ROBERT FIELD: That's the outline of what we're going to cover. But before we get started on that, I want to talk about a couple of things. First of all, last time, we talked about the two slit experiment. And it's mostly classical. There is only a little bit of quantum in it where we talk about momentum as being determined by h over lambda. OK. But what were the two surprising things about the two slit experiment? There are two of them, two surprises. Yes?

AUDIENCE: [INAUDIBLE] That a single particle can interfere with itself.

ROBERT FIELD: Yes. That's the most surprising thing. And when you go to really low intensity-- so there's only one photon, which is quantum, in the apparatus, somehow, it knows enough to interfere with itself. And this is the most mysterious aspect. But then there's one other aspect, which is how it communicates that interference with itself. What is that? You're hot. You want to do another one?

AUDIENCE: What do you mean by how it communicates?

ROBERT FIELD: Well, here we have the screen on which the information is deposited. And you have possibly some sort of probability distribution, which is a kind of a continuous thing. But you don't observe that. What do you observe? So you could say the state of a particle has amplitude everywhere on this screen. But what do you see in the experiment? Yes?

AUDIENCE: So you just see one [INAUDIBLE] point. [INAUDIBLE] thousands and thousands. And eventually, you see it mimic that probability [? sequence. ?]

ROBERT FIELD: That's exactly right. So this state of the system which is distributed collapses to a single point. We say that what we are observing-- and this is mysterious. And it should bother you now. We're seeing an eigenvalue of the measurement operator. So we have this sort of thing. It goes into the measurement operator. And the measurement operator says, this is one of the answers I'm permitted to give you.

And another aspect that's really disturbing or puzzling is that you do many identical

experiments. And the answer is always different. There is no determinant. It's all probabilistic. So this really should bother you. And eventually, it won't.

OK. Now, there's another question I have. When I described the two slit experiment, I intentionally put something up on the diagram that should bother you, that should have said, this is ridiculous. I used a candle in the lecture. And I used a light bulb in the notes. And why is that ridiculous? Yes?

AUDIENCE: You said many frequencies.

ROBERT FIELD: That's right. So the sources of light that I misled you with intentionally have a continuous frequency distribution. And the interference depends on the same frequency. So the only way you would see any kind of diffraction pattern, any kind of pattern on the two slit experiment, is if you had monochromatic light. It would all wash out. You'd still get dots. But the dots would never give you anything except perhaps a superposition of two or three or an infinite number of patterns. OK. This is, again, something that's really bothersome.

Now, I also use the crude illustration of an uncertainty principle, that the uncertainty in position z-axis and the uncertainty in the momentum along the z-axis was greater than h. And I froze. And I didn't realize that I had delta s with the slit. But it really is the same thing as the z. Because the slit is how you define the position in the z-axis. And so this is the first taste of the uncertainty principle. And I said I didn't like it.

OK. In quantum mechanics, you're not allowed to look inside small stuff. You're not allowed to see the microscopic structure. You're only able to do experiments, usually thought experiments, an infinite number of identical experiments. And they reveal the structure in some complicated, encoded way.

And this is really not what the textbooks are about. Textbooks don't tell you how you actually think about a problem with quantum mechanics. They tell you, here are some exactly solved problems. Memorize them. And I don't want you-- I don't want to do that.

OK. Last part of the introduction here-- suppose we have a circular drum, a square drum, and a rectangular drum. Have you ever seen a square drum or a rectangular drum? Do you have an idea how a square drum would sound? Yes. You do have an idea. It would sound terrible. Because the frequencies, you get are not integer multiples. It would just sound amazingly terrible.

But if you had a square drum or a rectangular drum, you could do an experiment with some kind of acoustic instrument to find out what the frequency distribution is of the noise you make. And you would be able to tell. It's not round. It might be square. Or it might have a certain ratio of dimensions.

This is what we're talking about as far as internal structure is concerned. And it's very much like what you would do as a musician. I mean, certainly, when a musical instrument is arranged correctly, it's not like a square drum. It sounds good. And that's because you get harmonics or you get integer multiples of some standard frequency.

OK. So now, we're going to talk about the classical wave equation, which is not quantum. But it's a very similar sort of equation to the Schrodinger equation. And so the methods for solving this differential equation are on display. And so the trick is-- well, first of all, where does this equation come from? And it's always force is equal to mass times acceleration in disguise.

