Wormlike Chains

- Thin Elastic Filaments

$$\frac{\text{Bending Energy}}{\text{Length}} = \frac{1}{2} E I \frac{1}{R^2}$$

E ← Young's modulus

I ← 2nd moment of inertia

R ← radius of curvature

- Persistence Length

$$\langle \underline{t}(s) \cdot \underline{t}(s+\Delta s) \rangle = \exp\left[-\frac{k T}{E I} |\Delta s| \right]$$

$$\frac{EI}{kT} = l_p \qquad \text{persistence length}$$

- Bending Energy

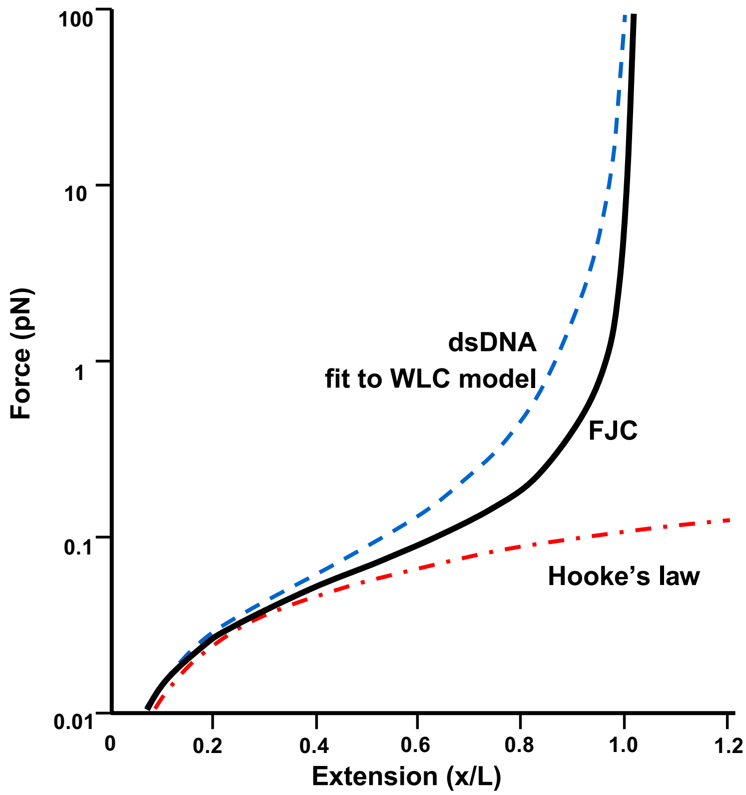
$$\frac{\text{Energy}}{\text{Length}} = \underbrace{\frac{k T l_p}{2}}_{k_F/2} \left(\frac{\partial \underline{t}}{\partial s} \right)^2$$

- Relation of WLC & FJC

$$\langle \underline{R} \cdot \underline{R} \rangle_{WLC} = 2 l_p^2 \left[\frac{L}{l_p} + \exp\left(\frac{-L}{l_p} \right) - 1 \right]$$

rigid $\quad l_p/L \gg 1 \qquad \langle R^2 \rangle \to L^2$

flexible $\quad l_p/L \ll 1 \qquad \langle R^2 \rangle \to 2 l_p L \quad \Rightarrow l_p = \frac{b}{2}$ (b ← Kuhn step)



After Bustamante et al., *Current Opinion in Structural Biology*, 2001

 (Macromolecules 1995)

Graph (Figure 2.) removed due to copyright considerations

WLC $$U = \int_0^L \frac{kT\, l_p}{2} \left(\frac{\partial \underline{t}(s)}{\partial s} \right)^2 ds$$

Difficult in general to solve ...

Marko & Siggia (1995) : limits for $l_p/L << 1$

trick: equipartition of energy

small force $$f_z \approx \frac{3}{2} \frac{kT}{l_p} \left(\frac{\langle z \rangle}{L} \right)$$

large force $$f_z \approx \frac{1}{4} \frac{kT}{l_p} \left(1 - \frac{\langle z \rangle}{L} \right)^{-2}$$

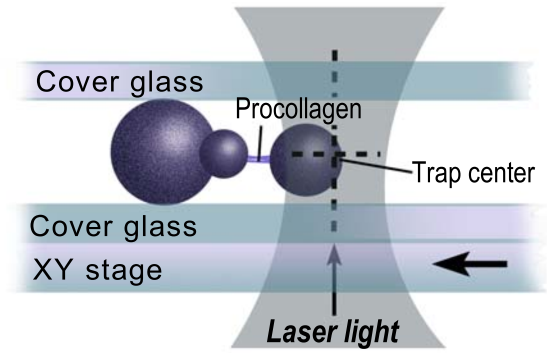
interpolation $$f_z = \frac{kT}{l_p} \left[\frac{1}{4\left(1 - \frac{\langle z \rangle}{L}\right)^2} + \frac{\langle z \rangle}{L} - \frac{1}{4} \right]$$

Images removed due to copyright considerations.

