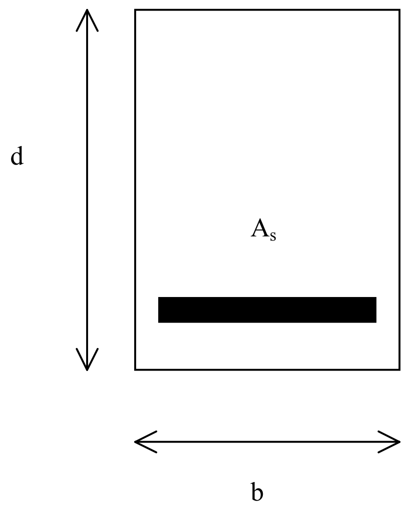
1.051 Structural Engineering Design

QUIZ 1 REVIEW

Example 1 (Flexural Strength of a Given Member)



b = 12”
d = 17.5”
A_s = 4.00 in^2
f_y = 60,000 psi
f_c’ = 4000 psi

Find M_n, M_u

$M_u \leq \phi M_n$; ϕ = 0.9 for flexure
Therefore, with M_n, M_u can be calculated

$$M_n = \rho f_y bd^2\left(1 - \frac{\rho f_y}{1.7 f_c'}\right)$$ → Equation (1)

$$\rho = \frac{A_s}{bd} = \frac{4.00}{12 \times 17.5} = 0.019$$ → Equation (2)

Check $\rho_{\min} = \dfrac{200}{f_y} = 0.0033$ → Equation (3)

$$\frac{3}{4}\rho_b = \frac{3}{4} \cdot \frac{0.85\beta_1 f_c'}{f_y} \cdot \frac{87000}{87000 + f_y} = 0.0214$$ → Equation (4)

$\therefore \rho_{\min} \leq \rho \leq \frac{3}{4}(\rho_b)$ → Equation (5)

$\therefore$ **M_n = 3487 kips.in** and **M_u = 0.9 x M_n = 3138 kips.in**

Example 2 (Section Design with a Given Moment)

Unknowns: b, d, h, A_s
Given: l = 15 feet
DL = 1.27 kips/ft
LL = 2.44 kips/ft
f_c' = 4000 psi
f_y = 60,000 psi
γ_c = 150 psf

1. Assume b and h for self-eight determination:

   Let b = 10 in and h = 18 in
   d = 18 – 2.5 = 15.5
   d/b = 1.5

   Minimum depth for simply supported beam = l/16 = 15/16 . 12 = 11.25; OKAY!

2. Find the applied moment to be resisted

   W = 150 . (10/12) . (18/12) . (1/1000) = 0.1875 kips/ft (this is to be revised)

   Therefore, W_u = 1.4(1.27 + 0.1875) + 1.7(2.44) = 6.19 kips/ft

   $M_u = w_u l^2/8 = 6.19\ (15)^2/8\ .\ 12$ (ft/in) = 2089 kips.in

3. Compute ρ_{min} & ρ_b; and choose ρ

   $\rho_{min} = 200/f_y = 0.0033$
   ¾ ρ_b = 0.0214; use ρ = 0.0214 (Not economical, but adequate for demonstration purpose)

   Find the required bd^2

   $$M_u = \phi \rho f_y (bd^2)\left(1 - \frac{\rho f_y}{1.7 f_c'}\right)$$
   $bd^2 = 2229\ in^2$

   Actual $bd^2 = 10.\ (15.5)^2 = 2403\ in^2$ > Required $bd^2 = 2229\ in^2$; **OKAY!**

4. Assign rebar arrangement

   $\rho = A_s/bd = 0.0214$

   $A_s = 0.0214.b.d = 0.0214.10.(15.5) = 3.32\ in^2$ (required)
   Provide 2#10 + 1#8

   ∴ A_s provide = $3.32\ in^2$

   Note: ρ can be smaller and a larger section may be needed to improve cost and deflection performance. However, if there is architectural restrictions on sizes, a ρ with a value closer to the upper bound is normally used (to reduce section size as much as possible)

