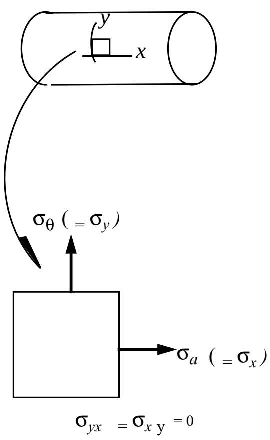
Problem Set #5 1.050 Solid Mechanics
Fall 2004

(Due Friday, 15 October)

Problem 5.1



A closed can of soda is under pressure equivalent to that in an automobile tire. We measured the wall thickness in lab on Thursday to be 0.0025 inches. Estimate the axial stress σ_a and the hoop stress σ_θ? acting at a point in this closed, thin cylindrical shell, away from the ends. C

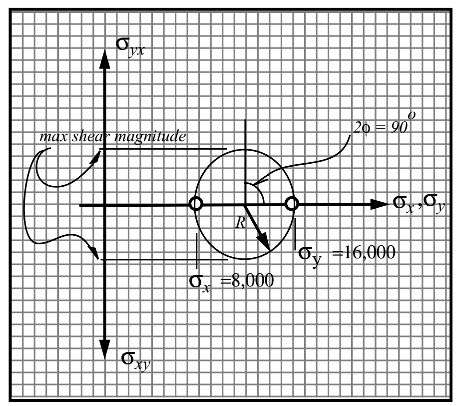
From class and textbook, p. 95.

$$\sigma_\theta = p_i \frac{R}{t} \qquad and \qquad \sigma_a = p_i \frac{R}{2t}$$

The pressure (within an automobile tire) I estimate to be 30- 40 psi. The thickness t is .0025 inches and the diameter is around 2 inches. With these we obtain $\sigma_\theta = 12{,}000 \text{ to } 16{,}000 \text{ psi}$ *and* $\sigma_a = 6{,}000 \text{ to } 8{,}000 \text{ psi}$

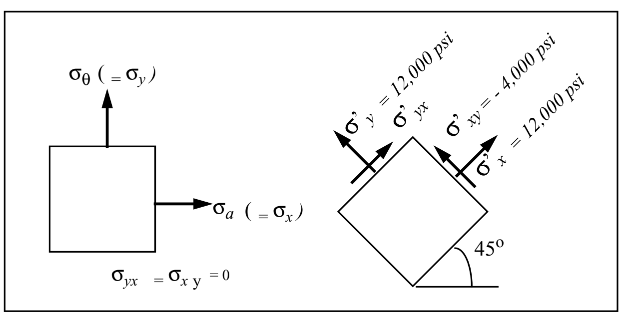
Construct a Mohr's circle for this state of stress with components with respect to the xy axis shown. ($\sigma_x = \sigma_a$; $\sigma_y = \sigma_\theta$).

We show the Mohr's circle at the right. Note how we identify the axial stress with σ_x and the hoop stress with σ_y. The shear stress components $\sigma_{yx} = \sigma_{xy}$ are zero so we plot the points corresponding to this orientation of the element on the line $\sigma_{yx} = \sigma_{xy} = 0$. The diameter of the Mohr's circle lies on this axis and its radius is just $R = (\sigma_y - \sigma_x)/2 = 4{,}000$ psi.



Determine the rotation of axis at the point which would bring you to planes of maximum shear stress. What is the magnitude of the maximum shear stress component at the point.

The rotation of axis to reach planes of maximum shear components is $2\phi = 90^o$ on Mohr's circle so $\phi = 45^o$ in the "material" or "physical" plane.



At the left we show the element at the surface of the cylinder before and after rotation of reference axes.

Note that the normal stress components w.r.t this rotated frame are equal, evident from the Mohr's circle

NB: We shall see that if we consider a three dimensional element at the point, the max shear acts on a x or y face, but in the z direction.

