

5.0 context and direction

In Lesson 4, we encountered instability. We think of stability as a mathematical property of our linear system models. Now we will embed this mathematical notion within the practical context of process operability. That is, we must not forget that our system models help us operate processes. Along the way, we will encounter a special category of instability/inoperability: the non-self-regulating process.

DYNAMIC SYSTEM BEHAVIOR**5.1 remember the stability criterion for linear systems**

In Section 4.9, we introduced a stability criterion for a linear system: non-negative poles in the transfer function (5.1-1) indicate that the system output $y(t)$ will not remain stable in response to a system input $x(t)$.

$$\frac{y'(s)}{x'(s)} = G(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + 1} \quad (5.1-1)$$

As a simple example, consider a first-order system:

$$\tau \frac{dy'}{dt} + y' = Kx' \quad y'(0) = 0 \quad (5.1-2)$$

We know that the Laplace transform representation is completely equivalent.

$$y'(s) = \frac{K}{\tau s + 1} x'(s) \quad (5.1-3)$$

The transfer function in (5.1-3) has a single pole at $-\tau^{-1}$. If the time constant τ is a positive quantity (as in our tank), the pole is negative and the response is stable (as we have seen in Lesson 3).

If the time constant were a negative quantity, however, the pole would be positive. As we saw in Section 4.9, the response would be unstable because of the exponential term in the solution of (5.1-2)

$$y'(t) \propto e^{-t/\tau} \quad (5.1-4)$$

This unbounded response could be in a positive or negative direction, depending on the sign of the gain K . We will address on another occasion what sort of system might have a negative time constant; for now we recognize that encountering an unstable linear system should cause us to look carefully at the process whose behavior it represents.

5.2 remember that feedback control can make stable systems go unstable

Recall from Section 4.21 that we took a perfectly stable second-order process, placed it in a feedback loop with a first-order valve, and applied proportional-mode control. By increasing the controller gain too far, we could drive the system to instability. We could use our linear stability criterion to predict the onset of instability as we did in Section 5.1. That is, we compute the poles of ... not the *process* transfer function, but the transfer function that represents the process in feedback loop with other components!

5.3 the special case of zero poles

Now consider a system with a single pole whose value is zero.

$$y'(s) = \frac{K}{\tau s} x'(s) \quad (5.3-1)$$

This is a non-negative pole; we claim this indicates an unstable system. If we apply a step disturbance to (5.3-1), we obtain upon inversion:

$$y' = K \frac{t}{\tau} \quad (5.3-2)$$

Certainly y' increases without bound, so that it qualifies as unstable. You should try different bounded disturbances in (5.3-1), though, to explore whether this is always the case.

5.4 the integrator and the non-self-regulating process

The system of Section 5.3, far from being an oddball case, is actually quite important. It is known as an integrator. An example is shown in Figure 5.4-1.

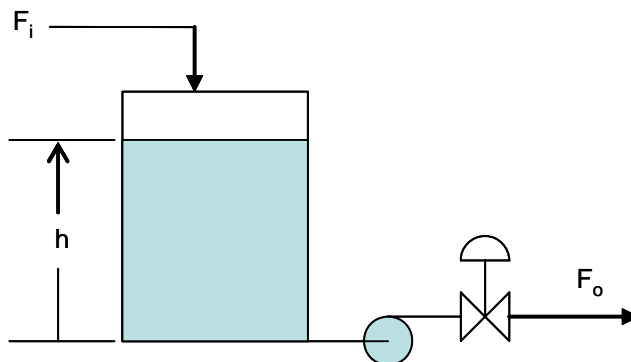


Figure 5.4-1: Tank with independent inlet and outlet flows

The inlet flow is simply given, out of our control, and the outlet is pumped. The material balance gives

$$A \frac{dh}{dt} = F_i - F_o \quad (5.4-1)$$

Both inlet and outlet flow disturbances affect the tank level h . We envision a steady reference condition in which the flows are balanced at F_r and the level is at h_r . Expressing (5.4-1) in deviation variables, we obtain