Example 3 (Crack Width Determination)

Given: $b = 12''$
$h = 20''$
$A_s = 4.00\ in^2$ (4 #9)
$f_y = 60{,}000$ psi
Exposure = external

$$w = 0.000091 f_s \sqrt[3]{d_c A}$$ → Equation 1

$f_s = 0.6f_y$ in kips = 0.6 x 60 = 36 kips/in

$d_c = 2.5$ in

$A = A_{eff} / N$ → Equation 2

A_{eff} = web width x 2 x d_c = 12 x 2 x 2.5 = 60 in^2

N = Total A_s / area of largest bar = 4.00 / 1.00 = 4

Therefore, A = 60 / 4 = 15 in^2

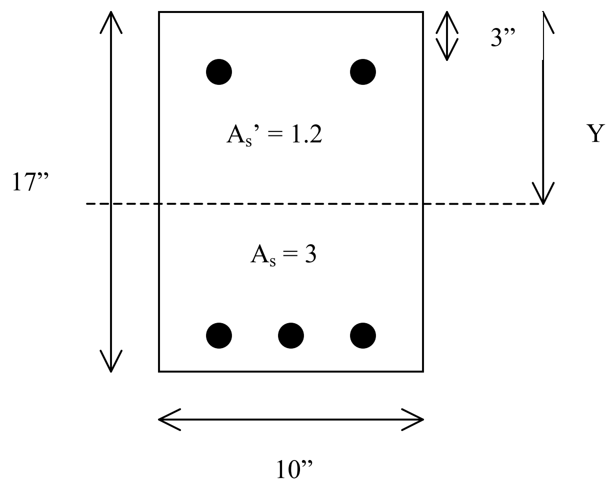
W = 0.011 in

ACI Stipulation
External exposure: $W_{max} = 0.013$ in

Since, $W < W_{max}$; **OKAY!**

Example 4 (Neutral Axis Location of Cracked Section)

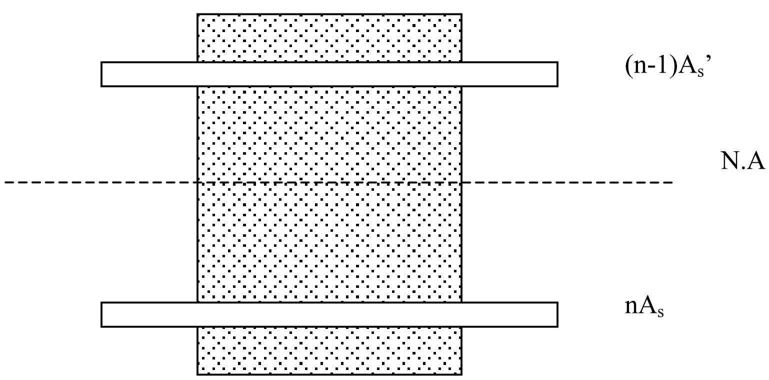
Given: $E_c = 3,625,000$ psi
$E_s = 29,000,000$ psi



