}$

d) $\varepsilon_{OLD}/D = 10^{-2} \Rightarrow f = 0{,}038\ (R.T.) \Rightarrow \Delta H_f = 2069\, Q^2$

$H_p = 3 + 2412\, Q^2\ (s.i.),\quad H_p(ft) = 9{,}84 + 3{,}30 \cdot 10^{-5}\left[Q\left(gal/min\right)\right]^2$

This relationship corresponds to the curve "OLD PIPE" plotted above.

The system works at point O, where:

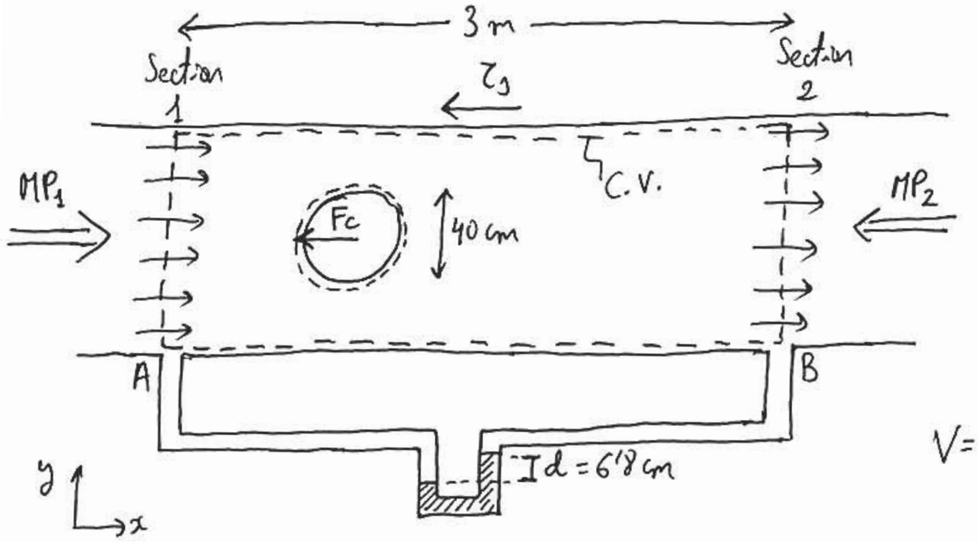
$\underline{\underline{Q}} = 1360\ gal/min = \underline{\underline{0{,}088\ m^3/s}}$ → smaller discharge

$\underline{\underline{H}} = 70\ ft \simeq \underline{\underline{21\ m}}$ → larger pump head

$\eta = 0{,}79$

$\underline{\underline{BHP}} = \dfrac{1000 \cdot 9{,}8 \cdot 0{,}088 \cdot 21}{0{,}79} \simeq \underline{\underline{23{,}0\ kW}} = 30{,}8\ HP$ → about the same energy consumption.

-PROBLEM N° 5:



F_C: Force from the cylinder on the fluid.

τ_s: Shear stress from the walls on the fluid.

$V = 0'80$ m/s

a)

From the manometer reading

$$\underline{p_A - p_B} = (SG_m - SG_w)\,\rho_w g d = (1'30-1)\cdot 1000 \cdot 9'8 \cdot 0'068 \approx \underline{\underline{200 \text{ Pa}}}$$

Since flow is well-behaved in sections 1 and 2,

$$p_{1,CG} - p_{2,CG} \underset{\substack{\uparrow \\ \text{hydrostatic} \\ \text{pressure in 1 \& 2}}}{=} p_A - p_B = 200 \text{ Pa}$$

b) This is a non-circular cross-section. So to calculate τ_s we replace D by 4Rh in Darcy-Weisbach's formulation.

