

Addendum to Lecture 7, 9.07

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10.31.16

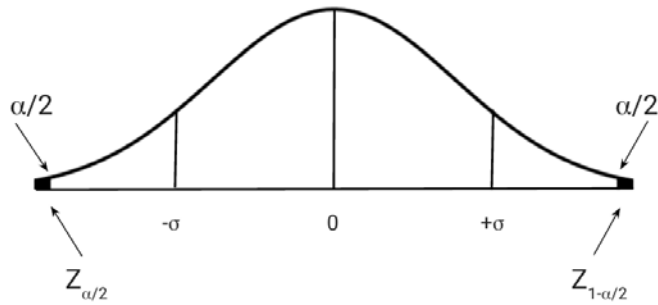
1 Confidence Interval

A range of value where a parameter is likely to lie with probability $1 - \alpha$ for $dt(0, 1)$ usually $0 < \alpha \ll 1$.

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{n^{\frac{1}{2}}(\bar{x} - \mu)}{\sigma} \text{ Pick } \epsilon \alpha \in (0, 1)$$

For example $\alpha = 0.05$, $\alpha = 0.01$

$$\begin{aligned} 1 - \alpha &= Pr(Z_{\frac{\alpha}{2}} \leq Z \leq Z_{1-\alpha}) \\ &= Pr\left(Z_{\alpha} \leq \frac{n^{\frac{1}{2}}(\bar{x} - \mu)}{\sigma} \leq Z_{1-\frac{\alpha}{2}}\right) \\ &= Pr\left(\frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \bar{x} - \mu \leq \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}}\right) \\ &= Pr\left(-\bar{x} + \frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq -\mu \leq -\bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}}\right) \\ &= Pr\left(\bar{x} - \frac{Z_{\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \mu \leq \bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}}\right) \\ &= Pr\left(\bar{x} - \frac{z_{\frac{1-\alpha}{2}}\sigma}{n^{\frac{1}{2}}} \leq \mu \leq \bar{x} + \frac{Z_{1-\frac{\alpha}{2}}\sigma}{n^{\frac{1}{2}}}\right) \end{aligned}$$



N. B. By symmetry $Z_{\frac{\alpha}{2}} = -Z_{1-\frac{\alpha}{2}}$

or

$$Z_{1-\frac{\alpha}{2}} = -Z_{\frac{\alpha}{2}}.$$

For example if $x = 0.05$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$Z_{\frac{\alpha}{2}} = Z_{0.025} = -1.96$$

The probability $1 - \alpha$ is interpreted in the long-run frequency sense. We will explain this in Lecture 8.

σ^2 is not known.

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