$n = E_s/E_c = 8$

Locate the neutral axis by using **C = T**

<u>Transformed Section</u>

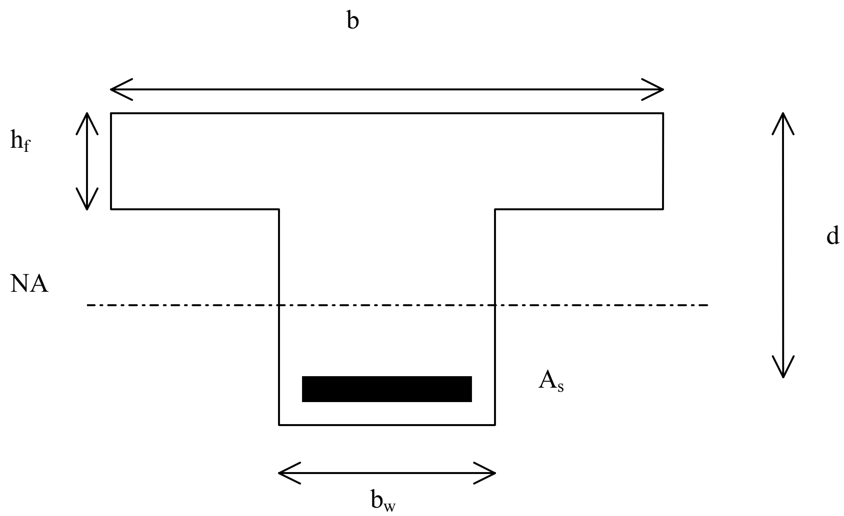


Therefore,
$Y\ (10)\ Y/2 + (n-1)\ A_s'\ (Y-3) = nA_s(17-5)$
$5Y^2 + 7(1.2)(Y-3) = 8(3)(17-Y)$
$Y = 6.62$ in

Example 5 (Moment of Cracked section0

$I_{cr} = bY^3/12 + (n-1)A_s'(Y-3)^2 + nA_s(17-Y)^2 = 10(6.62)^3/3 + 7(1.2)(6.62 – 3)^2 + 8(3)(17-6.62)^2 =$ **3663 in⁴**

Example 6 (Evaluate Stirrup Spacing for Different Shear Loads)



Given $f_c' = 3000$ psi
$f_y = 60{,}000$ psi
$A_v = 0.22$ in^2 (#3 stirrup) → $\Pi(3/8)^2/4$ x 2 legs

b = 30”
d = 16.5”
b_w = 10”
ϕ = 0.85 for shear design

Case 1: V_u = 12 kips
Case 2: V_u = 36 kips
Case 3: V_u = 42 kips

Equations to be used:

$$V_c = 2\sqrt{f_c'}\,b_w d$$

$$s = d/2; \quad s = \frac{A_w f_y}{50 b_w}$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$V_c = 2\,(3000)^{1/2}.10.(16.5) = 18.1$ kips

$\phi V_c/2 = 0.85 \times 18.1 / 2 = 7.7$ kips

Case 1

$V_u > \phi V_c/2$; since 12 > 7.7
But
$V_u < \phi V_c$ since 12 < 15.4

Therefore, use minimum reinforcement, spacing is the smallest of

s = d/2 = 16.5/2 = 8.25", $s = A_v f_y/(50b_w)$ = 0.22 (60000) / (50 x 10) = 26.4", s = 24"

Choose s = 8.25" → theoretical

Provide s = 8" → practical

Case 2

$V_u > \phi V_c$ since 36 > 15.4

$V_s = V_u/\phi - V_c = 36/0.85 - 18.1 = 24.3$ kips

and

$s = A_v f_y d/V_s = 0.22 \times 60 \times 16.5 / 24.3 = 8.96"$

But need to check if $V_s \leq 4\sqrt{f_c'}\, b_w d = 4\,(3000)^{1/2}\,.\,10\,.\,(16.5) = 36.1$ kips > 24.3

Therefore, $s_{max} = A_v f_y/50b_w$ = 26.4" or s_{max} = d/2 = 8.25" < 24"

→ s = 8.25"

Therefore, provide s = 8" as before

For V_u = 12 or 36 kips
Provide the same stirrup arrangement!!!

Case 1: For safety reason
Case 2: For need-based reason

Case 3

V_u = 42 kips

$V_s = V_u/\phi - V_c = 42/0.85 - 18.1 = 31.3$ kips $< 4\sqrt{f_c'}\, b_w d = 36.1$ kips

Therefore, provide $s = \frac{A_v f_y d}{V_s} = \frac{0.22 \times 60 \times 16.5}{31.3} = 6.96"$