$$A \frac{dh'}{dt} = F_i' - F_o' \quad (5.4-2)$$

Integrating (5.4-2) from an initial deviation of zero, we obtain

$$h' = \frac{1}{A} \int_0^t (F_i' - F_o') dt \quad (5.4-3)$$

Thus the name integrator: the response variable is simply the integral of whatever inputs are fed to it. This can be a big problem. Suppose that this is your tank. You observe that F_i is running quite steadily, so you adjust F_o to match it and go home for the night. Just after you leave, F_i increases to a new steady value. We derive the results from (5.4-3):

$$h' = \frac{F_i' - F_o'}{A} t \quad (5.4-4)$$

In this simple case, the numerator is a positive constant. You can substitute numbers into (5.4-4) to help you estimate what time of night you will receive a telephone call.

The integrator has a zero pole; if we generalize from (5.4-2) we see a corollary of this property:

$$\frac{dy}{dt} \neq f(y) \quad (5.4-5)$$

Recall that y is a quantity that can accumulate in a system; it could be total mass, amount of a chemical species, or energy, for example. Equation (5.4-5) tells us that the rate of accumulation of y is not affected by the inventory: your tank level may be rising fast, but the flow keeps on coming. An equivalent statement of (5.4-5) is that the net inflow of y across the system boundary does not depend on how much y is within the boundary.

Processes that contain an integrator are known as non-self-regulating. This term comes directly from the property expressed by (5.4-5). By contrast, the first order system in (5.1-2) will come to a new steady value

after a disturbance. It can do this because its rate of accumulation does depend on the inventory y , and we call it self-regulating. Clearly a non-self-regulating process will require frequent attention.

5.5 getting practical - when self-regulation is not

We have identified formal stability with negative poles in the transfer function. Of the non-negative poles, we have given a special name - integrator - to systems with zero poles, and we know to watch out for them. These results of linear stability theory are quite useful. However, we must never forget that our models are only approximations of the real processes.

As an illustration, we cite Shinskey (2002), who asserts that non-self-regulating behavior is a matter of degree, not strictly confined to systems with zero poles. To see this, we rearrange the first-order system of (5.1-2).

$$\frac{\tau}{K} \frac{dy'}{dt} + \frac{1}{K} y' = x' \quad y'(0) = 0 \quad (5.5-1)$$

We suppose that a particular system features a time constant and gain that are both large. Under these circumstances, the term involving the inventory y' might be neglected in comparison to the others, leaving

$$\frac{\tau}{K} \frac{dy'}{dt} = x' \quad y'(0) = 0 \quad (5.5-2)$$

In the limit of (5.5-2), our system is an integrator. The fact that it is not strictly an integrator is irrelevant if its behavior approaches that of (5.5-2): in the matter of midnight phone calls, the process might well be considered non-self-regulating.

Let us illustrate by a tank with gravity-driven outlet flow.

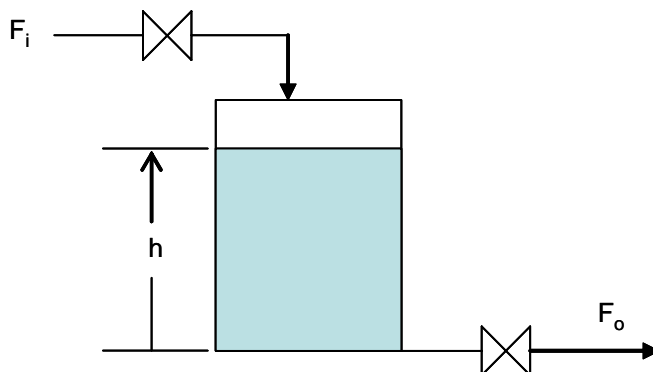


Figure 5.5-1: Tank with gravity-driven outlet flow

We take the simplest course and assume that the outlet flow is directly proportional to the liquid level h . The proportionality constant k is small when the outlet valve is constricted, and increases as the valve is opened. A material balance gives

$$\frac{A}{k} \frac{dh'}{dt} + h' = \frac{1}{k} F_i' \quad (5.5-3)$$

where we base deviation variables on a steady reference condition.

