Homework #2
Quality Loss Functions

<u>Due Date:</u> Thursday, June 11, 1:05PM

<u>Objectives:</u>

- Understand the mathematical properties of quality loss functions
- Explore the various formulations of the quality loss function
- Apply quality loss functions to engineering systems

<u>Note:</u> In this course, if I ask for a *proof*, I'm looking for a derivation of the requested result from given formulae. If I ask for a *demonstration*, you should provide an instance of the phenomenon or property described using any appropriate means (e.g., a numerical example or a simulation).

<u>Assignment</u>

1. (20%) Prove that the expected value of the nominal-the-best loss function is proportional to the second moment of the quality characteristic about the target value.

2. (60%) A plant manufactures 100 Ohm resistors. The acceptable tolerances are $\pm 5\%$ of the nominal value. The cost to scrap the product (A_o) is \$0.10.

   Explore the effect of bias (mean shift) and variance on quality loss. You may do this with closed form mathematics or via Monte Carlo simulation (you may wish to use the MathCad sheet under Assignment #2 on the 16.881 web-site HW2_quality_loss.mcd). Address the following questions as a minimum:

   a) Assume a normally distributed variation in the resistors and nominal-the-best quality loss. If the manufacturing process puts the mean on target, and the process capability index (C_p) is one (3σ equals Δ_o), what is the average quality loss?

   b) Motorola's and GE's "Six Sigma" Programs have a goal of reducing variance to $1/6$ of Δ_o. What reduction in quality loss will be achieved as compared to the case in (a) if this target is met? Comment on the size of the change.

   c) It is often said that bias (mean shift from the target value) is typically 1.5σ. Which process is more robust to this mean shift, the 3σ process in (a) or the 6σ process in (b)?

   d) Estimate the quality loss if the process were so poor that the mean was off by Δ_o and the process capability index (C_p) dropped to $2/3$ (2σ equals Δ_o), How do you interpret the physical or economic meaning of this result? Would you modify the quadratic loss function in light of this and, if so, how?

e) What effect will a change in the distribution shape from normal to uniform have on the result in (a) if the variance is the same? Demonstrate your conclusion. Comment on the relationship to the result of problem 1. Bonus geek points to anyone who can derive the width of the uniform distribution from first principles.

f) If the quality loss is asymmetric, with losses twice as great for resistance above the target than for resistance below the target, what is the lowest quality loss achievable with a process capability index of one? Extra credit for the most precise answer.

3. (20%) Provide an engineering example of a system which approximates
   a) A smaller-the-better loss function
   b) A larger-the-better loss function
   c) An asymmetric loss function
   d) A discontinuous loss function such as the "fraction defective" loss function

Explain each example with a paragraph or so and perhaps a few hand sketches.