Need to check

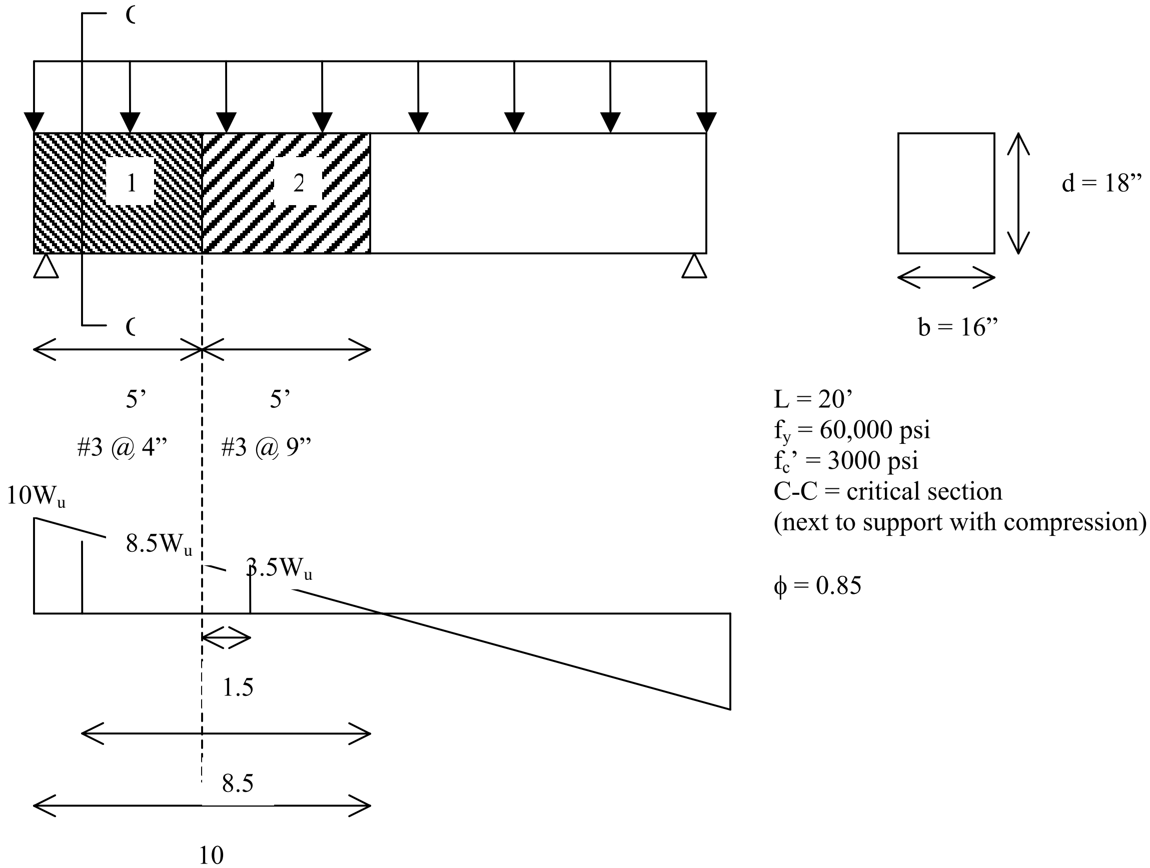
$$s_{\max} = \frac{A_v f_y}{50 b_w} \; or \; \frac{d}{2} \; or \; 24"$$

Therefore, s = 6.96" controls

Provide s = 6.5"

Example 7 (Determine Maximum Load Based on Shear Design)

Given:



$L = 20'$
$f_y = 60{,}000$ psi
$f_c' = 3000$ psi
C-C = critical section
(next to support with compression)

$\phi = 0.85$

Region 1

Check V_c

$V_c = 2\sqrt{f_c'}\, b_w d = 2 \times (3000)^{1/2}\ 16 \,.\, 18 = 31.55$ kips

Find V_s

$V_s = A_v f_y d / s = 0.22\ (60)\ (18) / 4 = 59.4$ kips

Find V_u

$V_u = \phi\ (V_c + V_s) = 0.85\ (31.55 + 59.4) = 77.3$ kips

Find W_u allowed

At the critical section $V_u = 8.5\ W_u$
Therefore, $W_u = 77.3 = 9.1$ kips/ft

Region 2

V_c = 31.55 kips (same)

$V_s = A_v f_y d / s$ = 59.4 . (4/9) = 26.4 kips

$V_u = \phi (V_c + V_s)$ = 0.85 (31.55 + 26.4) = 49.3 kips

Find W_u allowed

At the transition, the interface will be taken care of by the last stirrup in Region (1).
Therefore, consider d from the interface.