OK. And then you have tricks for how you solve this. And one of the most frequently used and powerful tricks is separation of variables. You need to know how that works. Then once you solve the problem, you have the general solution. And you then say, well, OK, for the specific case we have, like a string tied down at both ends, we have boundary conditions. And we impose those boundary conditions.

And then we have basically what we would call the normal modes of the problem. And then we would ask, OK, well, suppose we're doing a specific experiment or doing a specific preparation of the system. And we can call that the pluck of the system. And you might pluck several normal modes.

You get a superposition state. And that superposition state behaves in a dynamic way. And you want to be able to understand that dynamics. And the most important thing that I want you to do is, instead of trying to draw the solutions to a differential equation, which is a mathematical equation, I want you to draw cartoons, cartoons that embody your understanding of the problem. And I'm going to be trying to do that today.

OK. In this course, for the first half of the course, most of what we're going to be doing is solving for exactly soluble problems-- the particle in a box, the harmonic oscillator, the rigid rotor, and the hydrogen atom. With these four problems, most of the things that we will encounter in quantum mechanics are somehow related to these.

And in the textbooks, they treat these things as sacred. And they say, OK, well, now that you've solved them, you understand quantum mechanics. But these are really tools for understanding more complicated situations. I mean, you might have a particle in a box. Instead of with a square bottom, it might have a tilted bottom. Or it might have a double minimum.

But if you understand that, you then can begin to build an understanding of, what are the things in the experiment that tell you about these distortions of the standard problem? And the same thing for a harmonic oscillator. Almost everything that's vibrating is harmonic approximately. But there's a little bit of distortion as you stretch it more. And again, you can understand how to measure the distortions from harmonicity by understanding the harmonic oscillator. We did rotor, H atom. It's all the same.

So I would like to tell you that these standard problems are really important. But nothing is like that. And what's important is how it's different from that. And this is my unique perspective. And you won't get that from McQuarrie or any textbook. But this is MIT. So there are templates for understanding real quantum mechanical system.

And the big thing, the most important technique for doing that is perturbation theory. And so perturbation theory is just a way of building beyond the oversimplification. And it's mathematically really ugly. But it's tremendously powerful. And it's where you get insight.

OK. Now, many people have complained that they found 5.61 hard. Because it's so mathematical. And maybe this is going to be the most mathematical lecture in the course. But I don't want it to be hard. Now, chemists usually derive insights from pictorial rather than mathematical views of a problem.

So what are the pictures that describe these differential equations? How do we convert what seems to be just straight mathematics to pictures that mean something to us? And that's my goal, to get you to be drawing freehand pictures that embody the important features of the solutions to the problems.

OK. So we're going to be looking at a differential equation. And one of the first questions you ask, well, where did that equation come from? And you're not going to derive a differential equation ever in this course. But you're going to want to think, well, I pretty much understand what's in this differential equation. And then we'll use standard methods for solving that

equation.

And one such differential equation is this-- second derivative of some function with respect to a variable is equal to a constant times that function. Now, that you know. You know sines and cosines are solutions to that. And you know that exponentials are solutions to that. Now, that pretty much takes you through a lot of problems in quantum mechanics.

But now, one of the important things is this is a second-order differential equation. And that means that there are going to be two linearly independent solutions. And you need to know both of them. I'll talk about this some more later.

Now, sometimes, the differential equations look much more complicated than this. And so the goal is usually to rewrite it in a form which corresponds to a differential equation that is well known and solved by mathematicians whose business is doing that. But we won't be doing that.

OK. But usually, when you have a differential equation, the function is of more than one variable. And frequently, it's position and time. And so the first thing you do is you try to separate variables. And so that's what we're going to do.

So we have a differential equation. And the first thing is a general solution. And one of the things that this solution will have is nodes. And the distance between nodes-- here's a node. Here's a node. That's half the wavelength. And we know that in quantum mechanics, if you know the wavelength, you know the momentum. So nodes are really important. Because it's telling you how fast things are moving.

We can also look at the envelope. And this would be some kind of classical, as opposed to a quantum mechanical, probability distribution. And so it might look like this. But the important thing about the envelope is that it's always positive. Because it's probability, as opposed to a probability amplitude, which can be positive and negative. Interference is really important in quantum mechanics. But sometimes, the envelope tells you all you need to know.

And the other thing is the velocity of a stationary phase. So you have a wave. And it's moving. And you sit at a point on that wave. And you ask, how fast does that point move? And I did that last time. And OK. So I've already talked a little bit about what we do next.