Formally, (5.5-3) represents a self-regulating process. However, consider for a tightly constricted valve, k is small. In system terms, this means that gain $1/k$ and time constant A/k are both large. The large gain means that small disturbances F_i' have a large effect on response h' . The large time constant means that the response will be slow.

In physical terms, the constricted outlet forces the liquid level to rise significantly to respond to relatively small increases in the inlet flow. It will take a while to reach a level sufficient to push the flow out, so that there will have been a lot of accumulation in the tank. If the constriction is sufficiently severe, you will still receive your midnight telephone call, even though your linear system model (5.5-3) has a negative pole. The process is effectively non-self-regulating.

5.6 we want our processes to be operable

When we speak of “instability”, we speak of unbounded change in the response variable, as in (5.1-4), (5.3-2), or (5.4-4). Of course, nothing really became unbounded: the tank level rose only until it spilled over, or drained until it was empty. What really matters to us is process operability. A process that wakes us at midnight is not operable; it will always need attention, or else run into troublesome limits. We prefer that our processes be operable, and process control should contribute to that end; therefore, a non-self-regulating process should be placed under automatic control.

Linear system stability calculations are useful indications of operability; however, Section 5.5 taught us that operability depends on the context of the operation. Thus “operability” is “stability” made practical. A control engineer must ask, “Under what conditions can this process become inoperable?”

- Part of the answer may come from applying stability theory, either through formal calculations, or by knowing what to look for while tuning a controller. Most practical control loops will have a stability limit. Stability calculations in Section 4.21 helped us select a

controller gain for operability. In this way, we seek to determine a degree of control.

- Another part of the answer may come from an intuitive examination of the process, seeking to understand what affects the inventory of each conserved quantity (mass, species, energy). This is especially useful for processes that contain multiple operations. Such an examination can reveal both where control is needed, and where control might cause a problem. Here we are seeking a scheme of control.

Engineering often requires a blend of rigorous analysis and informed intuition. For the remainder of Lesson 5 we will examine several process features that affect operability. Much of this treatment is based on Downs (1992).

5.7 we must control the liquid level integrator

Suppose that we desire to maintain a constant level in the tank of Figure 5.4-1. To make this process self-regulating, we must apply automatic control. We must choose a control scheme; perhaps conditioned by steady-state thinking, we consider managing inventory by managing the inlet and outlet flows: after all, doesn't $IN = OUT$ at steady state? We arrange in Figure 5.7-1 to measure the instantaneous inlet flow and adjust the outlet flow to match.

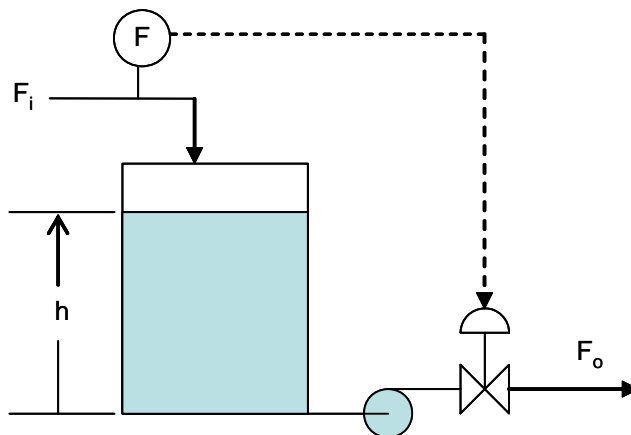


Figure 5.7-1: Feedforward control of inventory by manipulating outlet flow

Such a scheme may allow you to stay at home all night, because it will tend to reduce imbalance between flows. However, any discrepancy between the flow rates - through calibration error, instrument drift, shortcomings of the controller, insufficient adjustment of the pump motor speed - will contribute to accumulation through (5.4-3). If that discrepancy is sufficiently biased, over time the level will creep to an undesired value.

If our objective is to control level, then we should measure the level! The scheme of Figure 5.7-1 did not correct the basic fault of the integrator; in Figure 5.7-2 we apply a feedback loop to allow the inventory to affect the rate of accumulation. Thus the process (under control) becomes self-regulating: a rise in level will trigger an increase in the outlet flow rate, and so forth.