See Figures 1, 2, and 3 and Table 1 in Isambert, Herve, et al. "Flexibility of Actin Filaments Derived from Thermal Fluctuations." *Journal of Biological Chemistry* 270 (19) 12 May 1995, 11437-11444.

After Fig. 1 and 2 in Sun, Y. et al. "Mechanical Properties of Single Type II Collagen Molecule." Paper No: 0082, 48th Annual Meeting of the Orthopaoedic Research Society.

Stretching a procollagen II molecule with an optical tweezers



The force-extension curve of a single collagen II

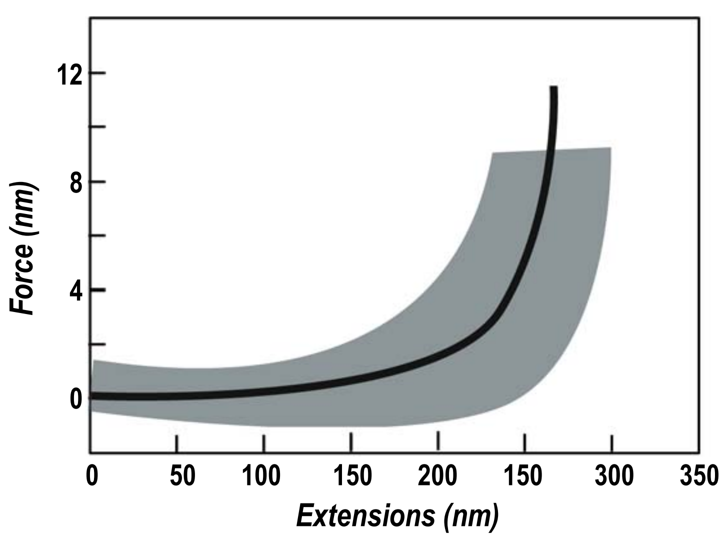


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Figure 1 in Fujii, T. et al. "Mechanical Properties of Single Hyaluronan Molecules." Paper No: 0390, 48th Annual Meeting of the Orthopaoedic Research Society.