But the important thing is always, at the end, you draw a cartoon. And you endow that cartoon with your insights. And that enables you to remember and to understand and to organize

questions about the problem. OK. So let's get to work on a real problem.

So we have a string that's tied down to two points. And so let's look at the distortion of that string. And so we chop this region of space up into regions. So this might be the region at x minus 1. And this might be the region at x0. And this might be the region of x1.

And we're interested in-- OK, suppose we have the value of the displacement of the wave here at x minus 1 and here and here. OK, so these would be the amount that the wave is displaced from equilibrium. And we call those u of x.

And so the first segment here, the minus 1 segment, this segment is pulling down on this segment of the string by this amount. And this one is pulling up on the segment by that amount. So we want to know, what is the force acting on each segment? And so we have the force constant times the displacement at x0 minus the displacement at x minus 1. So this is the difference between the displacements. And this is the force constant. We're talking about Hooke's law. Hooke's law is the force is equal to minus k times the displacement.

And so we collect the forces felt by each particle. And the forces felt by each particle are, again, the force constant times the difference in u at 0 and minus 1 minus the difference-- plus 1 minus the difference in u at 0 and plus 1. And this is a second derivative. This is the second derivative of u with respect to x.

So we've derived a wave equation. And we know it's going to involve a second derivative. So force is equal to mass times acceleration. Well, we already know the force is going to be related to the second derivative of u with respect to x. And now, this is something. And we know what this is. This is going to be the time derivative.

OK. And this is just something that gets the units right. And it has physical significance. But in the case of this particle on a string-- this wave on a string, it's related to the mass of the string and the tension of the string. And it's also related to the velocity that things move.

OK. So we have a differential equation that is related to forces equal to mass times acceleration. And the differential equation has the form second derivative of u with respect to x is equal to 1 over v squared times the second derivative of u with respect to t. That's the wave equation.

So it is really f is equal to ma. But OK. And now, the units of this-- this is x. And this is t. In

order to be dimensionally consistent, this has to be something that is x over t, OK? And so this may be a velocity. But it has units of velocity. That's the differential equation we want to solve.

OK. Well, the original differential equation that I wrote-- but I'm getting ahead of myself. OK. So this U of x and t-- we'd like it to be X of x times T of t. We think we could separate the variables in this way. So we try it. If we fail, it says you can't do that. Failure is usually going to be a result that the solution to the differential equation in this form is nothing. It's a straight line. Nothing's happening. So failure is acceptable. But if we're successful, we're going to get two separate differential equations.

OK. So what we do then is take this differential equation, substitute this in, and divide on the left. So we have 1 over X of x times T of t times the second derivative with respect to x of xt is equal to 1 over xt times the second derivative with respect to t of xt.

OK. Well, on this side of the equation, the only thing that involves time is here. This doesn't operate on time. And so we can cancel the time dependence from this side. And over on this side, this derivative operates on t but not x. And so we can cancel the x part. And so what we have now is an equation X of x second derivative with respect to x squared of x is equal to this constant, 1 over v squared, times 1 over t times the derivative of T with respect to little t, OK?

So this is interesting. We have a function of x on this side and a function of t on this side. They are independent variables. This can only be true if both sides are equal to the same constant.

So now, we have to differential equations. We have 1 over x second derivative with respect to x of x is equal to a concept. And we have 1 over v squared 1 over t second derivative with respect to t of T is equal to K. So now, we're on firmer ground. We know about solutions to this kind of equation.

And so there's two cases. One is this K is greater than 0. And the other is K less than 0. So let's look at this equation. If K is greater than 0, then if we plug in a sine or a cosine, we get something that's less than 0. Because the derivative of sine with respect to its variable is negative cosine. And then we do it again, we get back to sine. And so sines and cosines are no good for this equation if K is greater than 0.

But exponentials-- so we can have e to some constant x or e to the minus some constant x. Or here, for the negative value of K, we could have sine some constant x and cosine of some constant x. We know that. So we have two cases. So to make life simple, we say K is going to

be equal to lowercase k squared. Because we want to use this lowercase k in our solutions.

All right. So I may have confused matters. But the solution for the time equation and the position equation are clear. And so depending on whether K is positive or negative, we're dealing with sines and cosines or exponentials. OK. So I don't want to belabor this, but the next stage is boundary conditions.