HYDRAULIC RADIUS $\rightarrow$ $Rh = \dfrac{\text{area}}{\text{wetted perimeter}} = \dfrac{1 \cdot 2}{1+1+2+2} = 0'333$ m

$$\left.\begin{aligned} Re &= \frac{V\cdot(4Rh)}{\nu} = \frac{0'80\cdot(4\cdot 0'333)}{10^{-6}} = 1'07\cdot 10^6 \\ \varepsilon/(4Rh) &= 5'3\cdot 10^{-3}/(4\cdot 0'333) = 0'0040 \end{aligned}\right\} \xrightarrow{\text{MOODY}} f = 0'0285$$

$$\underline{\underline{\tau_s}} = \frac{1}{8}\rho f V^2 = \frac{1}{8}\cdot 1000 \cdot 0'0285 \cdot 0'80^2 = \underline{\underline{2'28 \text{ Pa}}}$$

c)

We apply conservation of linear momentum in x-direction on the control volume represented above:

$\sum F_x = 0 = MP_1 - MP_2 - F_C - \tau_s \cdot$ (Surface area of the walls)
$\hookrightarrow A_s = P \cdot \ell = 6 \cdot 3 = 18\, m^2$

$0 = (\rho V_1^2 + p_{1,CG}) A_1 - (\rho V_2^2 + p_{2,CG}) A_2 - F_C - \tau_s \cdot A_s$

As explained before, $p_{1,CG} - p_{2,CG} = p_A - p_B = 200$ Pa.

By continuity, $V_1 = V_2 = V$, since $A_1 = A_2 = A = 2\, m^2$.

$\underline{\underline{F_C}} = (p_A - p_B) \cdot A - \tau_s \cdot A_s = 200 \cdot 2 - 2'28 \cdot (6 \cdot 3) \simeq \underline{\underline{359\ N}}$

The force from the fluid on the cylinder has a magnitude of 359 N and acts in the (+x)-direction.

d)

Drag force: $F_D = 359\ N = \frac{1}{2} \rho C_D \overset{= D \cdot 2}{A_\perp} V^2$

$359 = \frac{1}{2} \cdot 1000 \cdot C_D \cdot (0'4 \cdot 2) \cdot 0'8^2 \Rightarrow \underline{\underline{C_D = 1'40}}$

e)

If the cylinder were at point X, the streamlines at section 2 would not be straight and parallel. This would mean that the flow would not be well-behaved, and therefore:

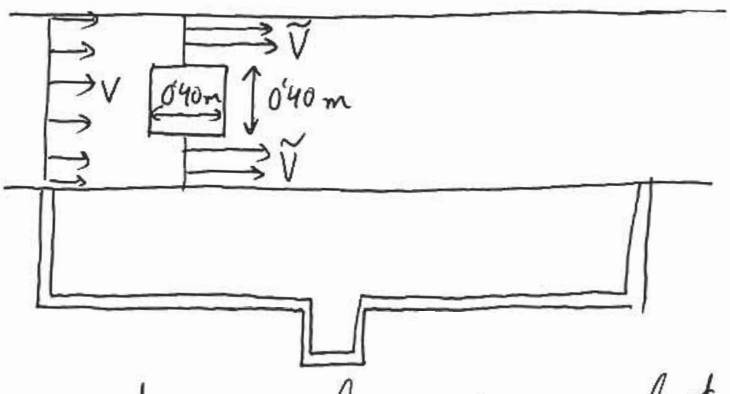
- Hydrostatic pressure at section 2 could not be assumed, and $p_{1,CG} - p_{2,CG} \neq p_A - p_B$
- You could not draw a control volume through section 2 to apply momentum conservation.

f)

It is an <u>upper bound</u> for the actual value of C_D. We calculated the shear stress neglecting the presence of the cylinder. The cylinder has the effect of reducing the local cross-sectional area, thus increasing the average velocity and the friction exerted by the walls. Therefore, the actual contribution of τ_s to the force balance is larger, so the real F_C is smaller and the real C_D is smaller than calculated.

g)

To calculate a lower bound, you could model the cylinder as a squared prism of 40x40 cm:



Now you have a larger average velocity $\tilde{V}$ in the narrow region, which will lead to a larger shear stress in this region and consequently to a smaller F_C when applying momentum conservation. Since the narrowing in this approximation is more strict than in the real case, the C_D you would obtain now is a lower bound.