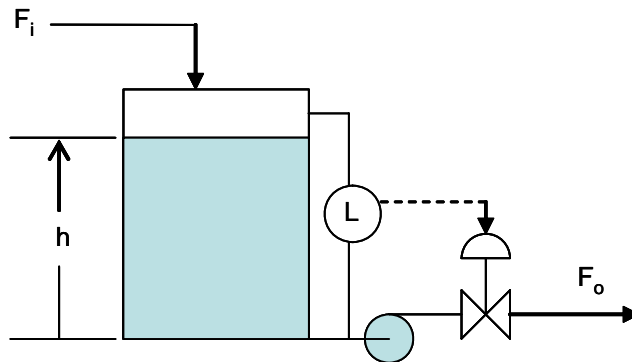


Figure 5.7-2: Feedback control of inventory

(By the way, in a later lesson we will return to feedforward control schemes and find them to be very useful when appropriately applied.)

5.8 inventory of individual chemical components

Downs (1992) recommends that a key to achieving good process operability is ensuring that the inventory of each chemical species is self-regulating. Thus we consider further the tank of Figure 5.7-2, letting the inlet flow comprise chemical species A and B. We write the component material balance for A, assuming that the tank is well-mixed:

$$\frac{d}{dt} VC_{Ao} = F_i C_{Ai} - F_o C_{Ao} \quad (5.8-1)$$

We saw this relationship in Lesson 3. Here, we notice particularly that the inventory of component A (VC_{Ao}) does influence the outlet path of A ($F_o C_{Ao}$) through the composition, so that (5.4-5) does not apply: the system is self-regulating. This is the behavior we recall from the disturbance responses we computed in Lesson 3.

When we explore more extensive processes, we may not be able to write the component balance in such concise form as (5.8-1). However, we can still apply the principle of (5.4-5) and examine qualitatively whether the inlet and outlet paths for a component depend on its inventory.

5.9 example of a solvent recovery process

Figure 5.9-1 shows a scrubber and distillation column used to recover volatile solvent A from an inert gas stream. We have already installed

plausible level control schemes (indicated by feedback loops with the control valves omitted) to ensure that individual vessels are self-regulating with respect to mass.

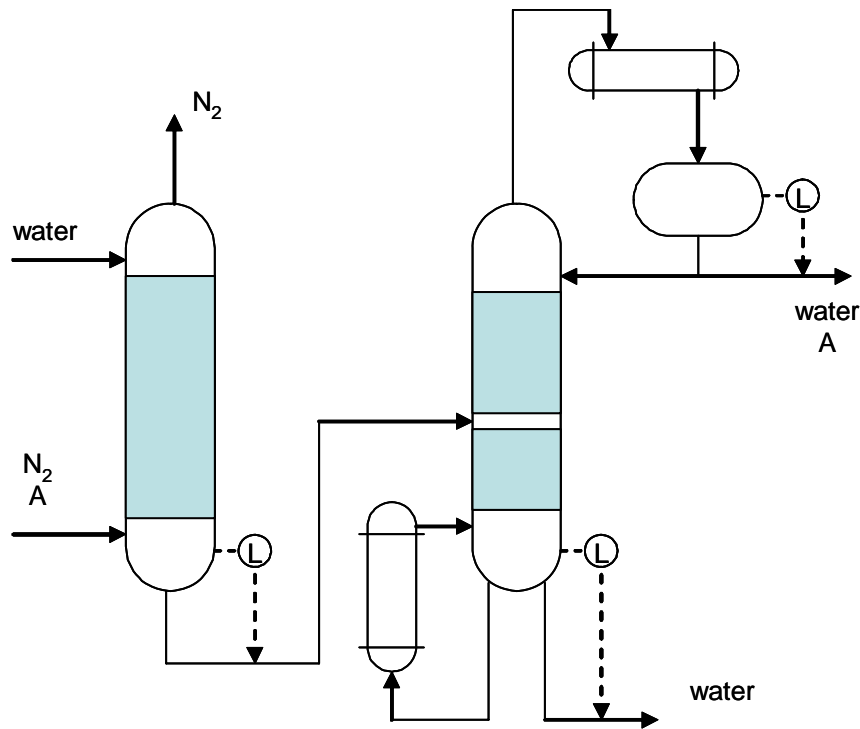


Figure 5.9-1 Basic process scheme for recovery of solvent A

We proceed methodically:

examine scrubber for component A

- inlet streams are independent of the inventory of A
- overhead gas stream is independent of A
- flow of A in the bottoms stream will increase as the bottoms concentration of A rises, as in Section 5.8