We don't know whether K positive or K negative is possible. But we do know boundary conditions. And so if we have a string which is tied down at the end-- so this is x equals 0. And this is x equals L. Then we input impose the boundary conditions. Well, the boundary conditions are u of 0t is equal to 0. And u of Lt is equal to 0.

So if we take the K greater than 0 case, u of 0t is equal to-- well, let's just do this again. 0t. We have x of 0 times T of t. OK, we don't really care about this. But x of 0 has to be 0-- I'm sorry. OK. So we have two solutions. If K is greater than 0, we have the exponential terms. So we have Ae to the 0 plus B to the minus 0. And this has to be equal to 0. That's x of 0.

Well, e to the 0 is 1. E to the minus 0 is 1. And so this is good. A has to be equal to minus B. And then we have the other boundary condition, X of L. We have A e to the L plus B e to the minus L. Well, I'm sorry. Let's just put what we already know. Minus A. So we can write this as A e to the L minus e to the minus L. And that has to be equal to 0. Can't do it. This can never be 0.

So that means the K greater than 0 solutions are illegal. Well, that's kind of bad news. Because it sounds like separation of variables is failing. But it doesn't. Because the K less than 0 solution works. So for K less than 0, X of 0 is equal to C sine 0 plus D cosine 0. And so that means D is equal to 0. Things are dying. And X of L, boundary condition, is C sine KL is equal to 0.

And this we can solve. So sine is equal to 0 when KL is equal to 0, 0 pi, 2 pi, et cetera. And so we have KL is equal to n pi. So we can write this as Kn is equal to n pi over L. We get quantization. This isn't quantum mechanics. But there are certain allowed values of this K constant. And we have a bunch of solutions.

And so what do they look like? So n equals 0, n equals 1, n equals 2. So what does the 0 solution look like? Yes?

AUDIENCE: No node.

ROBERT FIELD: Nothing. So we don't even think about this. We say, n equals 0 is not a solution. Because the wave isn't there. No nodes, one node. And if we look at this carefully, the node is always at a cemetery point. It's in the middle. If we have the next one, we'll have two nodes. And they'll be at the 1/3 2/3 point. And so we know where the nodes are.

We also know that the amplitude of each loop of the wave function is the same. But it alternates in sine. So you can draw cartoons now at will. Because this spatial part of the solution to this wave is clear. Any value of the quantum number, or the n, gives you a picture that you can draw in seconds. And there are a lot of quantum mechanical problems like that.

But sometimes, you have to keep in mind that the node separation, in other words-- well, let's just say node separation is lambda over 2. And lambda over 2 is equal to 1/2 h over p. So if we know what the momentum is, or we know what the kinetic energy is, we know what the momentum is. We know how momentum is encoded in node separations.

So everything we want to know about a one-dimensional problem is expressed in the spacing of nodes and the amplitude between nodes. And the amplitude between nodes have something to do with the momentum, too. Because if you're going from here to here at some high velocity, there's not much amplitude. And at a lower velocity, you get more amplitude. And so the amplitude in each of these node to node separate sections is related to the average momentum of the classical particle in that section.

So the classical mechanics is going to be extremely important in drawing cartoons for quantum mechanical systems. Not in the textbooks. We supposedly know classical mechanics pretty well, and especially here at MIT. So you might as well use that in order to get an idea of how all of the quantum mechanical problems you're facing will be behaving.

OK. So the next thing we want to do is finish the job. And so I can simply write down the time-dependent solutions. They are E sine vkn t plus F cosine vkn t. And we can say that-- rather than carrying around all this stuff, we can say omega n is vkn. Isn't that nice? So we have a frequency for the time-dependent part, which is an integer multiple of this constant V times this [? vector. ?] Or this you can think of as just k sub n.

So we can rewrite this in a frequency and phase form. We have now the full solution. We have A n sine n pi L over x. And then this e N sine n pi-- I'm so used to the pictures I don't even want to look at the equations anymore-- n omega t plus F n cosine n omega t. OK.

So we can also take this and rewrite it in a simpler form, E n prime cosine n omega t plus phi n. So we can combine these two terms as a single cosine with a phase vector. OK. So now, we're ready to actually go to the specific thing that you do in a real experiment or a real musical instrument.

We say, OK, here is the actual initial condition, the pluck of the system. And the pluck usually occurs at t equals 0. But I'll just specify it here. And what we have is now a sum over as many normal modes as you want. We have A n E n prime times sine n pi over L x times cosine N omega t plus phi N.

