MIT 16.90 Spring 2014: Problem Set 9

Qiqi Wang, Karen Willcox

Due: Monday May 5, in class

Problem 9. 1 *Quadrature*

Consider the function

$$g(\xi) = 2 + 2\xi - 3\xi^2 + \xi^3 + \beta \cos \frac{\pi \xi}{2},$$

where β is a constant.

We are interested in evaluating the following integral:

$$I = \int_{-1}^{1} g(\xi) d\xi.$$

1. Evaluate the integral analytically.
2. Compute the approximation to the integral I using a one-point Gaussian quadrature rule.
3. Compute the approximation to the integral I using a two-point Gaussian quadrature rule.
4. For what value of β is your two-point Gaussian quadrature approximation to the integral most accurate? Explain your result.
5. Explain (3–4 sentences) why quadrature is important and useful in the implementation of finite element methods. Give an example of how it might be used.

Problem 9. 2 *Monte Carlo Estimators*

We wish to estimate $\mathbb{P}\{A\}$, the probability of an event A occurring. A Monte Carlo simulation is performed, and the estimate of $\mathbb{P}\{A\}$ is computed as the fraction of times the event A occurs out of the total number of trials,

$$\hat{p}(A) = \frac{N_A}{N},$$

where $\hat{p}(A)$ is the estimate of $\mathbb{P}\{A\}$, and N_A is the number of times A occurred in the Monte Carlo simulation of sample size N.

1. Through the central limit theorem, we know that for large N, $\hat{p}(A)$ is normally distributed. Prove that this distribution has mean

$$E[\hat{p}(A)] = \mathbb{P}\{A\},$$

and standard error

$$\sigma_{\hat{p}} = \sqrt{\frac{\mathbb{P}\{A\}(1 - \mathbb{P}\{A\})}{N}}.$$

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