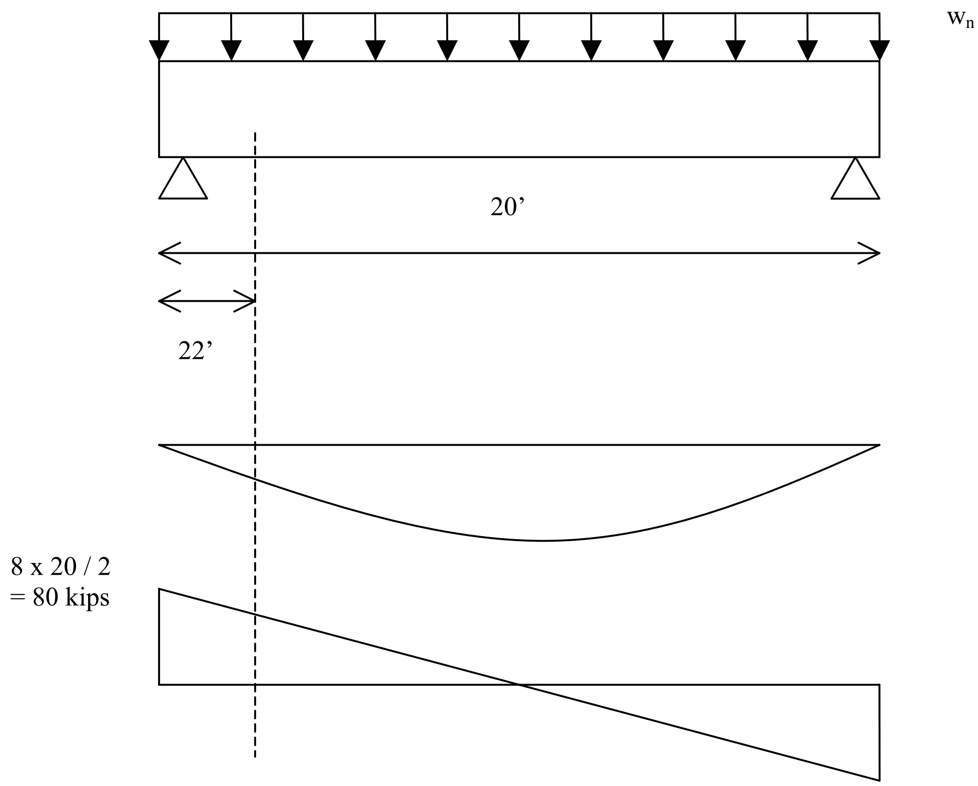
$V_u = 3.5W_u = 49.3$
W_u = 14.1 kips/ft

Obviously Region (1) controls

W_u (max) = 9.1 kips/ft

Example 8 (Design of Stirrups with Moment-Shear Coupling Consideration)

Given:



DL = 1.45 kips/ft (Include Self-Weight)
LL = 3.5 kips/ft
A_s = 6.06 in^2
f_c' = 2500 psi
f_y = 50,000 psi
b = 16"
d = 22"

1. Compute factored load

W_u = 1.4 DL + 1.7 LL = 1.4 x 1.45 + 1.7 x 3.5 = 7.98

Use W_u = 8.0 kips/ft

2. Compute M_u, V_u at critical section, d from the support

d = 22' = 22/12
V_u = 80 – 8(22/12) = 65.3 kips
M_u = (80 + 65.3)/2 . (22/12) = 133.19 kips.ft

3. Compute nominal shear strength

$$V_c = \left(1.9\sqrt{f_c^{'}} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 3.5\sqrt{f_c^{'}} b_w d$$