The same results could have been obtained without the use of Mohr's circle from the relationships which define the transformation of components of stress (in 2D) which appear on p. 118 in the text.

$$\sigma'_x = \left[\frac{(\sigma_x + \sigma_y)}{2}\right] + \left[\frac{(\sigma_x - \sigma_y)}{2}\right] \cdot \cos 2\phi + \sigma_{xy} \sin 2\phi$$

$$\sigma'_y = \left[\frac{(\sigma_x + \sigma_y)}{2}\right] - \left[\frac{(\sigma_x - \sigma_y)}{2}\right] \cdot \cos 2\phi - \sigma_{xy} \sin 2\phi$$

$$\sigma'_{xy} = -\left[\frac{(\sigma_x - \sigma_y)}{2}\right] \cdot \sin 2\phi + \sigma_{xy} \cos 2\phi$$

To find the orientation of axes which shows a maximum shear stress component we set the derivative of σ'_{xy} to zero. This gives

$$\tan 2\phi = -\left[\frac{(\sigma_x - \sigma_y)}{2\sigma_{xy}}\right]$$

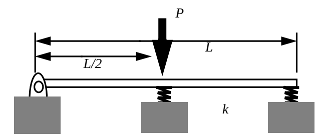
For our original components, with σ_x - σ_y finite and σ_{xy} zero, or approaching zero, the tangent of 2ϕ approaches a negative, very large number. This will be the case when 2ϕ approaches - 90^o. And this, as before is the angle we rotate in the Mohr's circle plane. Setting $2\phi = 90^o$ in the other two relations, we obtain that σ'_x and σ'_y are both equal to

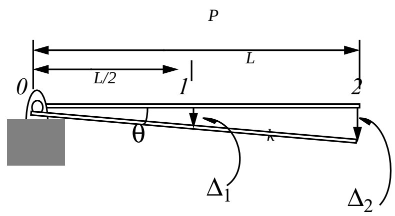
$$\left[\frac{(\sigma_x + \sigma_y)}{2}\right] = 12{,}000psi$$

Problem 5.2

A rigid beam is pinned supported at its left end and at midspan and the right end by two springs, each of stiffness k (force/displacement). The beam supports a weight P at mid span.

i) Construct a compatibility condition, relating the displacements of the springs to the rotation of the rigid beam.





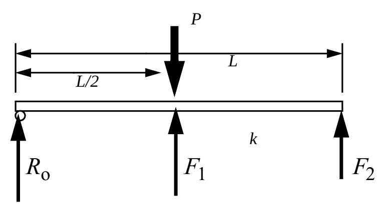
We draw the system in its deflected state. From the geometry, ie. similar triangles, we have , assuming small rotations and displacements

$$\frac{\Delta_1}{L/2} = \frac{\Delta_2}{L} = \theta$$

ii) Draw an isolation of the rigid beam and write out the consequences of force and moment equilibrium.

This requires drawing another, quite different diagram - a free body diagram or isolation of the beam showing all the forces acting.

Force and moment equilibrium yield



$$\sum F = 0 \qquad R_o + F_1 + F_2 - P = 0$$

$$\sum M|_{about0} = 0 \qquad F_1 \cdot L/2 + F_2 \cdot L - (P \cdot L/2) = 0$$

iii) Using the force/deformation relations for the linear springs, express equilibrium in terms of the angle of rotation of the beam.

$$F_1 = k \cdot \Delta_1 = (kL/2) \cdot \theta \qquad F_2 = (k \cdot L) \cdot \theta$$

so moment equilibrium gives

$$[(kL/2) \cdot L/2 + (k \cdot L) \cdot L] \cdot \theta = P \cdot L/2$$

or

$$\left[\frac{5}{4} \cdot kL\right] \cdot \theta = P \cdot \frac{1}{2}$$

iv) Solve for the rotation, then for the forces of reaction at the three support points.

From above, we solve for theta, then back substitute into the force/deformation relations for the springs

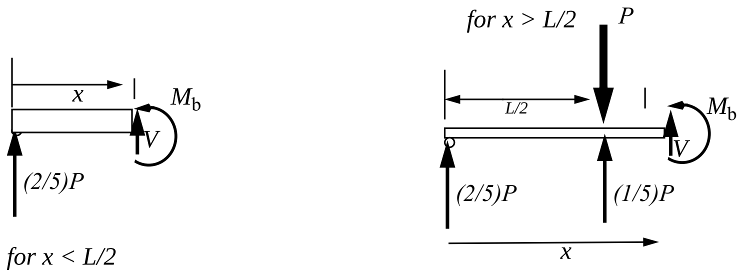
$$\theta = \left(\frac{2}{5}P\right)/(kL)$$

$$F_1 = kL/2 \cdot \theta = \frac{1}{5}P \qquad F_2 = k \cdot L \cdot \theta = \frac{2}{5}P$$

and, with these, the reaction at the pin is (2/5)P

v) Sketch the shear force and bending moment diagram.