So we have a bunch of terms like this, a spacial factor, and a temporal factor. And you can draw pictures of both. Now, but there is another simplification. From trigonometry, sine A cosine B-- we have sine and cosine-- can be written as 1/2 times sine n pi L x plus n omega t plus phi n plus sine n pi L x minus n omega t minus phi n.

So these are the two possible solutions. And we can write them now in terms of position factor at a time factor in the same sine or cosine function. So these are the things. Now, we're ready to make a picture.

OK. So these are the actual things that you make by exciting the system not in an eigenfunction. But it's a superposition of eigenfunctions. And again, there are certain things you learn. If you have a pure eigenfunction, you have standing waves. There's no left-right motion. There's no breathing motion. There's only up-down motion of each loop of the wave function.

If you have a superposition of two or more functions, which are all of even n, then what happens is you have no motions, you just have breathing. In other words, you have a function that might look sort of like this at one time and like that at another time. So amplitude is moving. So it can be moving. And in between, it's sort of like this.

Now, if you have a function which involves both even and odd n, you have left-right motion. This is true in quantum mechanics, too. So you only get motion if you're making superposition of eigen-- Yes?

AUDIENCE: What is the difference between breathing and the standing wave with no nodes?

ROBERT FIELD: Well, for this picture, this is over-simple. So I mean, you could have-- basically, what's

happening is amplitude is moving from middle to the edges and back. And so yes. But you want to develop your own language, your own set of drawings so that you understand these things.

And the important thing is the understanding, the ability to draw these pictures which contain the critical information about node spacings, amplitudes, shapes, and to anticipate when you're going to have left-right motion or when you're just going to have complicated up-down motions. Because there could be nodes. But there is no motion of the center of this wavepacket.

Now, this is fantastic. Because I just said wavepacket. Quantum mechanics-- eigenfunctions don't move. Superpositions of eigenfunctions do move. If we make a superposition of many eigenfunctions, it is a particle-like state. What that particle-like state will do is exactly what you expect from 8.01. The particle-like states-- the center of the wavepacket for a particle-like state moves according to Newton's equations.

So I'm saying I'm taking away your ability to look at microscopic stuff. And I'm going to give it back to you. By the end of the course, we're going to have the time-dependent Schrodinger equation. We're going to be able to see things move. And we're going to see why they move and how they encode that motion.

Not in the textbooks, but I think it's something you really want to be able to do. If you're going to understand physical systems and use it to guide your understanding, you have to be able to draw these pictures and build a step at a time. And so this way the equation, the classical wave equation, gives you almost all of the tools for artistry as well as insight.

OK. Now, in the notes, there is a time-lapse movie that shows what a two-state wave function-- what a two-state solution to the wave equation looks like if you have even and odd or only even and even terms.

OK. Now, I'm going to make some assertions at the end. We're coming back to the drum problem. And suppose we have a rectangular drum. Well, solving the differential equation for this rectangular drum gives you a bunch of normal mode frequencies that depend on two indices.

And so this is the geometric structure of the drum. And these are the quantum numbers. And these are the frequencies. And it's going to make a whole bunch of frequencies that are not

integer multiples of each other or of any simpler thing. And that's why it sounds horrible.

It's perhaps a little bit like playing a violin with a saw. It will sound terrible. You would never do it. But you would also never build a square or rectangular drum. But the noise that you make tells immediately not just what the shape of the instrument is but how it was played.

For example, suppose you had an elliptical drum. That'll sound terrible, too. But here are the two foci. I'm not so sure it'll sound terrible if you hit it here or here. And certainly, if you are a circular drum, if you hit it in the middle as opposed to on the edges, it'll sound different. The spectral content will be the same. But the amplitudes of each component will be different.

And so in quantum mechanics, you use the same sort of instinct as you develop as a musician in order to figure out how this system is going to respond to what you do to it. And that's pretty powerful. So many of you are musicians. And you know instinctively what's wrong when you do something that's not quite right, or your instrument is out of tune.

But in quantum mechanics, all of those insights will come to bear. Not in the textbooks. Because the textbooks tell you about exactly solved problems. And then they tell you how to do spectra that are too perfect for anybody else to observe. And you won't see those spectra. And they don't tell you what the spectra you will observe tell you about the system in question.

OK. So I should stop now. And I'm going to be generating Problem Set 2, which will be posted on Friday. Problem Set 1 is due this Friday. And-- Good. Thank you.