- PROBLEM N° 6 :

For $T = 25°C$ we obtain from Table 8.4 in Young et al.

$$\rho_{air} = \rho = 1'18 \ kg/m^3 \quad \text{and} \quad \nu_{air} = \nu = 1'56 \cdot 10^{-5} \ m^2/s$$

Thus, for $D = 0'074 m$ and $U_0 = 70 \ mph = 31'3 \ m/s$ we have

$$Re_0 = \frac{U_0 D}{\nu} = \frac{31'3 \cdot 0'074}{1'56 \cdot 10^{-5}} = 1'5 \cdot 10^5$$

From this information and $\varepsilon/D = 1'5 \cdot 10^{-3}$ we obtain from Fig. 9.18: $C_{D_0} \cong 0'41$

a)*

If we assume negligible change in velocity between mound and plate, then time of travel is

$$\underline{t_p} \simeq \ell/U_0 = \frac{60 \cdot 0'305}{31'3} = \underline{0'58 \ s}$$

(not a lot of time to swing the bat and hit the ball!)

If $U \simeq U_0 \simeq$ constant, then the ball would experience a drag force

$$F_D \simeq F_{D_0} = \frac{1}{2} C_{D_0} \rho \frac{\pi}{4} D^2 U_0^2 = 1'02 \ N$$

Thus, during its flight of ℓ, the ball has done work against the drag force,

$$\text{Work done} \cong F_{D_0} \ell = 1'02 \cdot 60 \cdot 0'305 = 18'7 \ J$$

This work done has been obtained from the ball's initial kinetic energy

$$E_{kin,0} = \frac{1}{2} m U_0^2 = 68'6 \ J$$

i.e., as the ball passes the plate, its kinetic energy

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* see end of problem for alternative solution to (a).

is reduced to

$$E_{kin,p} = \frac{1}{2} m U_p^2 = E_{kin,0} - \text{Work done} = 68'6 - 18'7 = 49'9 \text{ J}$$

or

$$\underline{\underline{U_p}} = 0'853\, U_0 = 26'7 \text{ m/s} = \underline{\underline{59'7 \text{ mph}}}$$

and our assumption of $U \sim U_0 \sim 70$ mph ~ constant to calculate t_p was reasonable. Note also: $U_p \simeq U_0 - 10$ mph

(Baseball rule of thumb: Pitch velocity crossing the plate is ~10 mph slower than when it leaves the pitcher's hand).

b) The vertical motion of the ball is governed by

$$m\ddot{z} = mg - F_{D,z}$$

where z is positive downward, and $F_{D,z}$ is the drag force component in vertical direction, i.e.,

$$F_{D,z} = \frac{1}{2} C_D \rho \frac{\pi}{4} d^2 \{U^2 + W^2\}^{1/2} W$$

where

$$C_D = C_D\left(Re = \frac{\{U^2+W^2\}^{1/2} D}{\nu},\ \frac{\varepsilon}{D} = 1'5 \cdot 10^{-3}\right)$$

with

$$W = \text{vertical (downward) velocity} = \dot{z}$$

Assuming $W = \dot{z} \ll U \simeq U_0$, we obtain

$$F_{D,z} \simeq \frac{1}{2} C_{D0} \rho \frac{\pi}{4} D^2 U_0 \dot{z} = F_{D0} \frac{\dot{z}}{U_0}$$

Thus, the vertical motion of the ball is governed by

$$m\ddot{z} = mg - F_{D0} \frac{\dot{z}}{U_0} \simeq mg$$

where the drag force was dropped since $F_{D0} = 1'02 \text{ N} < mg = 1'37 \text{ N}$ and $\frac{\dot{z}}{U_0} \ll 1$ is anticipated.