Therefore, $V_u d/M_u$ = 65.3 x 22 / (133.19 x 12) = 0.9 < 1.0; **OKAY!**

$\rho_w = A_s / b_w d$ = 6.06 / (16 x 22) = 0.0172

V_c = 47.06 kips

Check $3.5\sqrt{f_c'}\, b_w d$ = 61.6 kips > V_c; **OKAY!**

4. Stirrup provision

$\phi V_c/2$ = 0.85 (47.06) / 2 = 20 kips < V_u

Thus, need stirrups

$V_s = V_u/\phi - V_c$ = 65.3/0.85 – 47.06 = 29.76 kips

$s = A_v f_y d / V_s$ = 0.22 x 50 x 22 / 29.76 = 8.13”

Check s_{max}

i. d/2 = 11 in.
ii. $4\sqrt{f_c'} b_w d$ = 70.4 kips > V_s

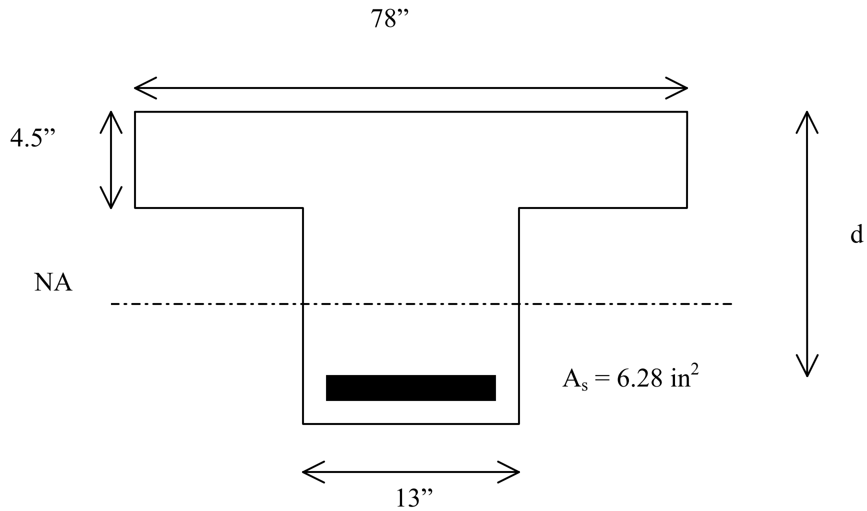
$s_{max} = A_v f_y / 50 b_w$ or d/2 or 24”
= 0.22 x 50,000/(50 x 16)
= 13.75

Therefore, s = 8.13” controls

Use s = 8”

5. Determine where to terminate by computing V_c and check against $\phi V_c/2$

Example 9



Determine the M_u for the given section

(a) $b_E = L/4 = 26/4 \times 12 = 78$”
(b) $b_E = b_w + 16t = 13 + 16 \times 4.5 = 85$”
(c) $b_E = 12 \times 13 = 156$”

if a = 4.5” = t
$c = 0.85 \times f_c' \times b_E \times a = 0.85 \times 3000 \times 78 \times 4.5 = 895$ kips

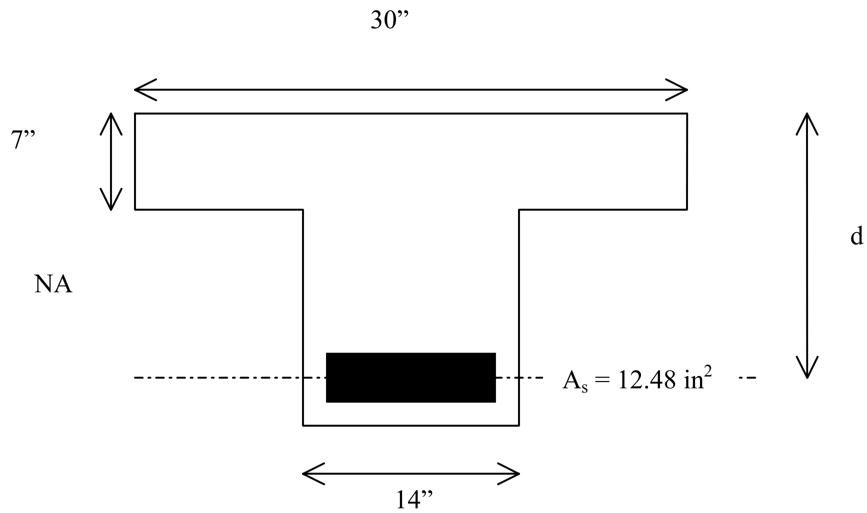
For equilibrium

$C = T = A_s f_y$
$A_s = 895{,}000/50{,}000 = 17.9$ in^2 (required)