By virtue of the bottoms stream, the scrubber is self-regulating for component A. As a practical illustration, a rise in the inlet composition of A will, over time, result in a higher flow of A from the bottom of the scrubber. (We remark here that this is a *qualitative* assessment that presumes good equipment operation, such as sufficient scrubber performance and a well-tuned level-control loop. Our purpose here is not to perform a detailed design or simulation; rather, it is to determine whether this process can, in principle, be self-regulating.)

examine scrubber for water

- the conclusion for water is identical to that for A

examine scrubber for N₂

- the inventory of N_2 is indicated by the pressure in the vessel. Both inlet and outlet flows of gas will be affected by the vessel pressure. Hence, the scrubber is self-regulating for N_2 .

examine the distillation unit (including accumulator)

- the flow of components in the bottoms and distillate streams depend on the composition of the column sump and accumulator inventories

As with the tank of Section 5.8, the level-control loops make the distillation unit self-regulating for both components.

examine the entire process

- the two inlet streams are independent of the process
- the scrubber overhead gas stream regulates N_2
- the distillate and bottoms streams regulate water and component A

The two unit operations are simply arranged in series, so that nothing about their combination affects the conclusions we had drawn from the individual units.

5.10 the process becomes non-self-regulating

A reasonable process objective would be to recover component A in a desired, not arbitrary, concentration. Hence, we install a composition controller on the distillate stream. In addition, we can reduce utility costs by recycling the bottoms stream to the scrubber. The results of these changes are shown in Figure 5.10-1.

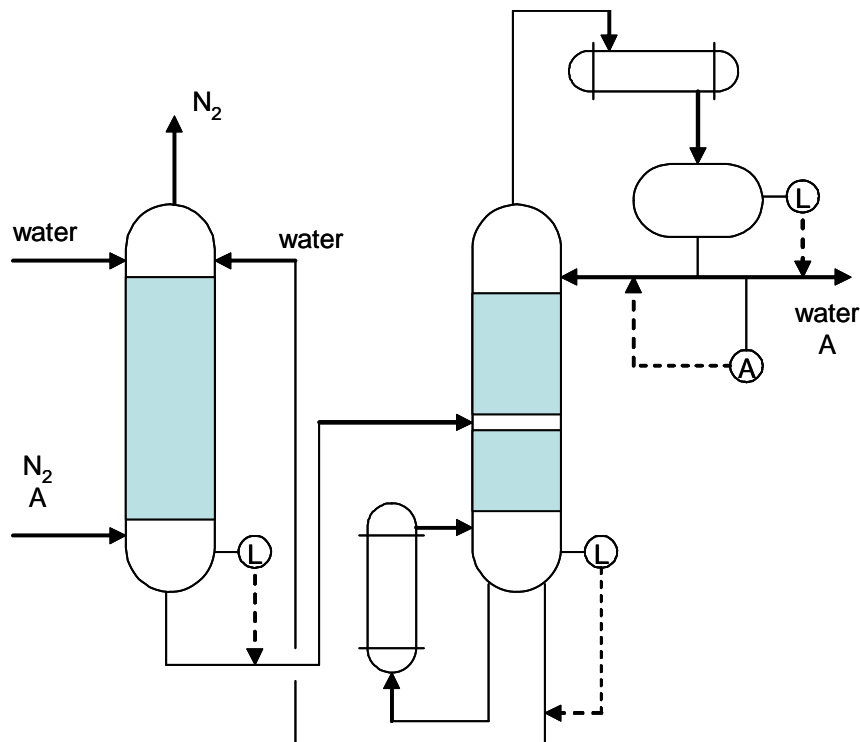


Figure 5.10-1 Recovery process with composition control and recycle

We repeat the examination of Section 5.9. We see quickly that the scrubber is unaffected: addition of a new independent inflow stream does not change the ability of the level-control loop to regulate the inventories of A and water.

examine the distillation unit (including accumulator) for A

- the inlet stream is independent of conditions in the unit
- the distillate composition is now fixed, but the removal of A can still vary in response to conditions because the distillate flow can vary.