We make two isolations, one for the portion of the beam left of center, another for right of center:

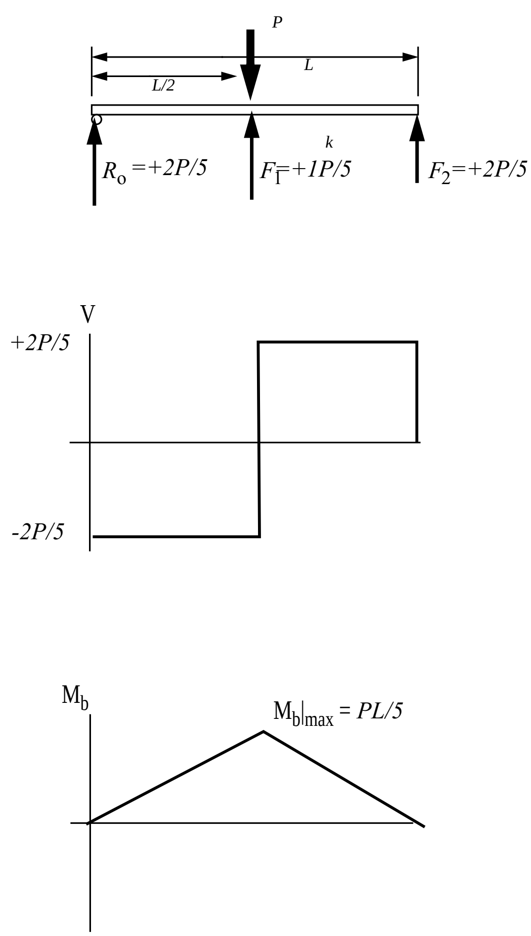


Force and moment equilibrium (about x, ccw > 0) gives

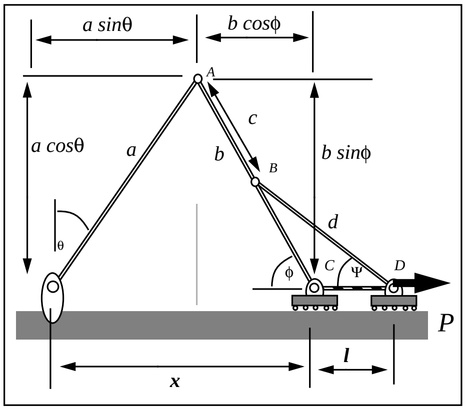
$$\left(\frac{2}{5}\right)P + V = 0 \qquad\qquad \left(\frac{2}{5}\right)P + V + \left(\frac{1}{5}\right)P - P = 0$$

$$M_b - \left(\frac{2}{5}\right)P \cdot x = 0 \qquad\qquad M_b - \left(\frac{2}{5}\right)P \cdot x + \left(P - \left(\frac{1}{5}\right)P\right) \cdot \left(x - \frac{L}{2}\right) = 0$$

We plot the shear force and bending moment distribution:



Problem 5.3



The figure shows one of the eight ribs of the umbrellas, the unfurling of which we have studied in Class Exercise #7, the results of which have been posted on our MIT server at two places under the heading of "Class Exercises"). We now want to finish our analysis and pick a size, a spring constant, for the spring connecting the "pins" C and D.

Set up a spread sheet, the final column of which computes the (nondimensional) value of the force P (as P/kb), the spring change-in-length, call it D, and the position of pin D relative to the top pin as functions of the angle θ.

Plot P/kb versus θ

In this use the following values for lengths:

a = 5.50 in. b = 5.25 in. c = 2.50 in. and d = 3.50 in.

Take the initial value for theta to be 20° , θ_0 = *20. deg. and increment by 2 to 5 degrees*

A plot of P/kb versus theta is shown. The full spread sheet is posted on our MIT server.