Modeling Extreme Extension: Ising Model

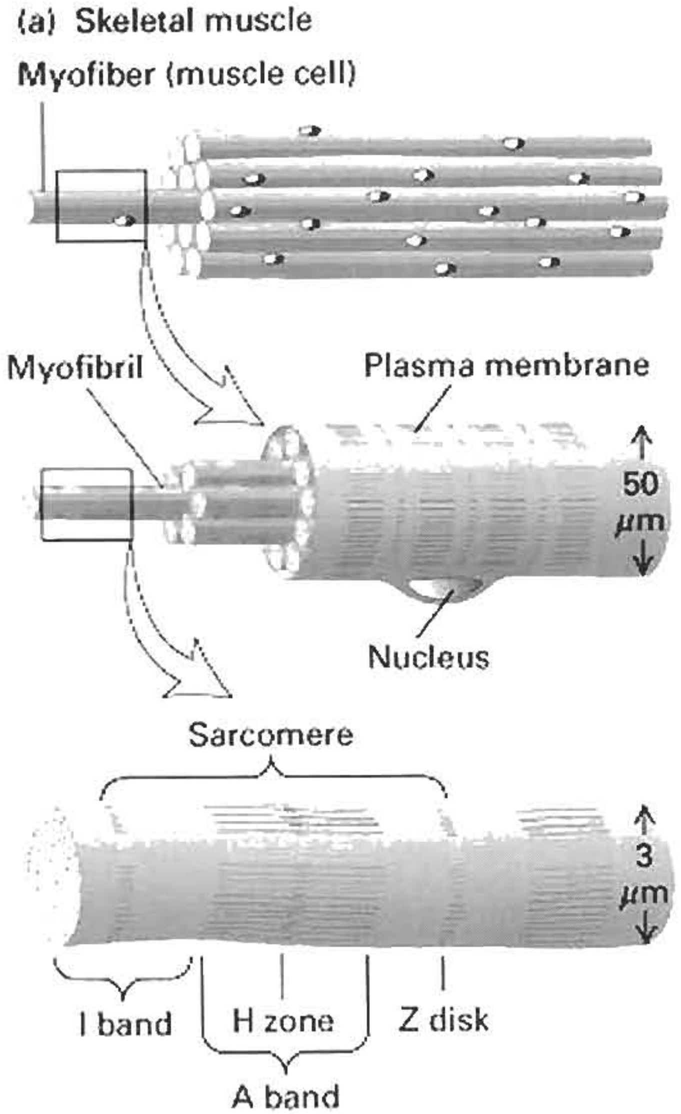
Cizeau & Viovy 1997

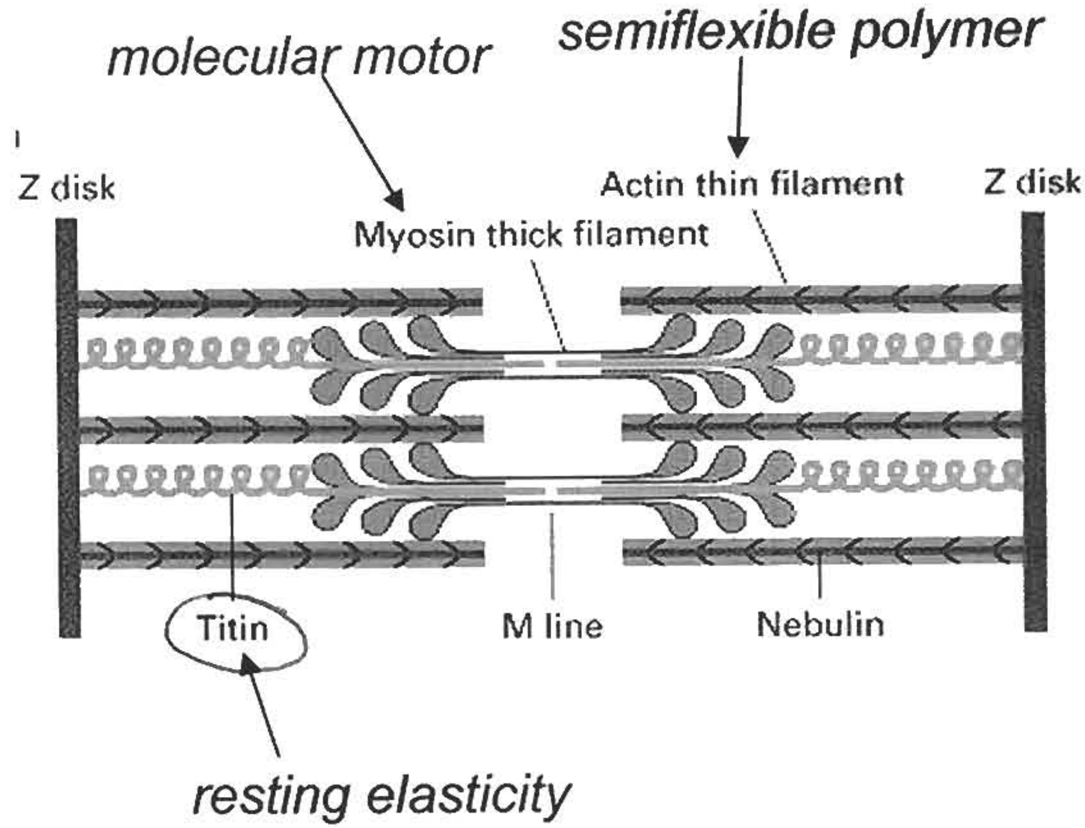
Images removed due to copyright considerations.

Image removed due to copyright considerations.
Figure 1 in Leger, J. F. et al. "RecA binding to a single double-stranded DNA molecule: A possible role of DNA conformational fluctuations."
Proc. Natl. Acad. Sci. USA - Biophysics **95** (Oct 1998) pp. 12295-12299.

Images removed due to copyright considerations.
Bennink, Martin L. et al. "Unfolding individual nucleosomes by stretching single chromatin fibers with optical tweezers." *Nature Structural Biology* **8** (7) July 2001.