The vertical equation is therefore approximately the free fall equation

$$m\ddot{z} \simeq mg \Rightarrow \ddot{z} = g$$

$$\dot{z} = W = gt + \dot{C} = gt \quad \text{since } w=0 \text{ at } t=0$$

$$z = \frac{1}{2}gt^2 + C = z_0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad (z_0 = 0 \text{ by choice})$$

When crossing the plate: $t = t_p = 0'58\,s$, so

$$\dot{z}_{max} = g\,t_p = 5'68\ m/s \ll U_0 = 31'3\ m/s$$

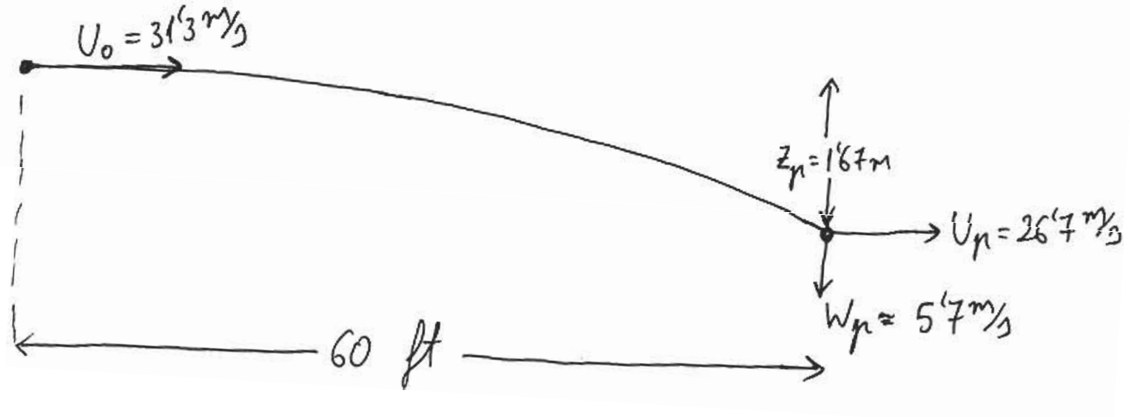
(assumption of neglect of drag force is valid!)

Since $x \simeq U_0 t \Rightarrow t = x/U_0$ and we have the trajectory

$$\underline{\underline{z = \frac{1}{2}\frac{g}{U_0^2}x^2}}$$ (parabola - most rapid descent as plate is crossed)

In particular,

$\underline{\underline{z_p = \text{drop from mound to plate}}} = z(x=\ell) = \underline{\underline{1'67\ m}}$



ALTERNATIVE SOLUTION TO QUESTION (a)

Equation governing horizontal motion of ball

$$m\ddot{x} = -F_{Dx} = -\frac{1}{2}\rho C_D \{U^2 + W^2\}^{1/2} U \frac{\pi}{4} D^2 \simeq$$

$$\simeq -\frac{1}{2}\rho C_{D0} \frac{\pi}{4} D^2 U_0 \dot{x} = -(F_{D0}/U_0)\dot{x}$$

when $C_D \simeq C_{D0}$ is assumed, along with $U^2 + W^2 \simeq U_0^2 \simeq$ constant

$$\ddot{x} + \frac{F_{D0}}{m U_0} \dot{x} = 0$$

ODE with solutions satisfying $\dot{x} = U_0$ at $t=0$

$$\dot{x} = U_0 \exp\left\{-\frac{F_{D0}}{mU_0} t\right\}$$

and, by integration using $x=0$ at $t=0$

$$x = (U_0^2 m / F_{D0})\left[1 - \exp\left\{-\frac{F_{D0}}{mU_0} t\right\}\right]$$

With $x = l$, equation for x gives t_p. And with $t = t_p$ equation for $\dot{x} = U$ gives $U_p = \dot{x}$ as ball passes plate. The results are:

$t_p = 0'63$ s
$U_p = 27'0$ m/s $= 60'4$ mph

} Similar to the results of the previous approach.