Steel reinforcement provided = 6.28 in^2 < 17.9 in^2; Therefore, a < t
This means that we should design according to a rectangular beam (simply reinforced)

$M_n = T \times (d - a/2)$
$T = A_s f_y = 6.28 \times 50 = 314$ kips
$A = T / (0.85 \times f_c' \times b_E) = 314 / (0.85 \times 3 \times 78) = 1.58$”

Example 10 (Determine M_n with the given section (isolated))



$A_s = 12.48$ in^2
$b_E = 30”$
$b_w = 14”$
$d = 36”$
$t = 7’$
$f_c’ = 3000$ psi
$f_y = 50{,}000$ psi

Check:
$4b_w = 56” > b_E$; **OKAY!**
$½\ b_w = 7” > t$; **OKAY!**

If $a = t = 7”$
$C = T \rightarrow 0.85f_c’b_Ea = 0.85\ f_c \times 30 \times 7 = 535.5 = A_sf_y$
$A_s = 10.71 < 12.48$ (provided);
Therefore, $a > t$ (i.e. Neutral axis is below the flange)

$C_1 = 0.85\ f_c’\ b_w\ a = 0.85 \times 3 \times 14 \times a = 35.7a$
$C_2 = 0.85\ f_c’\ (b_E – b_W)\ t = 0.85 \times 3 \times 930\text{-}14) \times 7 = 285.6$

$T = A_sf_y = 12.48 \times 50 = 624$
Therefore, $624 = 35.7a + 285.6$
$a = 9.48”$

$M_n = C_1(d\text{-}a/2) + C_2(d\text{-}t/2) = 35.7 \times 9.48 \times (36 – 9.48/2) + 285.6 \times (36 – 7/2) = 1155$ kips.ft

Example 11 (Design t-Beam with Given Load)

DL = 370 kips.ft b_E = 30"
LL = 520 kips.ft b_W = 14"
f_c' = 3000 psi d = 36"
f_y = 50,000 psi t = 7"

M_u = 1.4DL + 1.7LL = 1.4 x 370 + 1.7 x 520 = 1402 kips.ft
$M_n = M_u/\phi$ = 1402/0.9 = 1557.8 ≈ 1560 kips.ft

Find position of neutral axis (NA)

If a = t
T = C = 0.85 x 3 x 30 x 7 = 535 kips
M_n = C(d – a/2) = 535(36 – 7/2) = 1450 kips.ft < 1560 (required),

Therefore, a>t

C_1 = 0.85 x 3 x 14 x a = 35.7a
C_2 = 0.85 x 3 x (30-14) x 7 = 285.6

∴, 1560 x 12 = 35.7.a.(36-a/2) + 285.6(36-35)
18720 = 1285.2a – 17.85a^2 + 9282
a = 8.3 in
x = 8.3 / 0.85 = 9.765

T = $0.85f_c'b_wa + C_2$ = 0.85 x 3 x 14 x 8.3 + 285.6 = 582 kips
A_s = 582 / f_y = 11.64 in^2

$$a_b = 0.85\left(\frac{0.003}{0.003 + \frac{50}{29000}}\right) \times 36 = 19.4 in^2$$

$A_{s1b} = (0.85f_c'b_wa_b) / f_y$ = 13.85

$A_{s2b} = (0.85f_c' (b_E - b_w)t) / f_y$ = 5.71

Therefore, $A_{s,max} = 0.75 (A_{s1b} + A_{s2b})$ = 14.7 in^2 > 11.64 in^2; **OKAY!!!**