Muscles


(a) Skeletal muscle
Myofiber (muscle cell)
Myofibril
Plasma membrane
50 μm
Nucleus
Sarcomere
3 μm
I band
H zone
Z disk
A band


molecular motor
semiflexible polymer
Z disk
Actin thin filament
Z disk
Myosin thick filament
Titin
M line
Nebulin
resting elasticity

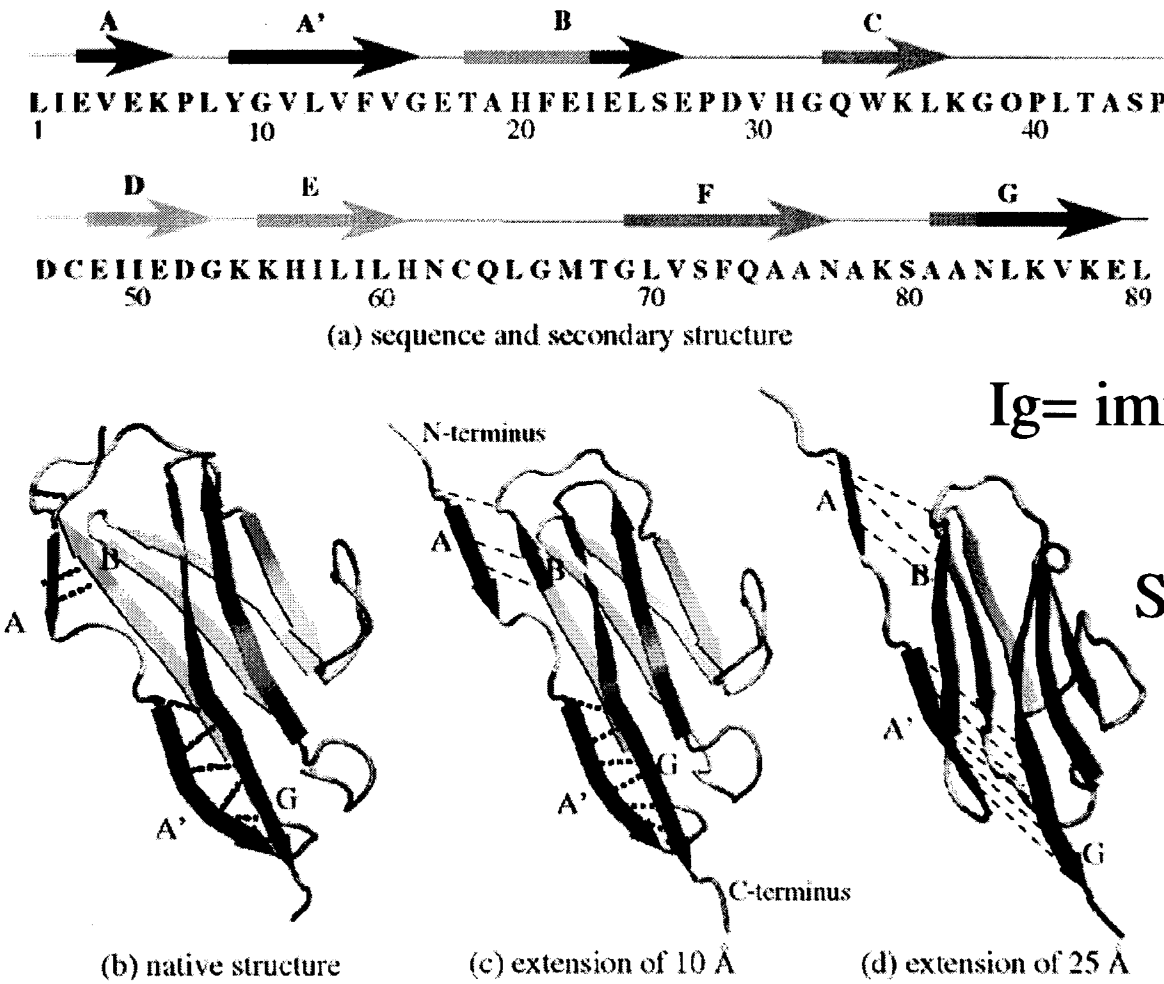
Pulling on Titin

"Sawtooth profile"

Image removed due to copyright considerations.
Figure 1 in Marszalek, Piotr E., et al. "Mechanical unfolding intermediates in titin modules." Nature 402, 100-103 (04 Nov 1999).

Importance of Ig DomainsTitin

http://www.ks.uiuc.edu/Research/titinIg/



(a) sequence and secondary structure

(b) native structure (c) extension of 10 Å (d) extension of 25 Å

Ig= immunoglobulin domains

Several hydrogen bonds

Image courtesy of the University of Illinois Theoretical and Computational Biophysics Group.
(NIH Resource for Macromolecular Modeling and Bioinformatics)

Images removed due to copyright considerations.
Figures 2 and 3 in Oberhauser, Andres F., et al. "The molecular elasticity of the extracellular matrix protein tenascin." *Nature* 393 (1998 May 14) 181-5.

Assembly of actin at the leading edge of migrating cells