Hence distillation is self-regulating for component A. For example, should the inflow of A increase to the column, the distillate flow can increase to remove it.

examine the distillation unit (including accumulator) for water

- the inlet stream is independent of conditions in the unit
- fixing the distillate composition means that water can be removed overhead only at a rate in proportion to the removal of A. Hence, this stream no longer depends on the inventory of water.
- water may still be removed in the bottoms, as required.

By virtue of the level-control loop on the bottoms, distillation is self-regulating for water.

examine the entire process

- the two inlet streams are independent of the process
- the scrubber overhead gas stream regulates N₂
- the distillate stream regulates component A
- the distillate stream does not regulate water.

Although each individual unit operation is satisfactory, the combination can no longer regulate the inventory of water. Water enters independently; water leaves only in proportion to the flow of A. To illustrate, suppose there is a step increase in water feed. The scrubber level controller responds by increasing the feed to the distillation column. The column responds by increasing the bottoms flow, and thus the recycle to the scrubber. The two sump controllers engage in a battle that one must lose, as the water inventory continues to rise.

To correct this problem, we must make at least one of these streams respond to the amount of water in the process. Downs (1992) gives the solution: as shown in Figure 5.10-2, control the scrubber sump with the water make-up flow. If we are designers, we still have further decisions to make about equipment sizing and control structure, but at this point we

have avoided one problem, at least, that no amount of post-design controller tuning could have solved.

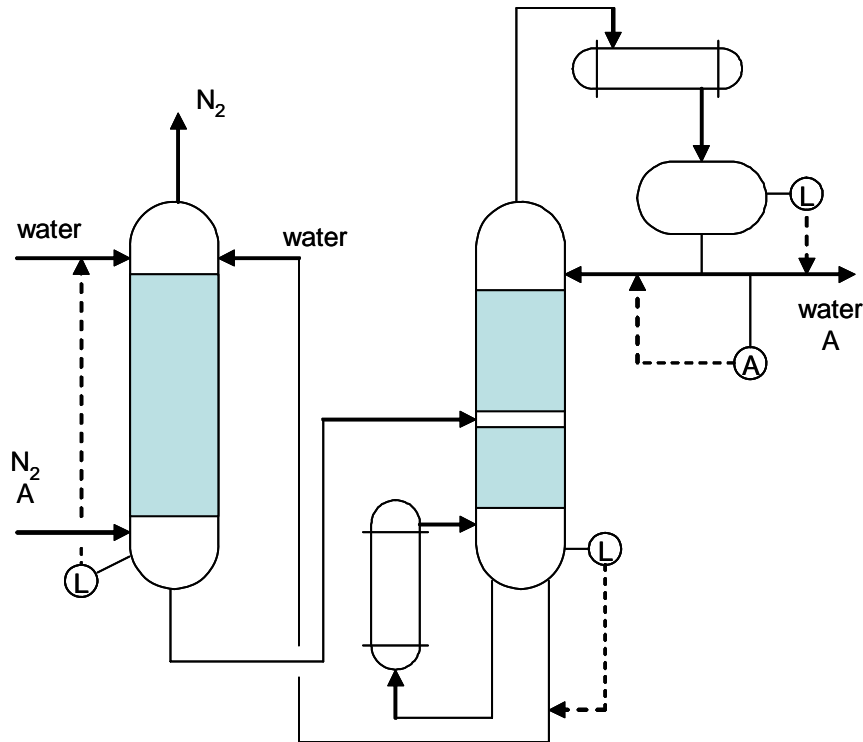


Figure 5.10-2 Control scrubber inventory with make-up water stream

Sections 5.9 and 5.10 have presented a lengthy example. Let us summarize the approach that was illustrated:

- ensure that overall mass inventory is regulated
- methodically examine each inlet and outlet stream for each process unit, as well as the whole process, with respect to each chemical species. Remember that good individual units can work poorly in combination.
- determine the qualitative relationship between the flow of each component and the inventory of that component. Certainly, there may be no relationship.
- we call the relationship *qualitative* because we are not considering the actual timing or magnitude of response to disturbances, nor details of equipment performance (e.g., whether the trays might flood, or if there is enough packing to remove all the solvent from the gas). These are matters of detailed design, to be addressed after we have established our basic process structure. Therefore, presume that all equipment works well, including any control loops that have already been specified.
- look out for composition controllers, because these restrict the flow of the impurity components. Look out for recycle streams, because

these eliminate an outlet path. These are not bad process features, but the control scheme must accommodate them.

- if any component has no stream that responds to its inventory, the process must be altered to supply one. Often this is accomplished by a change in control scheme.

5.11 chemical reactions provide paths in and out of a process

A reactant may not leave a process by any outlet flow path at all, but rather be entirely consumed within it. Therefore, in applying the method of Section 5.10 to a process with chemical reactors, we must extend the terms 'stream' and 'flow' to include consumption and generation. The general principle of (5.4-5) still applies: if the rate of reaction of a species does not depend on the inventory of that species, then the process may be non-self-regulating.

Kinetic expressions most often do depend on the concentration, so that problem may actually arise outside the reactor, and because of the control scheme. Downs (1992) gives an example of a reactor whose level, temperature, and composition are all controlled. The level is controlled by manipulating the outflow stream. The temperature is controlled by manipulating flow of a service fluid. The composition is controlled by manipulating the flow of a reactant recycle stream.

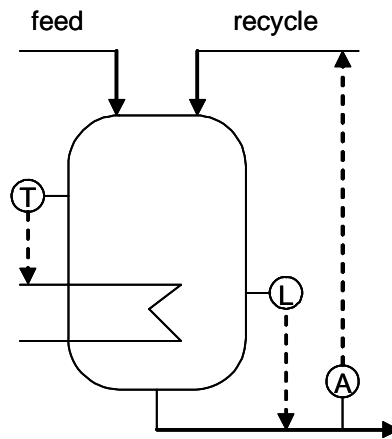


Figure 5.11-1 Reactor with three control loops

Let us examine each in/out path for a reactant A:

- the inflow of A in the feed is independent
- the inflow of A in the recycle stream does depend on the inventory of A, because it is manipulated to keep the concentration at set point
- the outflow of A varies to keep the inventory constant, because the level controller manipulates the flow rate

- the consumption of A depends on the volume of the reactor (set by the level controller), the temperature (set by the temperature controller), and the concentration of A (set by the composition controller). For well-behaved control loops, it is constant.

Everything seems fine. In fact, because of the level and composition controllers, we are actively working to keep the inventory of A constant.

The problem does not become apparent until we consider the reactor in the context of other process units. Suppose the concentration of A in the feed stream undergoes a step reduction. The level and composition controllers respond: the composition controller increases the supply of recycle to return the total inflow of A to normal, and the level controller adjusts the outlet flow as needed to keep the volume the same. Thus A reacts at the normal rate, and *therefore* excess A departs the reactor at the normal rate.

Somewhere downstream, a separation unit is being fed A at the normal rate, but being asked to return A at a higher than normal rate. It can do this only while its inventory lasts. The reactor has put the problem off onto other units, but there remains a problem nonetheless. We discover it by examining the component inventories, but doing so in the context of the larger process.

How would you solve this problem?

5.12 conclusion

We enlarged our notions of stability to develop a concept of what makes a process operable. In doing this, we have ranged qualitatively over a variety of chemical processes. In some cases these have included multiple operations and multiple control loops. In the next lessons, we will return to analysis of a single operation; as we deepen our understanding, try not to forget the broad perspective we have attempted here.

5.13 references

Downs, J. J. "Distillation Control in a Plant-wide Control Environment." In *Practical Distillation Control*. Edited by W. L. Luyben. New York, NY: Van Nostrand Reinhold, 1992. ISBN: 0442006012.

Shinskey, Greg F. *Process Control Systems: Application, Design, and Tuning*. 4th ed. New York, NY: McGraw-Hill, 1996. ISBN: 0070571015.