Addendum to Lecture 7, 9.07

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1 Confidence Interval

A range of value where a parameter is likely to lie with probability $1 - \alpha$ for $dt(0,1)$ usually $0 < \alpha << 1$.

$Z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{n^{\frac{1}{2}}(\bar{x}-\mu)}{\sigma}$ Pick ϵ α ϵ $(0,1)$

For example $\alpha = 0.05$, $\alpha = 0.01$

$1 - \alpha = Pr(Z_{\frac{\alpha}{2}} \leq Z \leq Z_{1-\alpha})$

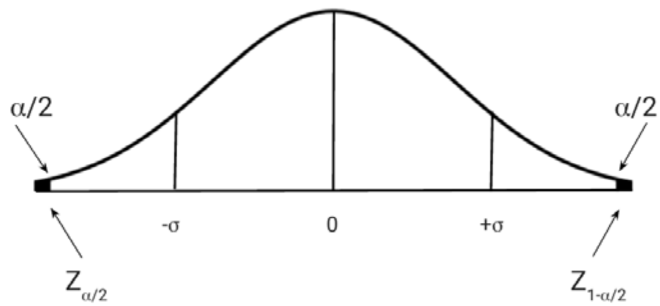
$= \text{Pr }(Z_{\alpha} \leq \frac{n^{\frac{1}{2}}(\bar{x}-\mu)}{\sigma} \leq Z_{1-\frac{\alpha}{2}})$

$= \text{Pr }(\frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \bar{x} - \mu \leq \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}})$

$= \text{Pr }(-\bar{x} + \frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq -\mu \leq -\bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}})$

$= \text{Pr }(\bar{x} - \frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \mu \leq \bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}})$

$= \text{Pr }(\bar{x} - \frac{z_{\frac{1-\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \mu \leq \bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}})$



N. B. By symmetry $Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}}$

or

$Z_{1-\frac{\alpha}{2}} = -Z_{\frac{\alpha}{2}}$.

For example if $x = 0.05$

$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$Z_{\frac{\alpha}{2}} = Z_{0.025} = -1.96$

The probability $1 - \alpha$ is interpreted in the long-run frequency sense. We will explain this in Lecture 8.

σ^2 is not known.