

9.65 May 5, 2004 Solving Problems Handout

Outline:

Approaches to understanding problem solving:

1. Different styles of problem-solving
2. How and why people get stuck
3. Prescriptive approaches
4. Simulations of problem-solving behavior

Introduction: Try this problem (from W. Kohler). [See overhead.]

A circle is divided into 4 sections. One radius is labelled "y". A rectangle is drawn in another section, in which a right triangle is embedded: d is the length of one side of the right angle, x the length of the other side of the right angle, and L is the length of the third side. Given d , x , and y , how long is L ?

Scientists and mathematicians have sometimes classified themselves as symbol-pushers versus visualizers. Which are you?

Now, classify yourself into one of 4 categories:

Visualizer Symbol-pusher

Insight solution to
circle problem

Formula (or no) solu-
tion to the circle..

1. Different styles of problem-solving:

The idea of classifying people as visualizers versus symbol-pushers is an example of this approach.

Wertheimer, **Productive thinking**, 1923:

The Altar window problem (lab)

Wertheimer proposed that there are 3 styles of problem-solving:

- a) Use deductive logic: standard algorithms; math; logical syllogisms.

- b) Retrieve the answer from memory: learn the steps in a proof. Improve performance by repetition, habit, frequency, recency, trial and error.

- c) Use productive thinking: analogies, application of knowledge from another field, and especially "structural analysis."

Another example that distinguishes among these styles or approaches:

Area of a parallelogram:

These are examples in which a visual) representation may help one to pick out the appropriate transformations to make the problem into a more familiar one: "insight" (or, in Reisberg, "illumination").

However, visualization can sometimes be misleading: as in folding a sheet of paper $1/100$ inches thick over, repeatedly: how thick will it be after 50 folds?

What is "insight"? A one-step recognition that a new problem is equivalent to a familiar problem? If so, insight will depend on the knowledge you already have.

The 13 problem of Duncker: Why are all 6-place numbers of the form 267,267 or 591,591, etc., all divisible by 13? (8% of subjects in Duncker's experiment solved with no further information; hints led to a higher percentage of solutions)

Can't have a feeling of "insight" unless the underlying principle is fully familiar and intuitively correct, once it comes to mind.

2. How people get stuck.

(a) Set or Einstellung: Luchin's water jars. [Lab 3, and Reisberg p.471-473]

(b) Duncker: "Functional fixedness." (See Reisberg. 469-471.) Problems with real objects; tacks in a box made it less likely the subject would think of using the box as part of the solution.

Other problems in which an object with a standard use is not seen as useable in another role (like pliers as a weight for a pendulum).

More abstract problems can be regarded as cases of "functional fixedness":

"A surgeon comes across an automobile accident, takes a look at the victim lying in the road, and exclaims..."

Related issue: transfer of training: recognition that another problem is analogous to one you already know the answer to.

It is often difficult to realize that two problems have the same underlying structure, so that if you've solved one, you can easily solve the other. See the discussion in Reisberg, 459-463.

3. Prescriptive approaches

-make inferences & associations

-look for analogies

-figure out subgoals

-hill-climb

-if stuck, look for new routes

-if necessary, leave the problem and return later

Question of incubation: Does your brain continue to work on the problem while you're not consciously thinking about it? See Reisberg, pp. 480-83 for a possible answer.

4. Simulations of problem-solving:

Newell & Simon (1972): The General Problem Solver (GPS) [Reisberg, pp. 448-453] They proposed the following.

A. There are just a few general characteristics of human problem solving:

-serial processing

-certain elementary processes: information transfer, elementary operations: they take of the order of 50-300 ms each

-Inputs and outputs are held in a small-capacity STM, measured in chunks that depend on experience

-storing in LTM takes about 5 s per chunk

--retrieval from LTM is fairly fast: e.g., 200 ms.

B. A task environment is represented as a problem space in which problem solving take place: a space of possible situations to be searched.

How do you set up the problem space? Task instructions and display; memory for similar or analogous tasks; metaprograms for setting up space-general task programs.

C. The task structures the space.

D. The structure determines the possible problem-solving program.

You start with an initial state, and you work toward a goal state, using operators. In setting up the problem space, you may have some path constraints. (R., p. 448.)

At any one time you are at a given node or knowledge state in the problem space; the knowledge state is what you have in STM. You can either choose an operator to apply to take you to a new node-- or can retreat to a different node (recover a different set of STM info--or look at one's external memory).

Heuristics include: hill-climbing (which doesn't always work, as you may have to go away from the goal at some point in the solution), means-end analysis, and working backward from the goal [R., pp. 450-453].

Newell & Simon's computer simulation is consistent with protocol analyses of subjects solving problems like the DONALD + GERALD = ROBERT problem: Each letter is a different digit: e.g., all D's are one digit: in this case, D = 5. The simulation predicts (approximately) the time for each step in solution, and the order of steps, the harder places, etc.

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      D O N A L D
      + G E R A L D
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      R O B E R T
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